Assignment 27: Functions of Two Variables (12.1-2) Name $\qquad$ Please provide a handwritten response.

1a. To graph the function $f(x, y)=\sin \left(y-x^{2}\right)$ change the MODE to 3D and enter $\mathbf{z 1}=\boldsymbol{\operatorname { s i n }}\left(\boldsymbol{y}-\boldsymbol{x}^{\wedge} \mathbf{2}\right)$ from $\bullet \mathbf{Y}=$. While still in the $\downarrow \mathbf{Y}=$ editor select $\mathbf{F} \mathbf{1}$ Tools, 9 Format. Set Coordinates to RECT, axes to BOX, Labels to ON, Style to HIDDEN SURFACE. Set Window to eye $\theta=-68.13$, eуe $\phi=50.89$, eye $\Psi=-2.47$, xgrid $=14$, ygrid $=14$,
$-\mathbf{2} \leq \boldsymbol{x} \leq 2,-\mathbf{2} \leq \boldsymbol{y} \leq \mathbf{2 , - 1 \leq z \leq 1}$. You can rotate the graph using the arrow keys. Record the general shape of the graph below.


1b. Graph $f(x, y)$ over a wider range and describe the general appearance of the resulting surface.

1c. Draw a contour plot of this function by going to Format (F1 Tools 9) and changing Style to CONTOUR LEVELS. Set eye $\boldsymbol{\theta}=\mathbf{9 0}$, еуe $\boldsymbol{\phi}=\mathbf{0}$, еуe $\boldsymbol{\Psi}=\mathbf{0}$ in the window. Sketch the result on the axes below.


2a. Contour plots can be used to show that $\lim _{(x, y) \rightarrow(0,0)} \frac{\boldsymbol{x}^{2} \boldsymbol{y}}{\boldsymbol{x}^{2}+\boldsymbol{y}^{2}}=\mathbf{0}$ and that $\lim _{(x, y) \rightarrow(0,0)} \frac{\boldsymbol{x}^{2}}{\boldsymbol{x}^{2}+\boldsymbol{y}^{2}}$ does not exist. Enter $\mathbf{z 1}=x^{\wedge} 2 y /\left(x^{\wedge} 2+y^{\wedge} 2\right)$ for $-.1 \leq x \leq .1,-.1 \leq y \leq .1$ and record the result below. Repeat with the window changed to $-.07 \leq x \leq .07,-.07 \leq y \leq .07$.


2b. How do these graphs support the conclusion that $\lim _{(x, y) \rightarrow(0,0)} \frac{x^{2} y}{x^{2}+y^{2}}$ exists?

2c. Now examine $\lim _{(x, y) \rightarrow(0,0)} \frac{x^{2}}{x^{2}+y^{2}}$ by graphing a contour plot of $z 3=x^{\wedge} 2 /\left(x^{\wedge} 2+y^{\wedge} 2\right)$.
Set your window to $-.01 \leq x \leq .01,-.01 \leq y \leq .01$. Repeat with the window changed to $-.001 \leq x \leq .001,-.001 \leq y \leq .001$. Record your results below.


2d. How do these graphs support the conclusion that $\lim _{(x, y) \rightarrow(0,0)} \frac{x^{2}}{x^{2}+y^{2}}$ does not exist?

2e. Based on contour plots, do you think that $\lim _{(x, y) \rightarrow(0,0)} \frac{x \boldsymbol{\operatorname { s i n }} \boldsymbol{y}}{\boldsymbol{x}^{2}+y^{2}}$ exists? Explain your answer.

