Preface

The wide-ranging debate brought about by the calculus reform movement has had a significant impact on calculus textbooks. In response to many of the questions and concerns surrounding this debate, we have written a modern calculus textbook, intended for students majoring in mathematics, physics, chemistry, engineering, and related fields.

Our intention is that students should be able to read our book, rather than merely use it as an encyclopedia filled with the facts of calculus. We have written in a conversational style that reviewers have compared to listening to a good lecture. Our sense of what works well with students has been honed by teaching mathematics for more than a combined 50 years at a variety of colleges and universities, both public and private, ranging from a small liberal arts college to large engineering schools.

In an effort to ensure that this textbook successfully addresses our concerns about the effective teaching of calculus, as well as others' concerns, we have continually asked instructors around the world for their opinions on the calculus curriculum, the strengths and weaknesses of current textbooks, and the strengths and weaknesses of our own text. In preparing this third edition, as with the previous editions, we enjoyed the benefit of countless insightful comments from a talented panel of reviewers that was selected to help us with this project.

OUR PHILOSOPHY

We agree with many of the ideas that have come out of the calculus reform movement. In particular, we believe in the **Rule of Four:** that concepts should be presented **graphically, numerically, algebraically** and **verbally,** whenever these are appropriate. In fact, we would add **physically** to this list, since the modeling of physical problems is an important skill that students need to develop. We also believe that, while the calculus curriculum has been in need of reform, we should not throw out those things that already work well. Our book thus represents an updated approach to the traditional topics of calculus. We follow a mainstream order of presentation, while integrating technology and thought-provoking exercises throughout.

One of the thrusts of the calculus reform movement has been to place greater emphasis on problem solving and to present students with more realistic applications as well as open-ended problems. We have incorporated meaningful writing exercises and extended, open-ended problems into **every problem set.** You will also find a **much wider range of applications** than in most traditional texts. We make frequent use of applications from students' experience both to **motivate the development of new topics** and to illustrate concepts we have already presented. In particular, we have included numerous examples from a wide range of fields to give students a familiar context in which to think of various concepts and their applications.

We believe that a conceptual development of the calculus must motivate the text. Although we have **integrated technology throughout,** we have not allowed the technology to drive the book. Our goal is to use the available technology to help students reach a conceptual understanding of the calculus as it is used today.

MOTIVATION AND UNDERSTANDING

Perhaps the most important task when preparing a calculus text is the actual *writing* of it. We have endeavored to write this text in a manner that combines an appropriate level of informality with an honest discussion regarding the difficulties that students commonly face in their study of calculus. In addition to the concepts and applications of calculus, we have also included many frank discussions about what is practical and impractical, and what is difficult and not so difficult to students in the course.

Our primary objectives are to find better ways to motivate students and facilitate their understanding. To accomplish this, we go beyond the standard textbook presentation and tell students **why** they are learning something, **how** they will use it, and **why** it is important. As a result students master problem-solving skills while also **learning how to think mathematically,** an important goal for most instructors teaching the calculus course.

This edition of our text incorporates an early introduction to all transcendental functions. Our students have seen these functions before they ever set foot in a calculus classroom, so we would like to take advantage of their familiarity. We introduce the calculus of these functions in Chapter 2, along with the other rules of differentiation. We have found that this early introduction allows for more varied examples and exercises in the applications of differentiation (including graphing), integration, and applications of integration.

In our view, techniques of integration remain of great importance. Our emphasis is on helping students develop the ability to carefully distinguish among similar-looking integrals and identify the appropriate technique of integration to apply to each integral. The attention to detail and mathematical sophistication required by this process are invaluable skills. We do not attempt to be encyclopedic about techniques of integration, especially given the widespread use of computer algebra systems. Accordingly, in section 6.5, we include a discussion of integration tables and the use of computer algebra systems for performing symbolic integration.

In addition to a focus on the central concepts of calculus, we have included several sections that are not typically found in other calculus texts, as well as expanded coverage of specific topics. This provides instructors with the flexibility to tailor their courses to the interests and abilities of each class.

- For instance, in section 1.7, we explore **loss-of significance errors.** Here, we discuss how computers and calculators perform arithmetic operations and how these can cause errors, in the context of numerical approximation of limits.
- In section 3.9, we present a diverse group of applications of differentiation, including **chemical reaction rates and heart rates.**
- Separable differential equations and logistic growth are discussed in section 7.2, followed by direction fields and Euler's method for first-order ordinary differential equations in section 7.3.
- In Chapter 8, we follow our discussion of power series and Taylor's Theorem with a section on **Fourier series.**
- In sections 9.1–9.3 we provide **expanded coverage of parametric equations.**
- In section 10.4 we include a discussion of **Magnus force.**

CALCULUS AND TECHNOLOGY

It is our conviction that graphing calculators and computer algebra systems must not be used indiscriminately. The focus must always remain on the calculus. We have ensured that each of our exercise sets offers an extensive array of problems that should be worked by hand. We also believe, however, that calculus study supplemented with an intelligent use of technology gives students an extremely powerful arsenal of problem-solving skills. Many passages in the text provide guidance on how to judiciously use—and not abuse—graphing calculators and computers. We also provide ample opportunity for students to practice using these tools. Exercises that are most easily solved with the aid of a graphing calculator or a computer algebra system are easily identified with a icon.

IMPROVEMENTS IN THE THIRD EDITION

Building upon the success of the Second Edition of *Calculus,* we have made the following revisions to produce an even better Third Edition:

Organization

- **All transcendental functions are introduced early,** and their calculus is covered with the calculus of algebraic functions, to accommodate instructors who prefer this approach.
- **Differential equations** receive substantially more coverage in Chapter 7.

Presentation

- A **thorough rewrite** of the book resulted in a **more concise and direct presentation** of all concepts and techniques.
- The entire text was redesigned for a **more open, clean appearance** to aid students in locating and focusing on essential information.

Exercises

- **More challenging exercises** appear throughout the book, and ***Exploratory Exercises* conclude every section** to encourage students to synthesize what they've learned.
- **Technology icons** now appear next to all exercises requiring the use of a computer algebra system.

Aesthetics and Relevance of Mathematics

- *NEW* ***Beyond Formulas*** boxes appear in every chapter to encourage students to **think mathematically** and go beyond routine answer calculation.
- *NEW* ***Today in Mathematics*** boxes appear in every chapter showing students that mathematics is a dynamic discipline with many discoveries continually being made by **people inspired by the beauty of the subject.**
- *NEW* The ***Index of Applications*** shows students of diverse majors the **immediate relevance** of what they are studying.

SUPPLEMENTS

INSTRUCTOR'S SOLUTIONS MANUAL (ISBN 978-0-07-327656-4)

An invaluable, timesaving resource, the Instructor's Solutions Manual contains comprehensive, worked-out solutions to the odd- and even-numbered exercises in the text.

STUDENT SOLUTIONS MANUAL (ISBN 978-0-07-286969-9)

The Student Solutions Manual is a helpful reference that contains comprehensive, worked-out solutions to the odd-numbered exercises in the text.

INSTRUCTOR'S TESTING AND RESOURCE CD-ROM (ISBN 978-0-07-286962-0)

Brownstone Diploma® testing software, available on CD-ROM, offers instructors a quick and easy way to create customized exams and view student results. Instructors may use the software to sort questions by section, difficulty level, and type; add questions and edit existing questions; create multiple versions of questions using algorithmically-randomized variables; prepare multiple-choice quizzes; and construct a grade book.

MathZone www.mathzone.com

McGraw-Hill's MathZone is a cutting-edge, customizable web-based system that offers a complete solution to instructors' online homework, quizzing and testing needs. MathZone guides students through step-by-step solutions to practice problems and facilitates student assessment through the use of algorithmically-generated test questions. Student activity within the MathZone site is **automatically graded** and accessible to instructors in an integrated, exportable grade book.

MathZone also provides a wide variety of **interactive student tutorials,** including **new applets for every section** in the book to give students interactive practice on important concepts and procedures; algorithmic practice problems; **e-Professor,** a collection of step-by-step animated instructions for solving exercises from the text; **Calculus Concepts Videos;** and **NetTutor,** a live, personalized tutoring service offered via the Internet.

CALCULUS CONCEPTS VIDEOS (978-0-07-312476-6)

Students will see **essential concepts** explained and brought to life through **dynamic animations** in this new video series available on DVD and on the Smith/Minton MathZone site. The **twenty-five key concepts,** chosen after consultation with calculus instructors across the country, are the most commonly taught topics that students need help with and that also lend themselves most readily to on-camera demonstration.

ALEKS PREP FOR CALCULUS

ALEKS (**A**ssessment and **LE**arning in **K**nowledge **S**paces) is an artificial intelligence-based system for mathematics learning, available online 24/7. Using unique adaptive questioning, ALEKS accurately assesses what topics each student knows and then determines exactly what each student is ready to learn next. ALEKS interacts with the students much as a skilled human tutor would, moving between explanation and practice as needed, correcting and analyzing errors, defining terms and changing topics on request, and helping them master the course content more quickly and easily. **New ALEKS 3.0** now links to text-specific videos, multimedia tutorials, and textbook pages in PDF format. ALEKS also offers a robust classroom management system that allows instructors to monitor and direct student progress toward mastery of curricular goals. See www.highed.aleks.com.

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A COMMITMENT TO ACCURACY

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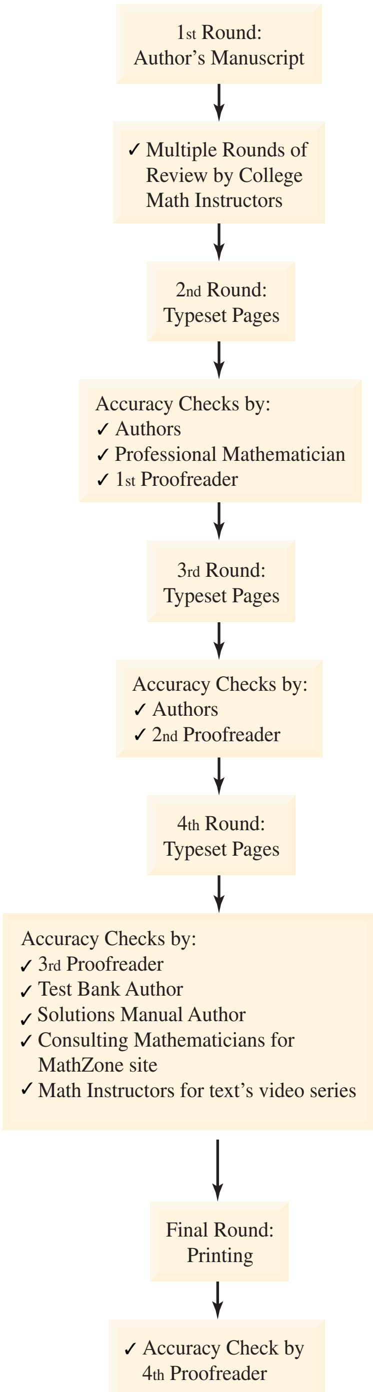

First Round

Step 1: Numerous **college math instructors** review the manuscript and report on any errors that they may find, and the authors make these corrections in their final manuscript.

Second Round

Step 2: Once the manuscript has been typeset, the **authors** check their manuscript against the first page proofs to ensure that all illustrations, graphs, examples, exercises, solutions, and answers have been correctly laid out on the pages, and that all notation is correctly used.

Step 3: An outside, **professional mathematician** works through every example and exercise in the page proofs to verify the accuracy of the answers.

Step 4: A **proofreader** adds a triple layer of accuracy assurance in the first pages by hunting for errors, then a second, corrected round of page proofs is produced.

Third Round

Step 5: The **author team** reviews the second round of page proofs for two reasons: 1) to make certain that any previous corrections were properly made, and 2) to look for any errors they might have missed on the first round.

Step 6: A **second proofreader** is added to the project to examine the new round of page proofs to double check the author team's work and to lend a fresh, critical eye to the book before the third round of paging.

Fourth Round

Step 7: A **third proofreader** inspects the third round of page proofs to verify that all previous corrections have been properly made and that there are no new or remaining errors.

Step 8: Meanwhile, in partnership with **independent mathematicians,** the text accuracy is verified from a variety of fresh perspectives:

- The **test bank author** checks for consistency and accuracy as they prepare the computerized test item file.
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- A **consulting group of mathematicians,** who write material for the text's MathZone site, notifies the publisher of any errors they encounter in the page proofs.
- A video production company employing **expert math instructors** for the text's videos will alert the publisher of any errors they might find in the page proofs.

Final Round

Step 9: The **project manager,** who has overseen the book from the beginning, performs a **fourth proofread** of the textbook during the printing process, providing a final accuracy review.

⇒ What results is a mathematics textbook that is as accurate and error-free as is humanly possible, and our authors and publishing staff are confident that our many layers of quality assurance have produced textbooks that are the leaders of the industry for their integrity and correctness.

Guided Tour

TOOLS FOR LEARNING

Real-World Emphasis

Each chapter opens with a real-world application that illustrates the usefulness of the concepts being developed and motivates student interest.

Parametric Equations and Polar Coordinates

CHAPTER 9

You are all familiar with sonic booms, those loud crashes of noise caused by aircraft flying faster than the speed of sound. You may have even heard a sonic boom, but you have probably never *seen* a sonic boom. The remarkable photograph here shows water vapor outlining the surface of a shock wave created by an F-18 jet flying supersonically. (Note that there is also a small cone of water vapor trailing the back of the cockpit of the jet.)

You may be surprised at the apparently conical shape assumed by the shock waves. A mathematical analysis of the shock waves verifies that the shape is indeed conical. (You will have an opportunity to explore this in the exercises in section 9.1.) To visualize how sound waves propagate, imagine an exploding firecracker. If you think of this in two dimensions, you'll recognize that the sound waves propagate in a series of ever-expanding concentric circles that reach everyone standing a given distance away from the firecracker at the same time.

In this chapter, we extend the concepts of calculus to curves described by parametric equations and polar coordinates. For instance, in order to study the motion of an object such as an airplane in two dimensions, we would need to describe the object's position (x, y) as a function of the parameter t (time). That is, we write the position in the form $(x, y) = (x(t), y(t))$, where $x(t)$ and $y(t)$ are functions to which our existing techniques of calculus can be applied. The equations $x = x(t)$ and $y = y(t)$ are called parametric equations. Additionally, we'll explore how to use polar coordinates to represent curves, not as a set of points (x, y), but rather, by specifying the points by the distance from the origin to the point and an angle corresponding to the direction from the origin to the point. Polar coordinates are especially convenient for describing circles, such as those that occur in propagating sound waves.

These alternative descriptions of curves bring us a great deal of needed flexibility in attacking many problems. Often, even very complicated looking curves have a simple description in terms of parametric equations or polar coordinates. We explore a variety of interesting curves in this chapter and see how to extend the methods of calculus to such curves.

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Definitions, Theorems and Proofs

All formal definitions and theorems are clearly boxed within the text for easy visual reference. Proofs are clearly labeled. Proofs of some results are found in Appendix A.

DEFINITION 2.2

The **derivative** of $f(x)$ is the function $f'(x)$ given by

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}, \tag{2.3}$$

provided the limit exists. The process of computing a derivative is called **differentiation.**

Further, f is differentiable on an interval I if it is differentiable at every point in I.

THEOREM 2.1

If $f(x)$ is differentiable at $x = a$, then $f(x)$ is continuous at $x = a$.

PROOF

For f to be continuous at $x = a$, we need only show that $\lim_{x\to a} f(x) = f(a)$. We consider

$$\begin{aligned}\lim_{x\to a}[f(x) - f(a)] &= \lim_{x\to a}\left[\frac{f(x)-f(a)}{x-a}(x-a)\right] && \text{Multiply and divide by } (x-a).\\ &= \lim_{x\to a}\left[\frac{f(x)-f(a)}{x-a}\right]\lim_{x\to a}(x-a) && \text{By Theorem 3.1 (iii) from section 1.3.}\\ &= f'(a)(0) = 0, && \text{Since } f \text{ is differentiable at } x=a.\end{aligned}$$

where we have used the alternative definition of derivative (2.2) discussed earlier. By Theorem 3.1 in section 1.3, it now follows that

$$\begin{aligned}0 &= \lim_{x\to a}[f(x) - f(a)] = \lim_{x\to a} f(x) - \lim_{x\to a} f(a)\\ &= \lim_{x\to a} f(x) - f(a),\end{aligned}$$

which gives us the result. ■

Examples

Each chapter contains a large number of worked examples, ranging from the simple and concrete to more complex and abstract.

Use of Graphs and Tables

Being able to visualize a problem is an invaluable aid in understanding the concept presented. To this purpose, we have integrated more than 1500 computer-generated graphs throughout the text.

Point	**(0, 0)**	$\left(-\frac{1}{6}, -\frac{1}{3}\right)$
$f_{xx} = 4$	4	4
$f_{yy} = -6y$	0	−2
$f_{xy} = -2$	−2	−2
$D(a, b)$	−4	4

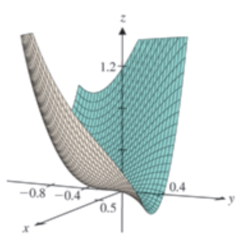


FIGURE 12.42
$z = 2x^2 - y^3 - 2xy$

EXAMPLE 7.2 Using the Discriminant to Find Local Extrema

Locate and classify all critical points for $f(x, y) = 2x^2 - y^3 - 2xy$.

Solution We first compute the first partial derivatives: $f_x = 4x - 2y$ and $f_y = -3y^2 - 2x$. Since both f_x and f_y are defined for all (x, y), the critical points are solutions of the two equations:

$$f_x = 4x - 2y = 0$$

and

$$f_y = -3y^2 - 2x = 0.$$

Solving the first equation for y, we get $y = 2x$. Substituting this into the second equation, we have

$$\begin{aligned}0 &= -3(4x^2) - 2x = -12x^2 - 2x\\ &= -2x(6x+1),\end{aligned}$$

so that $x = 0$ or $x = -\frac{1}{6}$. The corresponding y-values are $y = 0$ and $y = -\frac{1}{3}$. The only two critical points are then $(0, 0)$ and $\left(-\frac{1}{6}, -\frac{1}{3}\right)$. To classify these points, we first compute the second partial derivatives: $f_{xx} = 4$, $f_{yy} = -6y$ and $f_{xy} = -2$ and then test the discriminant. We have

$$D(0, 0) = (4)(0) - (-2)^2 = -4$$

and

$$D\left(-\tfrac{1}{6}, -\tfrac{1}{3}\right) = (4)(2) - (-2)^2 = 4.$$

From Theorem 7.2, we conclude that there is a saddle point of f at $(0, 0)$, since $D(0, 0) < 0$. Further, there is a local minimum at $\left(-\frac{1}{6}, -\frac{1}{3}\right)$ since $D\left(-\frac{1}{6}, -\frac{1}{3}\right) > 0$ and $f_{xx}\left(-\frac{1}{6}, -\frac{1}{3}\right) > 0$. The surface is shown in Figure 12.42. ■

As we see in example 7.3, the second derivatives test does not always help us to classify a critical point.

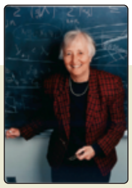
COMMENTARY AND GUIDANCE

Beyond Formulas

Beyond Formulas boxes appear in every chapter to encourage students to think mathematically and go beyond routine answer calculation.

BEYOND FORMULAS

The Mean Value Theorem is subtle, but its implications are far-reaching. Although the illustration in Figure 2.49 makes the result seem obvious, the consequences of the Mean Value Theorem, such as example 9.4, are powerful and not at all obvious. For example, most of the rest of the calculus developed in this book depends on the Mean Value Theorem either directly or indirectly. A thorough understanding of the theory of calculus can lead you to important conclusions, particularly when the problems are beyond what your intuition alone can handle. What other theorems have you learned that continue to provide insight beyond their original context?



TODAY IN MATHEMATICS

Cathleen Morawetz (1923–) A Canadian mathematician whose work on transonic flows and wave scattering greatly influenced the design of air foils. Both of her parents were mathematically trained, but her mathematics

Today in Mathematics

Today in Mathematics boxes appear in every chapter, showing students that mathematics is a dynamic discipline with many discoveries continually being made by people inspired by the beauty of the subject.

HISTORICAL NOTES

Johannes Kepler (1571–1630) German astronomer and mathematician whose discoveries revolutionized Western science. Kepler's remarkable mathematical ability and energy produced connections among many areas of research. A study of observations

Historical Notes

These biographical features provide background information on prominent mathematicians and their contributions to the development of calculus and put the subject matter into perspective.

CONCEPTUAL UNDERSTANDING THROUGH PRACTICE

Applications

The text provides numerous and varied applications that relate calculus to the real world, many of which are unique to the Smith/Minton series. Worked examples and exercises are frequently developed with an applied focus in order to motivate the presentation of new topics, further illustrate familiar topics, and connect the conceptual development of calculus with students' everyday experiences.

EXAMPLE 1.6 Steering an Aircraft in a Head Wind and a Crosswind

An airplane has an airspeed of 400 mph. Suppose that the wind velocity is given by the vector $\mathbf{w} = \langle 20, 30 \rangle$. In what direction should the airplane head in order to fly due west (i.e., in the direction of the unit vector $-\mathbf{i} = \langle -1, 0 \rangle$)?

Solution We illustrate the velocity vectors for the airplane and the wind in Figure 10.14. We let the airplane's velocity vector be $\mathbf{v} = \langle x, y \rangle$. The effective velocity of the plane is then $\mathbf{v} + \mathbf{w}$, which we set equal to $\langle c, 0 \rangle$, for some negative constant c. Since

$$\mathbf{v} + \mathbf{w} = \langle x + 20, y + 30 \rangle = \langle c, 0 \rangle,$$

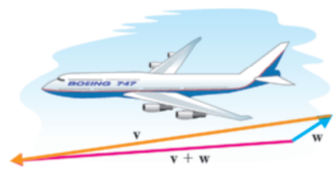


FIGURE 10.14
Forces on an airplane.

we must have $x + 20 = c$ and $y + 30 = 0$, so that $y = -30$. Further, since the plane's airspeed is 400 mph, we must have $\|\mathbf{v}\| = \sqrt{x^2 + y^2} = \sqrt{x^2 + 900} = 400$. Squaring this gives us $x^2 + 900 = 160{,}000$, so that $x = -\sqrt{159{,}100}$. (We take the negative square root so that the plane heads westward.) Consequently, the plane should head in the direction of $\mathbf{v} = \langle -\sqrt{159{,}100}, -30 \rangle$, which points left and down, or southwest, at an angle of $\tan^{-1}(30/\sqrt{159{,}100}) \approx 4°$ below due west. ■

33. A baseball is hit from a height of 3 feet with initial speed 120 feet per second and at an angle of 30 degrees above the horizontal. Find a vector-valued function describing the position of the ball t seconds after it is hit. To be a home run, the ball must clear a wall that is 385 feet away and 6 feet tall. Determine whether this is a home run.

34. Repeat exercise 33 if the ball is launched with an initial angle of 31 degrees.

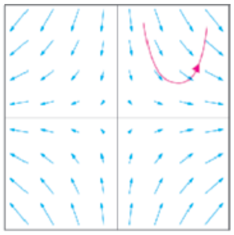
Balanced Exercise Sets

This text contains thousands of exercises, found at the end of each section and chapter. Each problem set has been carefully designed to provide a balance of routine, moderate and challenging exercises. The authors have taken great care to create original and imaginative exercises that provide an appropriate review of the topics covered in each section and chapter.

Technology Icon

Exercises in the section exercise sets and chapter review exercises that can most easily be solved using a graphing calculator or computer are clearly identified with this icon: .

42.



In exercises 43–52, use the formulas $m = \int_C \rho\, ds$, $\bar{x} = \frac{1}{m}\int_C x\rho\, ds$, $\bar{y} = \frac{1}{m}\int_C y\rho\, ds$, $I = \int_C w^2 \rho\, ds$.

43. Compute the mass m of a rod with density $\rho(x, y) = x$ in the shape of $y = x^2$, $0 \le x \le 3$.

44. Compute the mass m of a rod with density $\rho(x, y) = y$ in the shape of $y = 4 - x^2$, $0 \le x \le 2$.

45. Compute the center of mass $(\bar{x}, \bar{y})$ of the rod of exercise 43.

46. Compute the center of mass $(\bar{x}, \bar{y})$ of the rod of exercise 44.

47. Compute the moment of inertia I for rotating the rod of exercise 43 about the y-axis. Here, w is the distance from the point (x, y) to the y-axis.

48. Compute the moment of inertia I for rotating the rod of exercise 44 about the x-axis. Here, w is the distance from the point (x, y) to the x-axis.

56. Above the portion of $y = x^2$ from $(0, 0, 0)$ to $(2, 4, 0)$ up to the surface $z = x^2 + y^2$

57. Above the line segment from $(2, 0, 0)$ to $(-2, 0, 0)$ up to the surface $z = 4 - x^2 - y^2$

58. Above the line segment from $(1, 1, 0)$ to $(-1, 1, 0)$ up to the surface $z = \sqrt{x^2 + y^2}$

59. Above the unit square $x \in [0, 1]$, $y \in [0, 1]$ up to the plane $z = 4 - x - y$

60. Above the ellipse $x^2 + 4y^2 = 4$ up to the plane $z = 4 - x$

In exercises 61 and 62, estimate the line integrals (a) $\int_C f\, ds$, (b) $\int_C f\, dx$ and (c) $\int_C f\, dy$.

61.

(x, y)	$(0, 0)$	$(1, 0)$	$(1, 1)$	$(1.5, 1.5)$
$f(x, y)$	2	3	3.6	4.4

(x, y)	$(2, 2)$	$(3, 2)$	$(4, 1)$
$f(x, y)$	5	4	4

62.

(x, y)	$(0, 0)$	$(1, -1)$	$(2, 0)$	$(3, 1)$
$f(x, y)$	1	0	-1.2	0.4

(x, y)	$(4, 0)$	$(3, -1)$	$(2, -2)$
$f(x, y)$	1.5	2.4	2

Writing Exercises

Each section exercise set begins with writing exercises that encourage students to carefully consider important mathematical concepts and ideas in new contexts, and to express their findings in their own words. The writing exercises may also be used as springboards for class discussion.

EXERCISES 5.5

WRITING EXERCISES

1. In example 5.6, the assumption that air resistance can be ignored is obviously invalid. Discuss the validity of this assumption in examples 5.1 and 5.3.
2. In the discussion preceding example 5.3, we showed that Michael Jordan (and any other human) spends half of his airtime in the top one-fourth of the height. Compare his velocities at various points in the jump to explain why relatively more time is spent at the top than at the bottom.
3. In example 5.4, we derived separate equations for the horizontal and vertical components of position. To discover one consequence of this separation, consider the following situation. Two people are standing next to each other with arms raised to the same height. One person fires a bullet horizontally from a gun. At the same time, the other person drops a bullet. Explain why the bullets will hit the ground at the same time.
4. For the falling raindrop in example 5.6, a more accurate model would be $y''(t) = -32 + f(t)$, where $f(t)$ represents the force due to air resistance (divided by the mass). If $v(t)$ is the downward velocity of the raindrop, explain why this equation is equivalent to $v'(t) = 32 - f(t)$. Explain in physical terms why the larger $v(t)$ is, the larger $f(t)$ is. Thus, a model such as $f(t) = v(t)$ or $f(t) = [v(t)]^2$ would be reasonable. (In most situations, it turns out that $[v(t)]^2$ matches the experimental data better.)

8. The Washington Monument is 555 feet, $5\frac{1}{8}$ inches high. In a famous experiment, a baseball was dropped from the top of the monument to see if a player could catch it. How fast would the ball be going?
9. A certain not-so-wily coyote discovers that he just stepped off the edge of a cliff. Four seconds later, he hits the ground in a puff of dust. How high was the cliff?
10. A large boulder dislodged by the falling coyote in exercise 9 falls for 3 seconds before landing on the coyote. How far did the boulder fall? What was its velocity when it flattened the coyote?
11. The coyote's next scheme involves launching himself into the air with an Acme catapult. If the coyote is propelled vertically from the ground with initial velocity 64 ft/s, find an equation for the height of the coyote at any time t. Find his maximum height, the amount of time spent in the air and his velocity when he smacks back into the catapult.
12. On the rebound, the coyote in exercise 11 is propelled to a height of 256 feet. What is the initial velocity required to reach this height?
13. One of the authors has a vertical "jump" of 20 inches. What is the initial velocity required to jump this high? How does this compare to Michael Jordan's velocity, found in example 5.3?

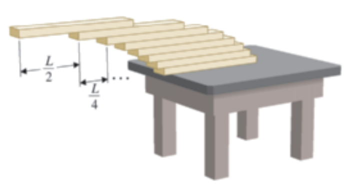
Exploratory Exercises

Each section exercise set concludes with a series of in-depth exploratory exercises designed to challenge students' knowledge.

49. For tennis rackets, a large second moment (see exercises 47 and 48) means less twisting of the racket on off-center shots. Compare the second moment of a wooden racket ($a = 9$, $b = 12$, $w = 0.5$), a midsize racket ($a = 10$, $b = 13$, $w = 0.5$) and an oversized racket ($a = 11$, $b = 14$, $w = 0.5$).
50. Let M be the second moment found in exercise 48. Show that $\frac{dM}{da} > 0$ and conclude that larger rackets have larger second moments. Also, show that $\frac{dM}{dw} > 0$ and interpret this result.

EXPLORATORY EXERCISES

1. As equipment has improved, heights cleared in the pole vault have increased. A crude estimate of the maximum pole vault possible can be derived from conservation of energy principles. Assume that the maximum speed a pole-vaulter could run carrying a long pole is 25 mph. Convert this speed to ft/s. The kinetic energy of this vaulter would be $\frac{1}{2}mv^2$. (Leave m as an unknown for the time being.) This initial kinetic energy would equal the potential energy at the top of the vault minus whatever energy is absorbed by the pole (which we will ignore). Set the potential energy, $32mh$, equal to the kinetic energy and solve for h. This represents the maximum amount the vaulter's center of mass could be raised. Add 3 feet for the height of the vaulter's center of mass and you have an estimate of the maximum vault possible. Compare this to Sergei Bubka's 1994 world record vault of $20'1\frac{3}{4}''$.
2. An object will remain on a table as long as the center of mass of the object lies over the table. For example, a board of length 1 will balance if half the board hangs over the edge of the table. Show that two homogeneous boards of length 1 will balance if $\frac{1}{4}$ of the first board hangs over the edge of the table and $\frac{1}{2}$ of the second board hangs over the edge of the first board. Show that three boards of length 1 will balance if $\frac{1}{6}$ of the first board hangs over the edge of the table, $\frac{1}{4}$ of the second board hangs over the edge of the first board and $\frac{1}{2}$ of the third board hangs over the edge of the second board. Generalize this to a procedure for balancing n boards. How many boards are needed so that the last board hangs completely over the edge of the table?



Review Exercises

Review Exercises sets are provided as an overview of the chapter and also include Writing Exercises, True or False, and Exploratory Exercises.

Review Exercises

WRITING EXERCISES

The following list includes terms that are defined and theorems that are stated in this chapter. For each term or theorem, (1) give a precise definition or statement, (2) state in general terms what it means and (3) describe the types of problems with which it is associated.

Sequence	Limit of sequence	Squeeze Theorem
Infinite series	Partial sum	Series converges
Series diverges	Geometric series	kth-term test for divergence
Harmonic series	Integral Test	
Comparison Test	Limit Comparison Test	p-Series
Conditional convergence	Absolute convergence	Alternating Series Test
Ratio Test	Root Test	Alternating harmonic series
Radius of convergence	Taylor series	Power series
Taylor's Theorem	Fourier series	Taylor polynomial

10. A series with all negative terms cannot be conditionally convergent.

11. If $\sum_{k=1}^{\infty} |a_k|$ diverges, then $\sum_{k=1}^{\infty} a_k$ diverges.

12. A series may be integrated term-by-term and the interval of convergence will remain the same.

13. A Taylor series of a function f is simply a power series representation of f.

14. The more terms in a Taylor polynomial, the better the approximation.

15. The Fourier series of x^2 converges to x^2 for all x.

81. $f(x) = \begin{cases} -1 & \text{if } -1 < x \le 0 \\ 1 & \text{if } 0 < x \le 1 \end{cases}$

82. $f(x) = \begin{cases} 0 & \text{if } -2 < x \le 0 \\ x & \text{if } 0 < x \le 2 \end{cases}$

83. Suppose you and your friend take turns tossing a coin. The first one to get a head wins. Obviously, the person who goes first has an advantage, but how much of an advantage is it? If you go first, the probability that you win on your first toss is $\frac{1}{2}$, the probability that you win on your second toss is $\frac{1}{8}$, the probability that you win on your third toss is $\frac{1}{32}$ and so on. Sum a geometric series to find the probability that you win.

84. In a game similar to that of exercise 83, the first one to roll a 4 on a six-sided die wins. Is this game more fair than the previous game? The probabilities of winning on the first, second and third roll are $\frac{1}{6}$, $\frac{25}{216}$ and $\frac{625}{7776}$, respectively. Sum a geometric series to find the probability that you win.

85. Recall the Fibonacci sequence defined by $a_0 = 1$, $a_1 = 1, a_2 = 2$ and $a_{n+1} = a_n + a_{n-1}$. Prove the following fact: $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{1+\sqrt{5}}{2}$. (This number, known to the

other occurrence of the golden ratio. Viscount Brouncker, a seventeenth-century English mathematician, showed that the sequence $1 + \frac{1^2}{2}, 1 + \frac{1^2}{2+\frac{3^2}{2}}, 1 + \frac{1^2}{2+\frac{3^2}{2+\frac{5^2}{2}}}$ and so on, converges to $\frac{4}{\pi}$. (See *A History of Pi* by Petr Beckmann.) Explore this sequence numerically.

88. For the power series $\frac{1}{1-x-x^2} = c_1 + c_2x + c_3x^2 + \cdots$, show that the constants c_i are the Fibonacci numbers. Substitute $x = \frac{1}{1000}$ to find the interesting decimal representation for $\frac{1,000,000}{998,999}$.

EXPLORATORY EXERCISES

1. The challenge here is to determine $\sum_{k=1}^{\infty} \frac{x^k}{k(k+1)}$ as completely as possible. Start by finding the interval of convergence. Find the sum for the special cases (a) $x = 0$ and (b) $x = 1$. For $0 < x < 1$, do the following: (c) Rewrite the series using the

Applications Index

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