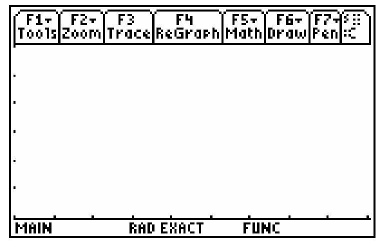
Assignment 7: Limits, Part III (1.7) **Name____________________**
Please provide a handwritten response.

1a. The function $f(x)=\dfrac{(x^3+4)^2-x^6}{x^3}$ can be used to illustrate the dangers of loss of significance errors. Graph $f(x)$ for $10000 \le x \le 100000$. Sketch the result below. Does this graph give any indication of the value of $\lim_{x\to\infty} f(x)$? Explain.



$10000 \le x \le 100000,\ 7.7 \le y \le 8.3$

1b. Now evaluate the function to complete the table below. Use $y_1(1000)$ to $y_1(10000000)$. You can shorten your typing by editing the current entry line (press the right arrow after an evaluation) to obtain the number in the table. ♦ **ENTER** will simplify your results.

x	$f(x)$
1000	
10000	
100000	
1000000	
10000000	

1c. Now evaluate the limit using $\textbf{limit}\left(((x \wedge 3+4)\wedge 2-x\wedge 6)/x\wedge 3, x, \infty\right)$ and record the result below. Is it likely that all of these results are correct? Which ones are not?

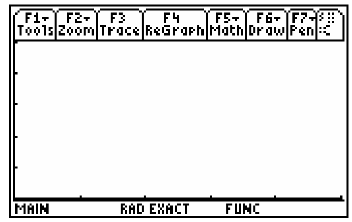
1d. $f(x)=\dfrac{(x^3+4)^2-x^6}{x^3}$ can be rewritten as $f(x)=\dfrac{8x^3+16}{x^3}$. Enter $f(x)$ in your calculator as y_1 and complete the table below with this new (but equivalent) formula for f. Do you think these new results are more trustworthy?

x	$f(x)$
1000	
10000	
100000	
1000000	
10000000	

1e. Evaluate $\textbf{limit}\left((8x\wedge 3+16)/x\wedge 3, x, \infty\right)$. Is this result more accurate?

2. Scientific notation is used to write very large or very small numbers in a convenient form; for example, **.000000000002673** would be written in scientific notation as $\mathbf{2.673X10^{-12}}$. Set **Mode** to **Float 12**. Enter $\mathbf{2.673*10\wedge(-12)}$ into your calculator and record the result below.

3a. Find a value of ***x*** for which loss of significance occurs in $\lim\limits_{x\to\infty}\sqrt{x}\left(\sqrt{x+4}-\sqrt{x+2}\right)$. Graph $y=\sqrt{x}\left(\sqrt{x+4}-\sqrt{x+2}\right)$ on the axes below. Based on this graph, what value would you give for $\lim\limits_{x\to\infty}\sqrt{x}\left(\sqrt{x+4}-\sqrt{x+2}\right)$?



$\mathbf{0 \le x \le 100000,\ 0 \le y \le 1}$

3b. Now complete the table below by evaluating $y_1(1.0*10\wedge 8)$, $y_1(1.0*10\wedge 9)$, etc. Where does loss of significance occur?

x	***g*(*x*)**
$\mathbf{1X10^{8}}$	
$\mathbf{1X10^{9}}$	
$\mathbf{1X10^{10}}$	
$\mathbf{1X10^{11}}$	
$\mathbf{1X10^{12}}$	

3c. You can rewrite ***y*** to avoid loss of significance; you can check that multiplying ***y*** by $\dfrac{\sqrt{x+4}+\sqrt{x+2}}{\sqrt{x+4}+\sqrt{x+2}}$ gives $y=\dfrac{2\sqrt{x}}{\sqrt{x+4}+\sqrt{x+2}}$. Enter $y=\dfrac{2\sqrt{x}}{\sqrt{x+4}+\sqrt{x+2}}$ as y_1 and complete the table below as in part **b**. Do these results seem more reliable?

x	***g*(*x*)**
$\mathbf{1X10^{8}}$	
$\mathbf{1X10^{9}}$	
$\mathbf{1X10^{10}}$	
$\mathbf{1X10^{11}}$	
$\mathbf{1X10^{12}}$	

3d. Evaluate $\lim\limits_{x\to\infty}\sqrt{x}\left(\sqrt{x+4}-\sqrt{x+2}\right)$ record the result below. Does it seem to be correct?

Evaluate $\lim\limits_{x\to\infty}\dfrac{2\sqrt{x}}{\sqrt{x+4}+\sqrt{x+2}}$ and record the result below. Do you get the same result?

3e. Repeat parts **a** and **b** for $\lim\limits_{x\to\infty}x\left(\sqrt{x^3+8}-x^{3/2}\right)$ and record a value of ***x*** at which loss of significance occurs.