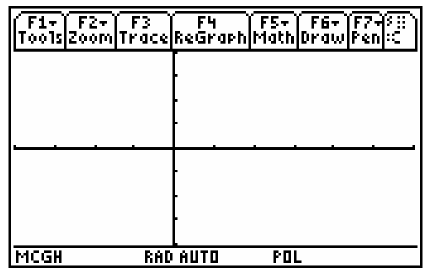
Assignment 23: Polar Cöordinates (9.4-7) **Name**______________________
Please provide a handwritten response.

1. Graph the limaçon $r = f(\theta) = 4\cos\theta - 2$ and the ellipse $r = g(\theta) = \dfrac{4}{2+\cos\theta}$ by setting the calculator **MODE** to **POLAR** and entering $r(\theta)$ from ♦**Y=.** **(**On the TI-89 θ is found from ♦**^.** On all three calculators θ can be found from the **CHAR** Greek menu.) Sketch the results below. How many points of intersection do these two curves have?



$-4 \le x \le 6, -4 \le y \le 4$

2a. You can use the **Solve** command on your calculator to find the points of intersection between these two curves. Record the values returned by the **Solve** command below. Note that the calculator has returned more values than you have points of intersection showing. Why? Remember, you have restricted your values to $0 \le \theta \le 2\pi$ and that $2\pi \approx 6.28318530718$. Now, find the points of intersection as ordered pairs (r,θ) corresponding to these values (remember you can evaluate $r_1(\theta)$ to find a corresponding r). List these points below.

2b. To be absolutely certain you have all points of intersection you will want to solve both $r_1(\theta) = r_2(\theta)$ and $r_1(\theta) = -r_2(\theta+\pi)$. Solve the second of these equations and record the results below. Also list all points of intersection (from **2a** and **2b**) on the graph above.

3a. Write down a sum of definite integrals that gives the area of the region lying both inside the ellipse and outside the limaçon. Also shade this area on the graph above.

3b. Evaluate these integrals and record the total area below. Based on the graph, is your answer plausible?

3c. Repeat parts **a** and **b** for the region lying inside the large loop of the limaçon (ignore the small loop) but outside the ellipse. Be sure to keep track of what parts of each curve correspond to which values of θ.

4a. The slope of a line tangent to a polar curve at $\theta = a$ is given by the formula: $\left.\frac{dy}{dx}\right|_{\theta=a} = \frac{f'(a)\sin a + f(a)\cos a}{f'(a)\cos a - f(a)\sin a}$. Use this formula with **Solve** to find all values at which the tangent line to the limaçon is horizontal. Record your results below and mark these points on the graph.

4b. The graph suggests that the two curves might be orthogonal at two of their intersection points. You may use the formula above to decide whether this is the case or you may also use your calculator to find the required slopes from the graph. From the **Graph** screen press **F5 (Math)** and choose **option 6 (Derivatives), option 1** $\left(\frac{dy}{dx}\right)$. (Recall that perpendicular lines have negative reciprocal slopes unless they are horizontal and vertical.)
At which points of intersection (if any) are the two curves orthogonal? Record your answer below.