Assignment 6: Limits, Part II (1.5) **Name**_____
Please provide a handwritten response.

1a. The **`Limit`** command can be used even when the answer is $\pm\infty$. Example 5.1 explains that whereas $\lim_{x\to 0}\frac{1}{x}$ does not exist, it is nonetheless true that $\lim_{x\to 0^+}\frac{1}{x}=\infty$ and that $\lim_{x\to 0^-}\frac{1}{x}=-\infty$.

Execute the command **`Limit[1/x, x->0, Direction->-1]`** to find $\lim_{x\to 0^+}\frac{1}{x}$ and record the result below. Is *Mathematica*'s result correct?

1b. Likewise execute the command **`Limit[1/x, x->0, Direction->1]`** to find $\lim_{x\to 0^-}\frac{1}{x}$ and record the result below. Is *Mathematica*'s result again correct?

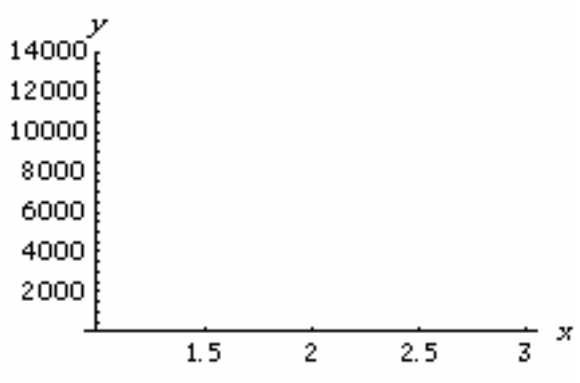
2a. Now find the value of $\lim_{x\to 2^+}\frac{4-x}{(x-2)^2}$. First execute the command

```
f[x_] = (4 - x)/(x - 2)^2
```

and then the command

```
Plot[f[x], {x, 1, 3}]
```

to see the graph near $x=2$. Sketch the result on the axes at right.



2b. Based on this graph, what do you think is the value of $\lim_{x\to 2^+}\frac{4-x}{(x-2)^2}$?

2c. Based on this graph, do you think that $\lim_{x\to 2}\frac{4-x}{(x-2)^2}$ exists? If so, then what is its value?

2d. Execute the command **`Limit[f[x], x->2, Direction->-1]`** to find $\lim_{x\to 2^+}\frac{4-x}{(x-2)^2}$, and record the result below. Does *Mathematica*'s result appear to be correct?

3. The **Limit** command can also be used when $x \to \infty$ or $x \to -\infty$; in this case we refer to ∞ as **Infinity** . For instance, to find the limit in Example 5.7, execute the command

```
Limit[(5x - 7)/(4x + 3), x->Infinity]
```

and record the result below. Is this answer correct?

4a. Now find the value of $\lim_{x \to -\infty} \frac{x + \cos x}{3x + 2}$.

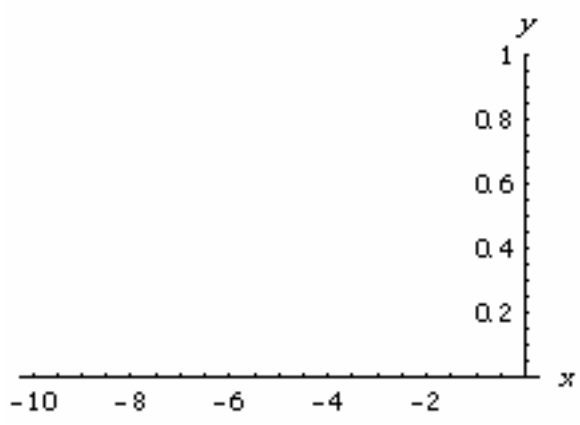
First execute the command

```
g[x_] = (x + Cos[x])/(3x + 2)
```

and then the command

```
Plot[g[x], {x, -10, 0}]
```

to how the graph looks when x is large and negative. Sketch the result on the axes at right.

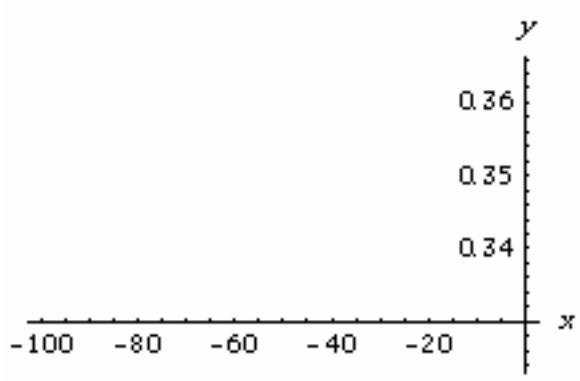


4b. Based on this graph, how accurately can you tell the value of $\lim_{x \to -\infty} \frac{x + \cos x}{3x + 2}$? What do you think it is?

4c. Zoom out further by executing

```
Plot[g[x], {x, -100, 10}]
```

Sketch the result on the axes at right. Can you now be more specific about the value of $\lim_{x \to -\infty} \frac{x + \cos x}{3x + 2}$? Why was the graph in part **a** so much smoother than this one?



4d. Try executing **Limit[g[x], x->-Infinity]** to find our limit; is the result surprising? This is *Mathematica*'s way of saying "I don't know"! Sometimes we can "help" *Mathematica* by putting the question differently. Execute **Clear[g]** and then write **g** in a different way by executing

```
g[x_] = x/(3x + 2) + Cos[x]/(3x + 2)
```

Now execute **Limit[g[x], x->-Infinity]** again. Is the result correct this time? In the same way use *Mathematica* to find $\lim_{x \to -\infty} \frac{x^{3/2} + \sin x}{\left(x^2 + 4\right)}$ and record the result below.