Assignment 16: Integration Techniques (6.1-6) Please provide a handwritten response.

Name_____

1a. The text notes that using identities we can often show that two different–looking results for an integral are both correct. Evaluate $\int \cos^3 x \sin^2 x \, dx$ by hand and record the result below.

1b. Evaluate this integral in *Mathematica* by executing

Integrate[Cos[x]^3*Sin[x]^2, x]

and record the result below. Does it look the same as your answer in part **a**?

1c. The **TrigReduce** command applies identities to change the form of trigonometric expressions; execute

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TrigReduce[(1/3)Sin[x]^3 - (1/5)Sin[x]^5]
```

to transform your result in part **a** and record the result below. Was *Mathematica*'s result correct after all?

2a. As Exercise 30, Section 6.5 suggests, multiplication in *Mathematica* can be denoted using simply a space rather than the multiplication operator *; to find $\int x \sin x \, dx$ execute both

Integrate[x*Sin[x], x]

and

Integrate[x Sin[x], x]

and record the result below; was there any difference between the two?

2b. Now repeat the last command without the space between \mathbf{x} and $\mathbf{Sin}[\mathbf{x}]$, and record the result below. What does this result mean?

3a. The inverse tangent function is denoted in Mathematica by ArcTan; execute

Integrate[Exp[x]*ArcTan[Exp[x]], x]

to evaluate the integral $\int e^x \tan^{-1} e^x dx$ of Exercise 27, Section 6.5 and record the result below.

3b. The \$ symbol (found above the "5" on your keyboard) refers in *Mathematica* to the <u>im-</u><u>mediately preceding output</u>, and is useful provided you don't lose track of what the last output was! Execute \$ and compare it to your answer in part **a**.

3c. We can differentiate *Mathematica*'s result in part **a** using the **D** command introduced earlier; execute **D**[%, **x**] and record the result below. Was *Mathematica*'s integral correct?

4a. Exercise 32, Section 6.5 investigates *Mathematica*'s ability to evaluate $\int x^3 e^{5x} \cos 3x \, dx$;

execute

Integrate[x^3*Exp[5x]*Cos[3x], x]

and record below just the denominator of the leading fraction in Mathematica's result.

4b. Now check your result by executing **D**[%, **x**] as in Question **2c**; is your answer surprising? Do you think that *Mathematica* has made a mistake somewhere?

4c. Bearing in mind that % now refers to the output you just obtained, execute **Simplify**[%] and record the result below. What lesson should we learn here?

5a. The Apart command performs partial fraction decompositions. Begin Exercise 34, Section 6.5 by executing

Apart[$(x^2 + 2x - 1)/(((x - 1)^2)*(x^2 + 4))$]

and recording the result below. Check your result by executing **Together**[%]; does everything look correct?

5b. Use the **Integrate** command to find an antiderivative of the expression in part **a**, and record the result below.

5c. Now proceed according to Question **4b**,**c** to check *Mathematica*'s result. Does it appear to be correct at first? At last?

6. Go through the three steps in Question **4** for the integral in Exercise 6, Section 6.5. Are you able to confirm that *Mathematica*'s antiderivative is correct? Explain.