The multiplier effect

The "multiplier" tells us the change in equilibrium GDP from a given change in desired spending, for example, a change in desired investment spending brought about by either a shift of the investment demand curve or a change in real interest rates. Mathematically, we desire a formula for $\frac{\Delta Y_e}{\Delta I_g}$ where Y_e is equilibrium GDP.

Suppose GDP is given by the formula $Y = C + I_g$ where C takes the linear formula $C = \mathrm{a} + bY$. As in math notes 27.1, "Consumption and saving," a is autonomous consumption spending and b is the MPC. Solving for equilibrium GDP, we obtain the general result $Y_e = \left(\frac{1}{1-b}\right) \times (a + I_g)$.

If I_g changes by some amount ΔI_g, then Y_e will change by some amount ΔY_e: $Y_e + \Delta Y_e = \left(\frac{1}{1-b}\right) \times (a + I_g + \Delta I_g) = \left(\frac{1}{1-b}\right) \times (a + I_g) + \left(\frac{1}{1-b}\right) \times \Delta I_g$. We can simplify this by subtracting Y_e and its equivalent $\left(\frac{1}{1-b}\right) \times (a + I_g)$ from both sides to obtain $\Delta Y_e = \left(\frac{1}{1-b}\right) \times \Delta I_g$. Now divide ΔI_g on both sides to obtain our desired result: $\frac{\Delta Y_e}{\Delta I_g} = \left(\frac{1}{1-b}\right)$.

Since b is the MPC and $(1 - b)$ is the MPS, this can be expressed alternatively as $\frac{\Delta Y_e}{\Delta I_g} = \left(\frac{1}{1-\text{MPC}}\right) = \left(\frac{1}{\text{MPS}}\right)$. For example, if the MPC is .75, the multiplier is $\left(\frac{1}{1-.75}\right) = \left(\frac{1}{.25}\right) = 4$, so that if $\Delta I_g = \$5$ billion, $\Delta Y_e = \$20$ billion.

Incidentally, the term $\left(\frac{1}{1-b}\right)$ has an alternate interpretation. Suppose we have an infinite sum of the form $1 + b + b^2 + b^3 + b^4 + \ldots$. We don't yet know what, if anything, this sum is equal to, but suppose it converges to some value Z. That is, $Z = 1 + b + b^2 + b^3 + b^4 + \ldots$. If we multiply each side of this equation by b, we would have $bZ = b + b^2 + b^3 + b^4 + b^5 + \ldots$. Next, subtract bZ from Z. We are left with $Z - bZ = 1$, as every term except the first term of Z cancels out. We can factor out Z from the term on the left and as long as b does not equal 1, divide both sides by $1 - b$ to obtain $Z = \left(\frac{1}{1-b}\right)$. That is, if the infinite sum converges at all, it will converge to $\left(\frac{1}{1-b}\right)$. Note that for $b > 1$ the sum is infinite. However, if $b < 1$ it can be shown that the sum converges to the value $\left(\frac{1}{1-b}\right)$ which you will recognize as the multiplier. This has an economic interpretation: the change in equilibrium GDP from a given change in investment can be seen as the sum of successive "rounds" of additional spending, with the amount at each round a constant proportion (equal to the MPC) of the prior round.