Assignment 8: Derivatives of Explicit Functions (2.1–9) **Name____________________**
Please provide a handwritten response.

1a. In Example 2.2 the derivative of the function $f(x) = 3x^3 + 2x - 1$ was found to be $f'(x) = 9x^2 + 2$. To carry out this calculation in *Mathematica*, first execute the command

```
f[x_] = 3x^3 + 2x - 1
```

followed by the command `f'[x]` . Record the result below; did *Mathematica* find the derivative correctly?

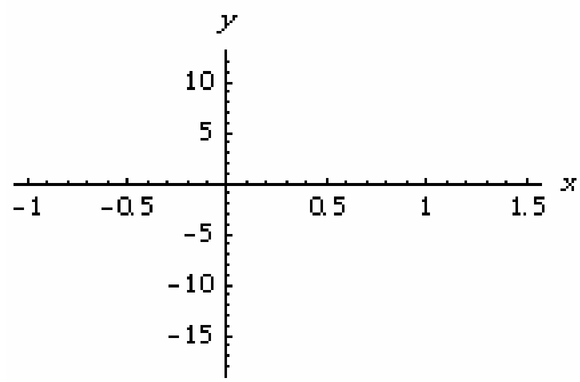
1b. The slope $m_{\tan}$ of the line tangent to the graph of *f* at, say, $x = 1$ is given by $f'(1)$; execute `f'[1]` to see that $m_{\tan} = 11$ in this case. Also execute `f[1]` to see that $y = 4$ when $x = 1$. The equation of our tangent line is therefore $y = 11(x-1) + 4 = 11x - 7$; execute

```
t[x_] = 11x - 7
```

followed by

```
Plot[{f[x], t[x]}, {x, -1,
        1.5}]
```

to graph both *f* and the tangent line together. Sketch the result on the axes at right. Does the tangent line really look as though its slope is 11 ? Why?



2a. To find the second derivative f'' of *f*, execute the command `f''[x]` . (The double–prime symbol `''` consists of the single–quote (or apostrophe) twice, not the double–quote once!) Record the result below; is it correct?

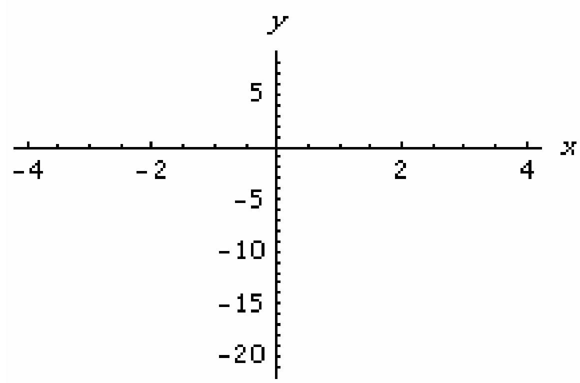
2b. Next execute the commands `g[x_] = Sin[2x/(x + 1)]` followed by `g''[x]` , and record the result below; would you care to work this out by hand?!

3a. Execute `Clear[f]` and `f[x_] = x^2*Exp[Sin[x]]` to define the function $f(x) = x^2 e^{\sin x}$, followed by `f'[x]`. Record the result below; what rules and formulas presented in this chapter did *Mathematica* need to find $f'(x)$?

3b. Execute the command

```
Plot[f'[x], {x, -4, 4}]
```

to plot the <u>derivative</u> $f'(x)$ over $-4 \le x \le 4$; sketch the result on the axes at right.



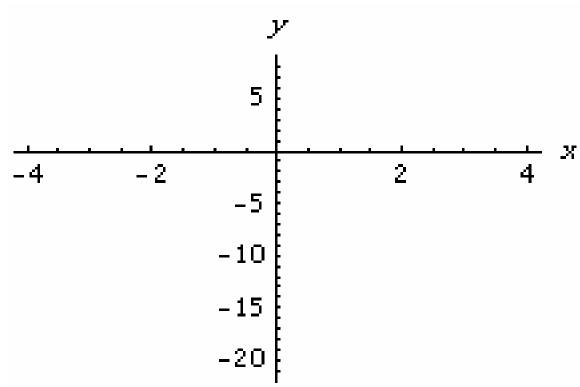
3c. According to the definition of derivative, if *h* is a small fixed number, then the difference quotient $\dfrac{f(x+h) - f(x)}{h}$ should be close to $f'(x)$, and so their graphs should lie close together. For the moment let's choose $h = 0.5$; execute

```
r[x_] = (f[x + 0.5] - f[x])/0.5
```

to define $r(x)$ as the quotient $\dfrac{f(x+0.5) - f(x)}{0.5}$, and then execute

```
Plot[{f'[x], r[x]}, {x, -4, 4}]
```

to graph both $f'(x)$ and $r(x)$ over $-4 \le x \le 4$ on the same axes. Sketch the result on the axes at right, using a dotted curve for the graph of $r(x)$.



3d. Execute `Clear[r]`, change the `0.5` to `0.4` in the definition of $r(x)$ in part **c**, and execute the commands in part **c** again. Are the two graphs closer? Can you still tell them apart?

3e. Experiment with smaller and smaller values of *h* until the graphs of $f'(x)$ and $r(x)$ over $-4 \le x \le 4$ become indistinguishable on your computer screen. How small does *h* have to be for this to happen?