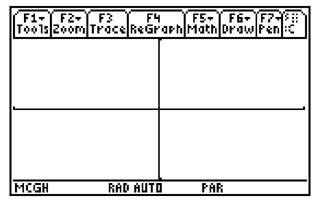
Assignment 25: Vector-Valued Functions, Part 1 (11.1-3)   Name_________________
Please provide a handwritten response.

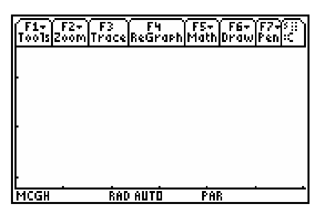
1a. The vector function $\vec{r}(t)=\langle cos(3t), sin(2t)\rangle,\quad 0\le t\le 2\pi$ can be defined by entering the command **Define** $\vec{r}(t)=[xt1(t), yt1(t)]$ in the entry line of the home screen. You can graph this function by setting the **MODE** to **PARAMETRIC** and letting $\begin{cases} xt1 = cos(3t) \\ yt1 = sin(2t) \end{cases}$. Sketch the resulting curve, known as a "Lissajous curve" on the axes provided below.



$-1\le x\le 1, -1\le y\le 1$

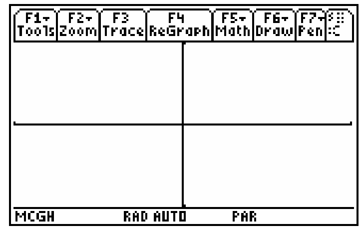
1b. You can easily evaluate the points $\vec{r}(0), \vec{r}\left(\frac{\pi}{4}\right),..., \vec{r}\left(\frac{n\pi}{4}\right)\quad 0\le n\le 8$ from the graphing screen on your calculator by pressing **F5 Math, 1 Value** and entering the θ value. Mark these coordinates on the graph above and draw arrows to show the orientation of the curve.

1c. The velocity vector $\vec{v}(t)=\vec{r}\,'(t)$ can be defined as **Define** $\vec{v}(t)=d(\vec{r}(t),t)$. It can then be evaluated by entering $\vec{v}(t)$. Speed is given by $\|\vec{v}(t)\|=\sqrt{\vec{v}(t)\cdot\vec{v}(t)}=norm(\vec{v}(t))$. (***norm(*** is found from **2nd 5 MATH, 4 Matrix** and selecting the **Norm** option. Find the speed and record the result below. Sketch the graph of $\|\vec{v}(t)\|=\begin{cases} xt2 = t \\ yt2 = norm(\vec{v}(t)) \end{cases}$ over $0\le t\le 2\pi$ on the axes below. Based on this graph, does the moving point ever stop?



$0\le x\le 6, .8\le y\le 3.6$

1d. Define the reparameterization of $\vec{r}_1(t)=\vec{r}(t+3\sin t)$, $0\le t\le 2\pi$ and graph the $\vec{r}_1(t)$ over $0\le t\le 2\pi$. Define $\begin{cases} xt3(t) = xt1(t+3\sin t) \\ yt3(t) = yt1(t+3\sin t) \end{cases}$ as you defined $\vec{r}(t)$ in **1a** and graph them. What is the subtle difference between this graph and that in part **a**? Record the graphs on the axes provided.

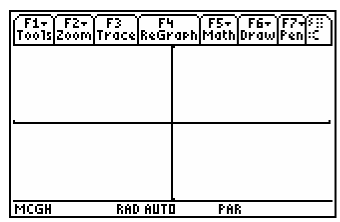


$\mathbf{-1 \le x \le 1, -1 \le y \le 1}$

$\mathbf{0 \le t \le 2\pi}$

1e. The velocity vector $\vec{v}(t) = \vec{r}\,'_1(t)$ is used to find the speed of a point moving under $\vec{r}_1(t)$. Plot the speed of $\vec{r}_1(t) = \langle xt1(t + 3\sin t), yt1(t + 3\sin t)\rangle$ by imitating part **c.** Note the approximate values of **t** where the speed is zero. Use the **Solve** command on your calculator to find more accurate values of $\vec{r}\,'(t) = \mathbf{0}$. What are the coordinates of the points where $\vec{r}_1(t)$ "stops"? Record your results below.

2a. Sketch the graph of $\vec{r}(t) = \langle 2\cos t + \sin 2t, 2\sin t + \cos 2t\rangle$, $0 \le t \le 2\pi$ on the axes provided below.



$\mathbf{-3 \le x \le 3, -3 \le y \le 3}$

$\mathbf{0 \le t \le 2\pi}$

2b. Find and mark on the graph any stationary points of $\vec{r}(t)$, as above.

2c. Define the reparameterization of $\vec{r}_1(t) = \vec{r}(t + \sin t)$, $0 \le t \le 2\pi$ and graph $\vec{r}_1(t)$ over $0 \le t \le 2\pi$. Check for stationary points. How do the results compare with part **b?**

2d. Repeat part **c** with $\vec{r}_1(t) = \vec{r}(t^2)$, $0 \le t \le \sqrt{2\pi}$.