Assignment 24: Vectors (10.1–5) **Name____________________**
Please provide a handwritten response.

1a. In *Mathematica* vectors (like many other things) are represented as lists; for example, execute `a = {3, -2}` and `b = {4, 1}` to represent the vectors $\mathbf{a} = \langle 3, -2 \rangle$ and $\mathbf{b} = \langle 4, 1 \rangle$. Then execute `5a - 3b` to calculate $5\mathbf{a} - 3\mathbf{b}$ and record the result below; is it correct?

1b. Execute `a.b` to calculate the dot product $\mathbf{a} \cdot \mathbf{b}$ of **a** and **b** and record the result below. (The `.` character is just a plain old period.) Is it correct?

1c. In *Mathematica* we find the magnitude of the vector $\|\mathbf{a}\|$ using the formula $\|\mathbf{a}\| = \sqrt{\mathbf{a} \cdot \mathbf{a}}$; execute `Sqrt[a.a]` and record the result below; is it correct?

2a. Similarly, three–dimensional vectors are represented by lists of length three; execute `c = {4, -1, 7}` and `d = {3, 3, -5}` to represent the vectors $\mathbf{c} = \langle 4, -1, 7 \rangle$ and $\mathbf{d} = \langle 3, 3, -5 \rangle$. Then execute `4c + d` and `(c + 2d).c` and record the results below.

2b. Execute `a.c` and describe the result below; what do you think is the problem?

2c. The cross product is represented in *Mathematica* by the `Cross` command; execute `Cross[c, d]` and record the result below. Are `c` and `d` parallel?

3a. To use *Mathematica* to assist with Exercise 16, Section 10.5 clear all variables used so far and execute `a = {-1, 0, 2}` and `b = {2, -3, 1}` to define vectors parallel to the two lines. Then find the dot and cross products of `a` and `b`; are the lines parallel, perpendicular or neither?

3b. Solve Exercise 12, Section 10.5. Note that in *Mathematica* $\cos^{-1} x$ is denoted `ArcCos[x]` and expresses the answer in radians and in exact form; executing `N[%]/Degree` will convert the output to degree measure.

4a. Once again clear your variables and define **`a`** and **`b`** to be vectors parallel to the two lines in Exercise 18, Section 10.5 and use the cross product to show that the two lines are not parallel. Thus, they either intersect or are skew. Record the cross product below.

4b. The **`Solve`** command can detect whether the lines meet, because this is equivalent to finding whether a system of three linear equations in the two unknowns *s* and *t* has a solution. Execute

```
Solve[{3 + t, 3 + 3t, 4 - t} == {2 - s, 1 - 2s, 6 + 2s}, {s,
t}]
```

and record the result below. To what point in space do these values of *s* and *t* correspond?

4c. Carry out Exercise 20, Section 10.5 and record your conclusions below.

5a. The **`Solve`** command is also very useful for finding parametric equations for the line of intersection, if any, of two given planes; here the situation is reversed and we are solving a system of two linear equations in the three unknowns *x* , *y* and *z* . To carry out Exercise 42, Section 10.5 execute

```
Solve[{3x + y - z == 2, 2x - 3y + z == -1}, {x, y, z}]
```

and record the result below as a set of parametric equations.

5b. Carry out Exercise 44, Section 10.5 and record your conclusions below.

6a. Confirm that the lines in Exercise 53, Section 10.5 intersect, and find the point of intersection.

6b. Use the cross product to find a normal vector for the desired plane.

6c. Write down an equation for the desired plane.