## Assignment 5: Limits, Part 1 (1.2) Please provide a handwritten response.

Name

1a. Many ordinary limits can be found with the TI-89 and the Voyage 200 using the limit command. For example, to evaluate $\lim _{x \rightarrow-3} \frac{3 x+9}{x^{2}-9}$ you can execute the command $\lim \operatorname{it}\left((3 x+9) /\left(x^{\wedge} 2-9\right), x,-3\right)$. Execute this limit and record the result below.

1b. Your text suggests that $\lim _{x \rightarrow 0} \frac{\sin x}{x}=1$. Execute $\lim \operatorname{it}(\sin (x) / x, x, 0)$ and record your result below. Does it agree with your text?

2a. You are asked for numerical and graphical evidence regarding $\lim _{x \rightarrow 0} \frac{\boldsymbol{\operatorname { t a n } x}}{\boldsymbol{\operatorname { s i n } x}}$. Graph $y=\frac{\boldsymbol{\operatorname { t a n }} \boldsymbol{x}}{\boldsymbol{\operatorname { s i n }} \boldsymbol{x}}$ on the axes below. What value for $\lim _{x \rightarrow 0} \frac{\boldsymbol{\operatorname { t a n }} \boldsymbol{x}}{\boldsymbol{\operatorname { s i n }} x}$ does this graph suggest?


2b. Next, evaluate $\boldsymbol{f}(\mathbf{- 0 . 1}), f(\mathbf{- 0 . 0 1})$, etc. to complete the table below. What value for $\lim _{x \rightarrow 0} \frac{\boldsymbol{\operatorname { t a n }} \boldsymbol{x}}{\boldsymbol{\operatorname { s i n }} \boldsymbol{x}}$ does the table suggest?

| $\boldsymbol{x}$ | $\boldsymbol{f}(\boldsymbol{x})$ |
| :---: | :---: |
| $-\mathbf{0 . 1}$ |  |
| $-\mathbf{0 . 0 1}$ |  |
| $-\mathbf{0 . 0 0 1}$ |  |
| $\mathbf{0 . 0 0 1}$ |  |
| $\mathbf{0 . 0 1}$ |  |
| $\mathbf{0 . 1}$ |  |

2c. Finally, evaluate $\lim \operatorname{it}(\boldsymbol{\operatorname { t a n }}(\boldsymbol{x}) / \sin (\boldsymbol{x}), \boldsymbol{x}, \mathbf{0})$ and record the result below. Did all three approaches lead you to the same conclusion?

3a. The example $\lim _{x \rightarrow 0} \frac{\boldsymbol{\operatorname { c o s }} \boldsymbol{x}-1}{\boldsymbol{x}^{2}}$ shows that round-off error can cause very misleading computed results. Enter $\boldsymbol{y}=\frac{\boldsymbol{\operatorname { c o s }} \boldsymbol{x}-\mathbf{1}}{\boldsymbol{x}^{2}}$ and complete the table below. (Be sure to count the zeros). Then evaluate $\lim \operatorname{it}\left((\boldsymbol{\operatorname { c o s }}(\boldsymbol{x})-\mathbf{1}) / \boldsymbol{x}^{\wedge} \mathbf{2}, \boldsymbol{x}, \mathbf{0}\right)$ and record the result below.

| $\boldsymbol{x}$ | $f(x)$ |
| :---: | :---: |
| $\mathbf{0 . 1}$ |  |
| $\mathbf{0 . 0 0 0 1}$ |  |
| $\mathbf{0 . 0 0 0 0 0 0 1}$ |  |
| $\mathbf{0 . 0 0 0 0 0 0 0 1}$ |  |
| $\mathbf{0 . 0 0 0 0 0 0 0 0 1}$ |  |

3b. Do you think that all of your calculator's results are correct in part a? If not, then which one(s) do you think are wrong, and why?

4a. To find one-sided limits you give the direction in the limit command by inserting $\mathbf{- 1}$ for the limit from the left. Graph the function $\boldsymbol{g}(\boldsymbol{x})=\frac{\boldsymbol{x}}{|\boldsymbol{x}|}$ on the axes provided below.
Evaluate $\lim _{x \rightarrow 0^{-}} \frac{\boldsymbol{x}}{|\boldsymbol{x}|}=\lim \operatorname{it}(\boldsymbol{x} / \boldsymbol{a b s}(\boldsymbol{x}), \boldsymbol{x}, \mathbf{0},-\mathbf{1})$ and record the result below.

$-2.5 \leq x \leq 2.5,-1.5 \leq y \leq 1.5$

4b. Now evaluate $\lim _{\boldsymbol{x} \rightarrow 0^{+}} \frac{\boldsymbol{x}}{\boldsymbol{x} \mid}=\boldsymbol{\operatorname { l i m }} \boldsymbol{\operatorname { i t }}(\boldsymbol{x} / \boldsymbol{\operatorname { a b s }}(\boldsymbol{x}), \boldsymbol{x}, \mathbf{0}, 1)$ and record the result below.

4c. Now evaluate $\lim _{x \rightarrow 0} \frac{\boldsymbol{x}}{|\boldsymbol{x}|}=\lim \operatorname{it}(\boldsymbol{x} / \boldsymbol{a b s}(\boldsymbol{x}), \boldsymbol{x}, \mathbf{0})$. Did you expect this result? Why?

