Elastic demand at the monopoly price

Before investigating the relationship between marginal revenue and elasticity of demand, we will need to digress a moment and recall the elasticity coefficient, E_d. By definition, E_d is the (absolute value of the) percentage change in quantity demanded divided by the percentage change in price: $E_d = \left|\frac{dQ/Q}{dP/P}\right|$.

A simple rearrangement of this formula shows that $E_d = \left|\frac{dQ}{dP}\cdot\frac{P}{Q}\right|$.

With this in mind, suppose a monopolist's demand curve is given by $P = f(Q)$, revenue is $R(Q) = QP = Qf(Q)$ and marginal revenue is $MR = R'(Q) = f(Q) + Qf'(Q)$. Suppose we now both divide and multiply the right-hand-side of MR by P: $MR = P(\frac{f(Q)}{P} + \frac{Q}{P}f'(Q)) = P(1 + \frac{Q}{P}f'(Q))$ where we make use of the fact that $P = f(Q)$ for the last equality.

Noting that $f(Q)$ is monotonically decreasing (so its inverse exists) and $f'(Q) = dP/dQ$, we can now assert that $\frac{dP}{dQ} = 1\Big/\frac{dQ}{dP}$. Making the substitution into our earlier expression for elasticity, $E_d = \left|\frac{dQ}{dP}\cdot\frac{P}{Q}\right| = -1\Big/\left(\frac{Q}{P}f'(Q)\right)$. (The negative sign is required since $f'(Q) < 0$ and E_d is expressed as a positive number.) Inverting both sides, we see that $\frac{Q}{P}f'(Q) = -1/E_d$. Substituting this into our equation for MR we obtain the following: $MR = P(1 - 1/E_d)$.

We now have an expression that relates MR to the elasticity of demand: $MR = P(1 - 1/E_d)$. If demand is inelastic, $E_d < 1$ and $MR < 0$. Alternatively, $MR > 0$ if and only if $E_d > 1$. As profit maximization requires that $MR = MC$ and MC is always positive, we see that a monopolist must always price in the elastic portion of the demand curve.