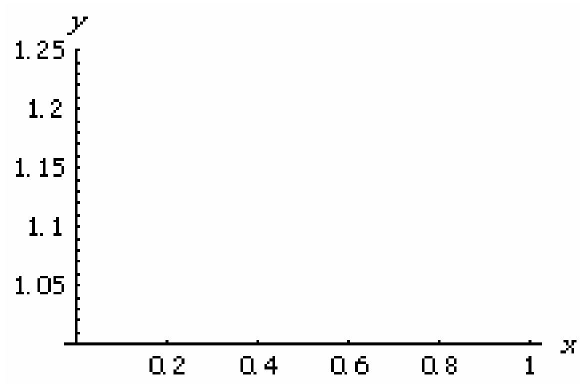
Assignment 13: Numerical Integration (4.7) **Name____________________**
Please provide a handwritten response.

1. **A**pply the estimation rules of this section to $\int_0^1 \sqrt[3]{x^2+1}\,dx$. Execute

```
f[x_] = (x^2 + 1)^(1/3)
```

to define $f(x) = \sqrt[3]{x^2+1}$ and then use the `Plot` command to graph *f* over $0 \le x \le 1$; sketch the result on the axes at right. Based on this graph, what would be your estimate of $\int_0^1 \sqrt[3]{x^2+1}\,dx$? (Be careful about where the origin is!)



2a. To apply the Midpoint Rule to this integral we must first define $x_i = a + i\Delta x$ just as in the preceding assignment. Execute in order the commands `a = 0.0` , `b = 1` , `n = 10` , `deltax = (b - a)/n` and `x[i_] = a + i*deltax` ; record below the result for x_i .

2b. The midpoint of each interval $[x_{i-1}, x_i]$ is given by $c_i = \frac{x_{i-1} + x_i}{2}$; execute

```
c[i_] = (x[i - 1] + x[i])/2
```

and then obtain the Midpoint approximation $\sum_{i=1}^{n} f(c_i)\Delta x$ by executing

```
mr = Sum[f[c[i]]*deltax, {i, 1, n}]
```

Is this result plausible? Enter it in the table on the next page.

3. To calculate the Trapezoid Rule approximation $\sum_{i=1}^{n} \frac{f(x_{i-1}) + f(x_i)}{2}\Delta x$, execute

```
tr = Sum[((f[x[i - 1]] + f[x[i]])/2)*deltax, {i, 1, n}]
```

and enter the result in the table.

4. According to the formula in Exercise 52, we can execute `tr/3 + 2mr/3` to calculate the Simpson's Rule approximation. Enter the result in the table.

n	Midpoint	Trapezoid	Simpson's
10			
20			
50			

5. Execute `Clear[a, b, n, deltax, x, mr, tr]`, replace `n = 10` with `n = 20` and re–execute all of the commands in Questions **2a**–**4** in order. Enter the results in the table. Which of the three approximations did not change when n was increased? Why?

6. Repeat Question **5** but with `n = 50` instead, and enter the results in the table. Are the three approximations drawing closer together as n increases?

7. As Remark 7.3 suggests, *Mathematica* has a sophisticated command called `NIntegrate` that accurately calculates difficult definite integrals like the one we are studying. Execute the command

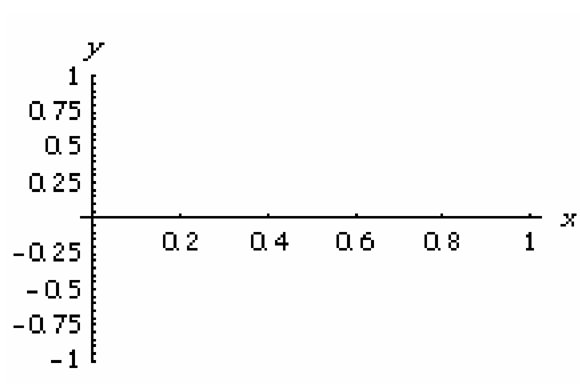
```
NIntegrate[f[x], {x, 0, 1}]
```

and record the result below. Based on this, which of the three approximation methods applied above was most accurate?

8a. You can almost always take the results of `NIntegrate` to be completely accurate. However, there are some unusual situations that cause trouble even for `NIntegrate`. Execute

```
g[x_] = Sin[1/x]
```

to define $g(x) = \sin\frac{1}{x}$ and then use the `Plot` command to draw the graph of g over $[0,1]$, and sketch the result (as best you can!) on the axes at right.



8b. Execute `NIntegrate[g[x], {x, .001, 1}]` to calculate $\int_{0.001}^{1} g(x)\,dx$ and describe what happens below. Do you think the numerical result given is trustworthy?