

Assignment 27: Functions of Two Variables (12.1–2)
Please provide a handwritten response.

Name _____

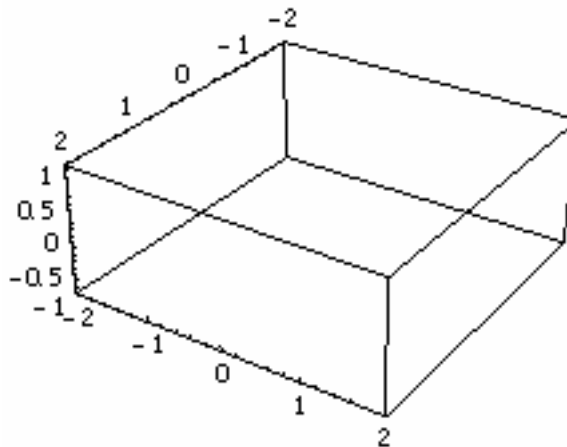
1a. To graph the function $f(x,y) = \sin(y - x^2)$ in Exercise 46, Section 12.1 execute

$$f[x_, y_] = \text{Sin}[y - x^2]$$

followed by

```
Plot3D[f[x, y], {x, -2, 2}, {y, -2, 2}, ViewPoint->{3, 2, 2}]
```

Sketch the result in the box at right; rather than try to copy every line drawn by *Mathematica*, just use general outlines and shading to give the overall shape.



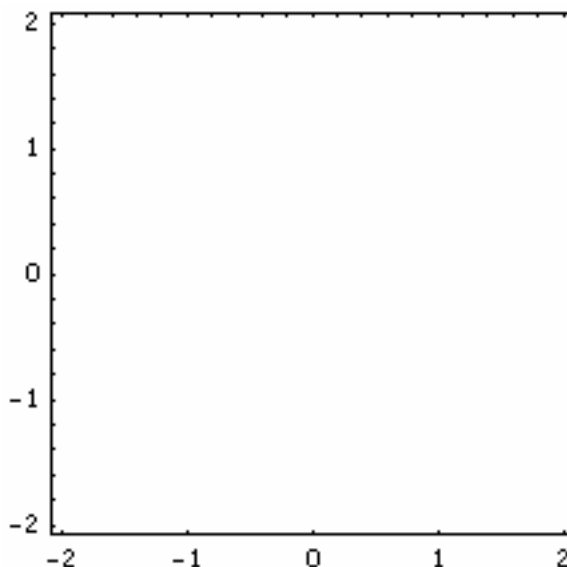
1b. Graph f over a wider range and describe the general appearance of the resulting surface.

1c. To draw a contour plot of f execute

```
ContourPlot[f[x, y], {x, -2, 2}, {y, -2, 2}]
```

What do we know about the level curves that makes the result look suspicious?

1d. Execute the preceding command with **PlotPoints** set to 25, and sketch the result in the frame at right. Is this graph more credible?



1e. Now execute the command from part **d** with **ContourPlot** replaced by **DensityPlot**; how is the result both more, and less, accurate than the preceding result?

2a. According to Exploratory Exercise 1, Section 12.2 the facts that $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^2 + y^2} = 0$ and that

$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2}{x^2 + y^2}$ does not exist can be detected using contour plots. Clear **f** and execute

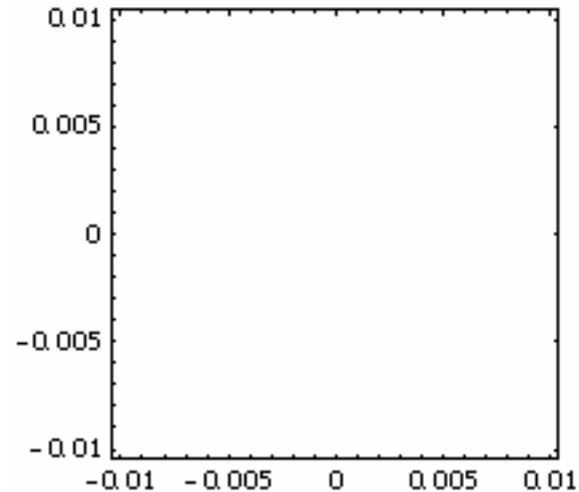
$$f[x_, y_] = x^2 y / (x^2 + y^2)$$

followed by

`ContourPlot[f[x, y], {x, -.01, .01}, {y, -.01, .01}, PlotPoints->50]`

and sketch the result in the frame at right.

Execute this command again with `.01` replaced throughout by `.001`; does the pattern seem to change?



2b. How do these graphs support the

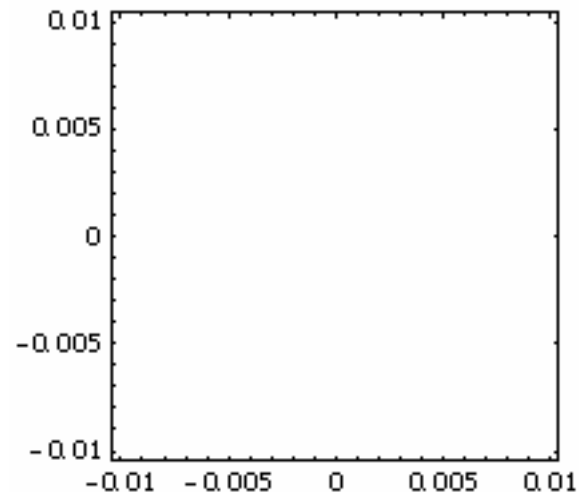
conclusion that $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^2 + y^2}$ exists?

2c. Now clear **f** and execute

$$f[x_, y_] = x^2 / (x^2 + y^2)$$

followed by the `ContourPlot` command in

part **a**, and sketch the result in the frame at right. Again, repeat this command with `.01` replaced throughout by `.001`; does the pattern seem to change here?



2d. How do these graphs support the

conclusion that $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2}{x^2 + y^2}$ does not exist?

2e. Based on contour plots, do you think that $\lim_{(x,y) \rightarrow (0,0)} \frac{x \sin y}{x^2 + y^2}$ exists? Explain your answer.