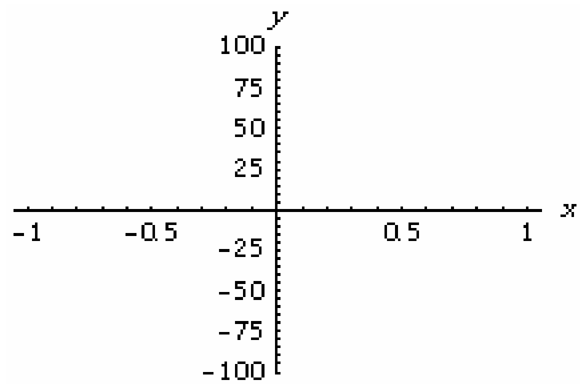
Assignment 17: Improper Integrals (6.6) **Name___________________**
Please provide a handwritten response.

1a. The integrals $\int_{-1}^{1} \frac{1}{x}\,dx$ and $\int_{-1}^{1} \frac{1}{x^2}\,dx$ are both improper and divergent. Execute

```
Plot[{1/x, 1/x^2}, {x, -1, 1}, PlotRange->{-100, 100}]
```

to sketch the functions $y = \frac{1}{x}$ and $y = \frac{1}{x^2}$ over $-1 \le x \le 1$ and sketch the results on the axes at right, labeling the graphs.



1b. To try to evaluate $\int_{-1}^{1} \frac{1}{x}\,dx$, execute

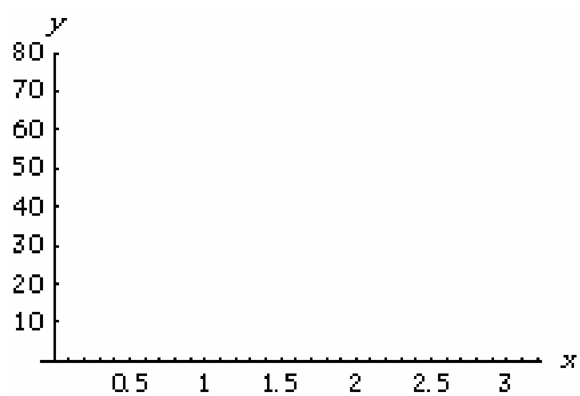
```
Integrate[1/x, {x, -1, 1}]
```

and record the result below. Does *Mathematica* give a value for this integral?

1c. Likewise evaluate $\int_{-1}^{1} \frac{1}{x^2}\,dx$ by executing `Integrate[1/x^2, {x, -1, 1}]` , and record the result below.

1d. Does *Mathematica* confirm that each of these integrals is divergent? Explain carefully below why *Mathematica* nevertheless gives very different results for them.

2a. Use the `Plot` command to sketch the graph of $f(x) = \frac{1}{\sqrt{1+\cos x}}$ over $0 \le x \le \pi$ on the axes at right, and explain why the integral $\int_{0}^{\pi} \frac{1}{\sqrt{1+\cos x}}\,dx$ is improper.



2b. Execute the command

```
Integrate[1/Sqrt[1 + Cos[x]], {x, 0, Pi}]
```

and record the result below; does this integral converge?

2c. Repeat part **a** but with the command

```
Integrate[1/(1 + Cos[x])^(.5), {x, 0, Pi}]
```

and record the result below; does this integral converge?

2d. Repeat part **a** but with the command

```
Integrate[1/(1 + Cos[x])^(1/2), {x, 0, Pi}]
```

and record the result below; so, does this integral converge?

2e. Apparently improper integrals can be difficult even for *Mathematica*! To decide which result is correct, note that when $\frac{\pi}{2} < x < \pi$, $-1 < \cos x < 0$, so that $1 < \sqrt{1-\cos x} < \sqrt{2}$, or $\frac{1}{\sqrt{2}} < \frac{1}{\sqrt{1-\cos x}} < 1$. For such x, therefore,

$$\frac{1}{\sqrt{1+\cos x}} > \frac{1}{\sqrt{1+\cos x}}\frac{1}{\sqrt{1-\cos x}} = \frac{1}{\sin x} = \csc x\ ,$$

so that if $\int_0^{\pi} \csc x\, dx$ diverges, then our integral diverges by comparison. Does it? Why? (Note that $\csc x$ is represented in *Mathematica* by `Csc[x]`.)

3a. To evaluate the integral $\int_0^{\infty} xe^{-2x}\,dx$ execute

```
Integrate[x*Exp[-2x], {x, 0, Infinity}]
```

and record the result below.

3b. In the same way evaluate $\int_0^{\infty} x^2e^{-2x}\,dx$ and explain how these two answers could be the same.