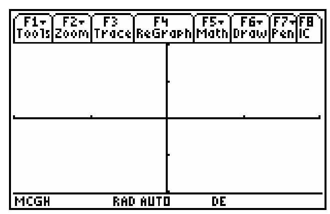
Assignment 31: Vector Fields in the Plane (14.1-5) Name_______________________
Please provide a handwritten response.

1a. Recall from Assignment 17 that vector fields can be drawn on your calculator. Draw the vector field for $\vec{v} = \langle -1, y^2 \rangle$. In order to draw this vector field on your calculator you must first express it in the form $\frac{dy}{dx} = -x^2$.

	TI-89	Voyage 200
DRAW A VECTOR FIELD	**Set MODE to DifEq** and enter the equation using **t** for **x** as $y1' = -t \wedge 2$ Set **F1 9 (FORMAT)** to **Euler and SlpFld .** Set **initial conditions** to **t0= 0,** WINDOW **to** $0 \le t \le 2$, $-2 \le x \le 2, -2 \le y \le 2$ **GRAPH**	**Define** $f(x, y) = -x^2$ **and** $y1 = f(x, y)$. **Highlight** and press **F4** to deselect ***y*1.** **Set WINDOW** values to $-2 \le x \le 2, -2 \le y \le 2$ Run the program **slopefld( )**

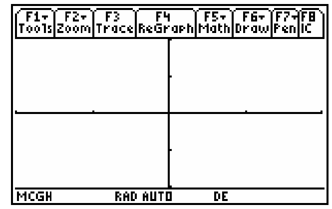
Sketch your results on the axes provided below.



1b. The flow lines of this vector field are the solutions of the separable differential equation, $\frac{dy}{dx} = -x^2$. Add flow lines to the vector field in **1a** by following the directions below.

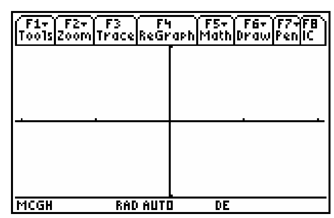
	TI-89	Voyage 200
ADD FLOW LINES	Without clearing your graph from **1a** enter a list {-3,-2,-1,0,1,2,3} in the ♦**Y=** editor as $yi1 = \{-3, -2, -1, 0, 1, 2, 3\}$	Without clearing your graph from **1a** run the program **slopegra(*x*,*y*)** multiple times for the pairs $(-3, 9), (-2, 8/3), (-1, 1/3)$, $(0, 0), (1, -1/3), (2, -8/3)$ and $(3, -9)$.

2a. To draw the gradient field corresponding to $f(x, y) = y \sin x$ on your calculator, first compute the gradient, $\nabla f(x, y) = [d(y * \sin(x), x), d(y * \sin(x), y)] = [y \cos x, \sin x]$ and express it in the form $\frac{dy}{dx} = \frac{\sin x}{y \cos x}$. You can then enter $y1' = \sin(t)/(y1 * \cos(t))$ with a window $0 \le t \le 2$, $-2 \le x \le 2, -2 \le y \le 2$, ***tstep* = .1** and graph the gradient field. Sketch your results on the axes provided below.



2b. The level curves of $f(x, y)$ can be added to the above sketch by entering the list $\{-1,1\}$ in $yi1$ as in **1b** above. What general connection between level curves and the gradient vector field does this graph bring out? Record your answer below.

3a. The line integral $\oint_C (x^2 - y)dx + y^2 dy$ where C is the circle $x^2 + y^2 = 1$ oriented counterclockwise can be investigated by defining $m(x, y) = x \wedge 2 - y,$ defining $n(x, y) = y \wedge 2,$ and defining $\vec{f}(x, y) = [m(x, y), n(x, y)]$. After forming $\frac{dy}{dx} = \frac{y^2}{x^2 - y}$ you can graph the vector field by entering $y1 = y1 * y1 / (t \wedge 2 - y1)$ over $-1 \le x \le 1, \quad -1 \le y \le 1, -1 \le t \le 1$, and pressing **F2 5 ZoomSqr.** Your **MODE** is still **DIFF EQUATIONS.** From the graph **save this picture** by pressing **F1,** selecting **2. Save Copy As ENTER Type 2:Picture Variable pic1.** Note that ZoomSqr has changed your ***xmin*** and ***xmax*** values. You can now define $\vec{r}(t) = [cos(t), sin(t)]$ to parameterize the curve C. Change your **MODE** to **PARAMETRIC** enter $xt1 = cos(t), yt1 = sin(t)$ and graph. Your calculator will keep the same window values for you. From the graph of ***r*(*t*)** press **F1** and select **1. Open ENTER Type 2. Picture Variable pic1** and press **ENTER**.
You now have the vector field and region ***C*** plotted together. Record your results on the axes below.



3b. To evaluate $\oint_C f(\vec{r}) \cdot dr$ you first need to separate the components of $\vec{r}(t)$. You need to use the command $mat\%list(\vec{r}(t)) \to \vec{q}(t)$. Then the commands $\vec{q}(t)[1]$ and $\vec{q}(t)[2]$ will give you the components of $\vec{r}(t)$. Form $dotP(f(\vec{q}(t)[1], \vec{q}(t)[2]), d(\vec{r}(t), t))$ and integrate the result over $0 \le t \le 2\pi.$ Record the result of this calculation below and tell whether Green's theorem gives the same result.

3c. Now repeat the integration from part **b** over $\frac{3\pi}{4} \le t \le \pi$. Would the graph lead you to expect a positive or negative result? Why? What result does your calculator give? Repeat the above for $\frac{3\pi}{2} \le t \le 2\pi$. Record both results below.