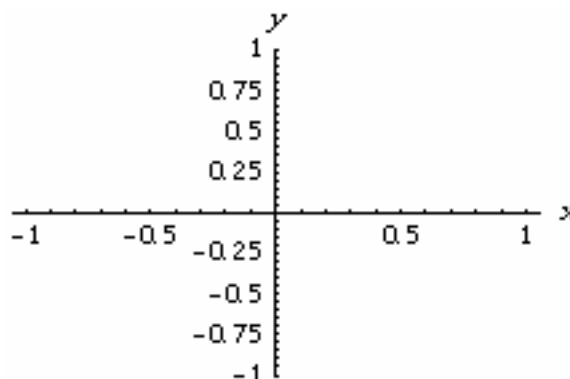


**Assignment 25: Vector-Valued Functions, Part I (11.1–3) Name \_\_\_\_\_**  
**Please provide a handwritten response.**

**1a.** Execute  $\mathbf{r}[t\_]=\{\text{Cos}[3t], \text{Sin}[2t]\}$  to define the vector-valued function  $\mathbf{r}(t)=\langle \cos 3t, \sin 2t \rangle$ ,  $0 \leq t \leq 2\pi$  and draw the graph of  $\mathbf{r}(t)$  by executing

```
ParametricPlot[Evaluate[r[t]], {t, 0, 2Pi}, AspectRatio->Automatic]
```

(The **Evaluate** command is needed here because of a technicality in *Mathematica*; try it without if you're curious.) Sketch the resulting "Lissajous curve" on the axes at right.



**1b.** To list the points  $\mathbf{r}(0)$ ,  $\mathbf{r}\left(\frac{\pi}{4}\right)$ , etc. execute

```
Table[r[n Pi/4], {n, 0, 8}]
```

Mark these coordinates, with their corresponding values of  $t$ , on the graph, and then draw arrows to show the orientation of the curve.

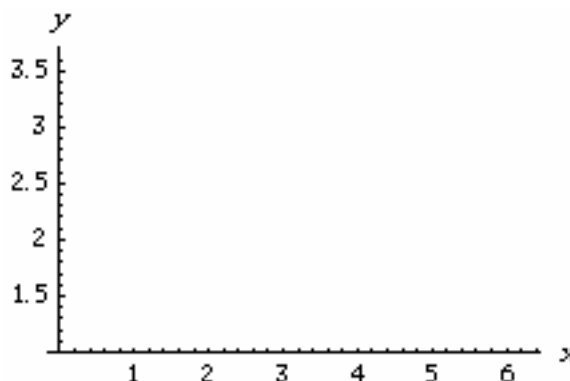
**1c.** Thinking of  $\mathbf{r}(t)$  as representing the position of a moving point, execute

```
v[t_] = r'[t]
```

to find the velocity vector  $\mathbf{v}(t)=\mathbf{r}'(t)$ , followed by

```
speed[t_] = Sqrt[v[t].v[t]]
```

to find the speed  $\|\mathbf{v}(t)\|=\sqrt{\mathbf{v}(t)\cdot\mathbf{v}(t)}$ . Sketch the graph of  $\|\mathbf{v}(t)\|$  over  $0 \leq t \leq 2\pi$  on the axes at right; based on this graph, does the moving point ever stop?



**1d.** Now execute  $\mathbf{r}_1[t\_]=\mathbf{r}[t+3\text{Sin}[t]]$  to define the reparameterization  $\mathbf{r}_1(t)=\mathbf{r}(t+3\sin t)$ ,  $0 \leq t \leq 2\pi$  and modify the **ParametricPlot** command in part **a** to draw the graph of  $\mathbf{r}_1(t)$  over  $0 \leq t \leq 2\pi$ . Why does this graph look "darker" in some places than that in part **a**?

**1e.** Clearing variables as needed, imitate part **c** to plot over  $0 \leq t \leq 2\pi$  the speed of a point moving under  $\mathbf{r}_1(t)$ , note the approximate values of  $t$  where the speed is zero, and apply **FindRoot** to **speed[t]** to find more accurate values. Then use **r1[t]** to find the coordinates of these points where  $\mathbf{r}_1(t)$  “stops”, and record the results below.

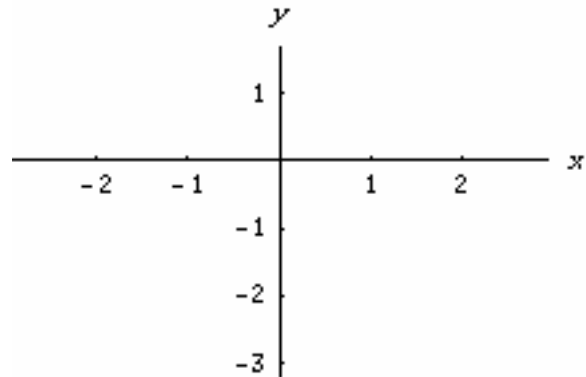
**1f.** Suppose you knew only the graph of a vector-valued function; so far, can we say for sure whether there are any points at which the function “stops”?

**1g.** Finding the points where this curve crosses itself, which amounts to finding pairs of numbers  $s$  and  $t$  such that  $\mathbf{r}(s) = \mathbf{r}(t)$ , would be difficult symbolically but is easy using **FindRoot** provided suitable starting values for  $s$  and  $t$  are given. For example, execute

**FindRoot[r[s] == r[t], {s, 0.6}, {t, 4}]**

to start near  $s = 0.6$  and  $t = 4$ , followed by **r[s] /. %**; record the result below. Now modify the starting values to find the exact coordinates of the point in the first quadrant where the curve crosses itself, and record the result below.

**2a.** Clear variables and redefine **r[t]** as  $\mathbf{r}(t) = \langle 2 \cos t + \sin 2t, 2 \sin t + \cos 2t \rangle$ ,  $0 \leq t \leq 2\pi$  and sketch the graph of  $\mathbf{r}(t)$  on the axes at right.



**2b.** Find and mark on the graph any stationary points of  $\mathbf{r}(t)$ , as above.

**2c.** Execute **r1[t\_] = r[t + Sin[t]]** to define the reparameterization  $\mathbf{r}_1(t) = \mathbf{r}(t + \sin t)$ ,  $0 \leq t \leq 2\pi$  and draw the graph of  $\mathbf{r}_1(t)$  over  $0 \leq t \leq 2\pi$ . Check for stationary points; how do the results compare with part **b**?

**2d.** Clearing variables as needed, repeat part **c** with  $\mathbf{r}_1(t)$  redefined by  $\mathbf{r}_1(t) = \mathbf{r}(t^2)$ ,  $0 \leq t \leq \sqrt{2\pi}$ .

**2e.** Would you now modify your answer to Question **1f**? How?