Chapter 13

TEMPERATURE AND THE IDEAL GAS

Problems

1. Strategy Use Eqs. (13-2b) and (13-3).

Solution Convert the temperature.

(a) $T_C = \dfrac{T_F - 32°F}{1.8°F/°C} = \dfrac{84°F - 32°F}{1.8°F/°C} = \boxed{29°C}$

(b) $T = 29\text{ K} + 273.15\text{ K} = \boxed{302\text{ K}}$

11. Strategy The hole expands just as if it were a solid brass disk. Use Eq. (13-6).

Solution Find the increase in area of the hole.

$$\Delta A = 2\alpha A_0 \Delta T = 2(1.9\times10^{-5}\,°C^{-1})(1.00\text{ mm}^2)(30.0°C - 20.0°C) = \boxed{3.8\times10^{-4}\text{ mm}^2}$$

25. Strategy Use Eqs. (13-4) and (13-6) and the given initial and final areas.

Solution Find the fractional change.

$$\frac{\Delta A}{A_0} = \frac{A - A_0}{A_0} = \frac{(s_0 + \Delta s)^2 - s_0^2}{s_0^2} = \frac{s_0^2 + 2s_0\Delta s + (\Delta s)^2 - s_0^2}{s_0^2} = \frac{2s_0\Delta s + (\Delta s)^2}{s_0^2} = \frac{\Delta s(2s_0 + \Delta s)}{s_0^2}$$

Now, since $s_0 >> \Delta s$, we have

$$\frac{\Delta A}{A_0} \approx \frac{\Delta s(2s_0 + 0)}{s_0^2} = \frac{2s_0\Delta s}{s_0^2} = \frac{2\Delta s}{s_0} = 2\alpha\Delta T \text{ since } \frac{\Delta s}{s_0} = \alpha\Delta T.$$

37. Strategy Use Eq. (13-10).

Solution Find the number of air molecules.

$$N = \frac{\rho V}{m} = (1.2\text{ kg/m}^3)(1.0\text{ cm}^3)\left(\frac{1\text{ m}^3}{10^6\text{ cm}^3}\right)\left(\frac{1}{29.0\text{ u}}\right)\left(\frac{1\text{ u}}{1.66\times10^{-27}\text{ kg}}\right) = \boxed{2.5\times10^{19}\text{ molecules}}$$

53. Strategy Use the microscopic form of the ideal gas law, Eq. (13-13).

Solution Find the number of air molecules released.

$N = \dfrac{PV}{kT}$, and V, k, and T are constant, so

$$\Delta N = \frac{V\Delta P}{kT} = \frac{(1.0\text{ m}^3)(15.0\text{ atm} - 20.0\text{ atm})(1.013\times10^5\text{ Pa/atm})}{(1.38\times10^{-23}\text{ J/K})(273\text{ K})} = -1.3\times10^{26}.$$

$\boxed{1.3\times10^{26}}$ air molecules were released.

65. Strategy The total internal kinetic energy of the ideal gas is equal to the number of molecules times the average kinetic energy per molecule. Use Eq. (13-20).

Solution Find the total internal kinetic energy of the ideal gas.

$$K_{\text{total}} = N\langle K_{\text{tr}}\rangle = N\left\langle \frac{3}{2}kT\right\rangle = \frac{3}{2}nRT = \frac{3}{2}(1.0\text{ mol})[8.314\text{ J/(mol}\cdot\text{K)}](273.15\text{ K}+0.0\text{ K}) = \boxed{3.4\text{ kJ}}$$

77. Strategy Form a proportion with the two reaction rates and solve for the temperature increase. Use Eq. (13-24).

Solution Find the temperature increase.

$$\frac{1.035}{1} = \frac{e^{-\frac{E_a}{kT_2}}}{e^{-\frac{E_a}{kT_1}}} = e^{\frac{E_a}{k}\left(\frac{1}{T_1}-\frac{1}{T_2}\right)}$$

$$\ln 1.035 = \frac{E_a}{k}\left(\frac{1}{T_1}-\frac{1}{T_2}\right)$$

$$\frac{k\ln 1.035}{E_a} = \frac{1}{T_1}-\frac{1}{T_2}$$

$$\frac{1}{T_2} = \frac{1}{T_1}-\frac{k\ln 1.035}{E_a}$$

$$T_2 = \left(\frac{1}{T_1}-\frac{k\ln 1.035}{E_a}\right)^{-1}$$

$$\Delta T = \left(\frac{1}{T_1}-\frac{k\ln 1.035}{E_a}\right)^{-1} - T_1$$

$$= \left[\frac{1}{273.15\text{ K}+10.00\text{ K}} - \frac{(1.38\times10^{-23}\text{ J/K})\ln 1.035}{2.81\times10^{-19}\text{ J}}\right]^{-1} - (273.15\text{ K}+10.00\text{ K}) = \boxed{0.14^\circ\text{C}}$$

93. Strategy and Solution The average of the test scores is $\frac{83+62+81+77+68+92+88+83+72+75}{10} = \boxed{78.1}$.

The rms value is $\sqrt{\frac{83^2+62^2+81^2+77^2+68^2+92^2+88^2+83^2+72^2+75^2}{10}} = \boxed{78.6}$. The most probable value is $\boxed{83}$, since it appears twice as often as any other score.

109. Strategy Use ideal gas law and Hooke's law.

Solution Find the pressure of the gas.

$PV = nRT$, so $P_{\text{gas}} = \frac{nRT}{V}$. The force with which the piston pushes on the spring is equal to $F = (P_{\text{gas}} - P_{\text{atm}})A_{\text{piston}}$. Set this equal to $F = k\Delta x$ to find the spring constant.

$k\Delta x = (P_{\text{gas}} - P_{\text{atm}})A_{\text{piston}} = \left(\frac{nRT}{V} - P_{\text{atm}}\right)A_{\text{piston}}$, so

$$k = \left[\frac{(6.50\times10^{-2}\text{ mol})[8.314\text{ J/(mol}\cdot\text{K)}](20.0\text{ K}+273.15\text{ K})}{(0.120\text{ m}+0.0540\text{ m})\pi(0.0800/2\text{ m})^2} - 1.013\times10^5\text{ Pa}\right]\frac{\pi(0.0800/2\text{ m})^2}{0.0540\text{ m}} = \boxed{7.4\times10^3\text{ N/m}}.$$