1a. We can use Mathematica to apply the curve-sketching techniques of this chapter to complicated functions such as $f(x)=\left(5-2 x^{3}\right) \sin x+5^{-x^{2}}$. Execute

$$
f\left[x_{-}\right]=\left(5-2 x^{\wedge} 3\right) * \sin [x]+5^{\wedge}\left(-x^{\wedge} 2\right)
$$

and then use the Plot command to draw the graph of $f$ over the interval $-5 \leq x \leq 5$; although this function displays interesting behavior throughout the $x y$-plane, in this assignment we will restrict ourselves to this interval. Sketch the result on the axes at right.

1b. Based on this graph, tell how many local maxima, local minima and inflection points $f$ appears to have over $-5 \leq x \leq 5$.


2a. It is not possible to solve the equation $f^{\prime}(x)=0$ for $x$ algebraically. However, we can use a graph of $f^{\prime}$ together with numerical equation solving to find the zeros of $f^{\prime}$. Execute

Plot[f'[x], \{x, -5, 5\}]
and sketch the result on the axes at right.
2b. According to this graph, how many zeros
 does $f^{\prime}$ have? Is this consistent with the number of local extrema you found in Question 1b? Record below the approximate values of the zeros of $f^{\prime}$.

2c. Execute FindRoot [f'[x], \{x, -2.1\}] to find the exact value of the zero of $f^{\prime}$ near $x=-2.1$, and record the result below; repeat using each of your approximate values in part $\mathbf{b}$ as starting values for FindRoot.

2d. Using these results, record below the complete set of intervals on which $f$ is increasing and decreasing. (Remember that we are considering only $-5 \leq x \leq 5$.)

3a. We can study the concavity of the graph of $f$ in the same way. Execute

$$
\text { Plot[f'י[x], }\{x,-5,5\}]
$$

and sketch the result on the axes at right.
3b. Because it is unclear from this graph how many zeros $f^{\prime \prime}$ has, execute
to get a closer look at the graph of $f^{\prime \prime}$ near the origin. Sketch the result on the axes at right.

3c. Altogether, how many zeros does $f^{\prime \prime}$ seem to have over $-5 \leq x \leq 5$ ? Tell roughly


```
Plot[f''[x], {x, -2, 1}]
```

```
Plot[f''[x], {x, -2, 1}]
``` where they are.


3d. Use FindRoot as you did in Question 2c to find the exact values of the zeros of \(f^{\prime \prime}\), and record the results below.

3e. Using these results, record below the complete set of intervals on which the graph of \(f\) is concave up and concave down over \(-5 \leq x \leq 5\).```

