Assignment 14: Solids of Revolution (5.1–4) **Name____________________**
Please provide a handwritten response.

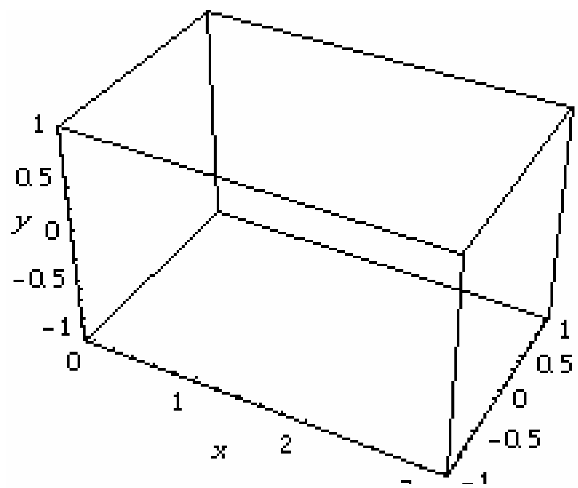
1. *Mathematica* can draw solids of revolution provided that we load in a package; execute

```
Needs["Graphics`SurfaceOfRevolution`"]
```

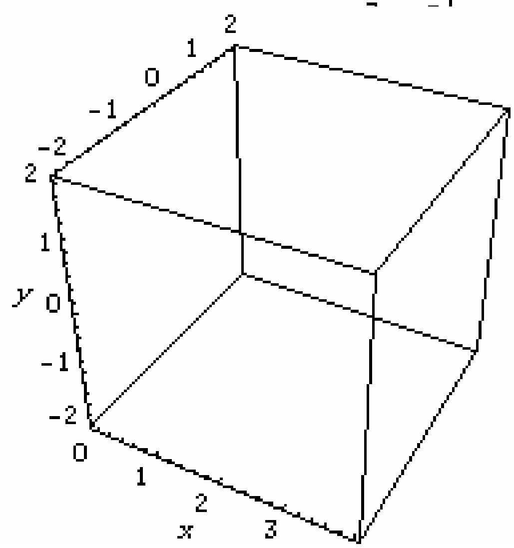
followed by `f[x_] = Sin[x]` and then

```
SurfaceOfRevolution[f[x], {x, 0, Pi}, RevolutionAxis->{1,0}]
```

to draw the solid formed when the graph of $f(x) = \sin x$ over $0 \le x \le 2\pi$ is revolved about the *x*–axis. Sketch the result in the box at right; one easy way to do this is to copy just the "mesh lines" forming the surface. (The `RevolutionAxis` option is needed here to specify the *x*–axis as the axis of revolution.)



2. As another example execute `Clear[f]` followed by `f[x_] = Sqrt[x]` , and then re–execute the `SurfaceOfRevolution` command in Question **1** with `Pi` replaced with `4` to draw the solid formed when the graph of $f(x) = \sqrt{x}$ over $0 \le x \le 4$ is revolved about the *x*–axis. Sketch the result in the box at right.



We can think of solids of revolution (like the ones you just drew) as using a certain amount of surface area to enclose a certain amount of volume. This leads to the question of what function *f* over what interval leads to a solid of revolution enclosing as much volume *V* as possible while using as little surface area *S* as possible. We can make this precise by studying the ratio $V / S^{3/2}$; it turns out that this fraction never goes above a certain limit, regardless of the function *f* that we use. Our goal is to use *Mathematica* to experiment with various possibilities for *f* to see how much volume a solid of revolution can enclose using a certain amount of area.

3a. To be specific, we assume that $f(x) \geq 0$ over $a \leq x \leq b$ and that the graph of f over this interval is being revolved about the x–axis to form a solid of revolution. Since the method of disks applies here, the volume is $\int_a^b \pi [f(x)]^2 dx$. Execute `a = 0` , `b = 4` and

```
v = NIntegrate[Pi*f[x]^2, {x, a, b}]
```

to find the volume of the solid in Question **2**, and record the result in the table below.

$f(x)$	$[a,b]$	V	S	$V / S^{3/2}$
$\sqrt{x}$	$[0,4]$			
$\sin x$	$[0,\pi]$			
$4-x^2$	$[-2,2]$			

3b. To find the total surface area for a given solid, we must include not only the "side" surface area given by $\int_a^b 2\pi f(x)\sqrt{1+[f'(x)]^2}\,dx$, but also the disks, if any, on the ends of the solids, whose areas are $\pi[f(a)]^2$ and $\pi[f(b)]^2$. (For instance, the solid in Question **2** has a disk with area $\pi[\sqrt{4}]^2 = 4\pi$ on the end at $x = 4$.) Execute

```
s = NIntegrate[2Pi*f[x]Sqrt[1+f'[x]^2],{x,a,b}] + Pi*f[a]^2 + Pi*f[b]^2
```

and record the result in the table.

3c. Next execute `v/(s^(3/2))` to find $V / S^{3/2}$ and enter the result in the table.

4. Now execute `Clear[f, a, b, v, s]` followed by `f[x_] = Sin[x]` , `a = 0` and `b = Pi` once again, and then re–execute the three commands above giving V , S and $V / S^{3/2}$. Record the results in the table. So far, which function gives the larger ratio?

5. Again execute `Clear[f, a, b, v, s]` followed by `f[x_] = 4 - x^2` , `a = -2` and `b = 2` , and repeat the procedure in Question **5**; enter the results in the table. (You can also sketch the solid as in Question **1**, to see what you're measuring.)

6. Now invent some functions of your own that you think might be strong contenders, run them as above and enter the results in the table. What do you think the maximum possible value of $V / S^{3/2}$ is, and what shape of curve gives it?