CHAPTER 15

Details of the Carnot Cycle

The four steps in the Carnot cycle are (Fig. 15.19):

$1 \rightarrow 2$: Isothermal expansion. Take in heat Q_H from the hot reservoir, keeping the gas at constant temperature T_H.

$2 \rightarrow 3$: Adiabatic expansion. The gas does work without any heat flow in, so the temperature decreases. Continue until the gas temperature is T_C.

$3 \rightarrow 4$: Isothermal compression. Heat Q_C is exhausted at constant temperature T_C.

$4 \rightarrow 1$: Adiabatic compression until the temperature is back to T_H.

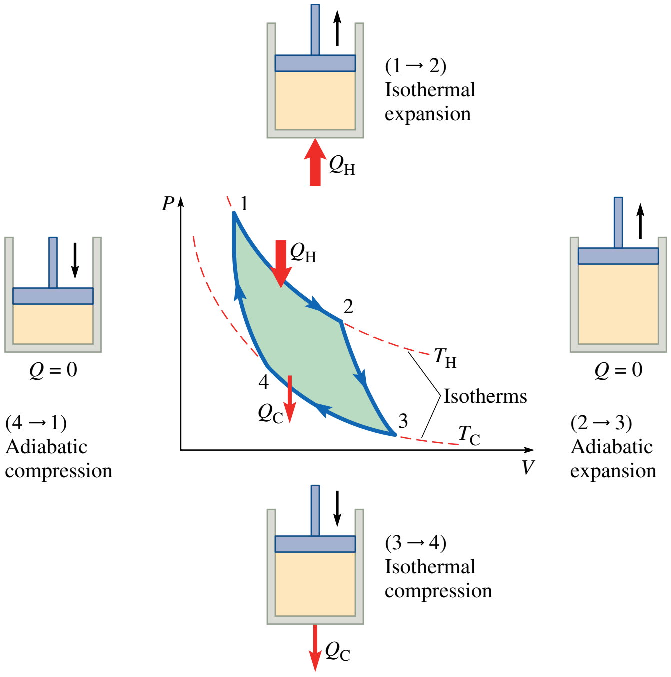

Figure 15.19 The Carnot cycle.

Example 15.10

Carnot Engine

A Carnot engine using 0.020 mol of an ideal gas operates between reservoirs at 1000.0 K and 300.0 K. The engine takes in 25 J of heat from the hot reservoir per cycle. Find the work done by the engine during each of the two isothermal steps in the cycle.

Strategy During the isothermal processes, the internal energy of the ideal gas stays the same, so

$$\Delta U = Q + W = 0 \quad \Rightarrow \quad |W| = |Q|$$

Solution $1 \rightarrow 2$: During the isothermal expansion, the work done by the gas is equal to the heat input—otherwise the temperature of the gas would change.

$$W_{1\rightarrow 2} = +25\text{ J (per cycle)}$$

$3 \rightarrow 4$: During the isothermal compression, the gas does negative work as it is compressed.

$$W_{3\rightarrow 4} = -Q_C$$

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Example 15.10 continued

The heats are proportional to the temperatures:

$$\frac{Q_\mathrm{C}}{Q_\mathrm{H}} = \frac{T_\mathrm{C}}{T_\mathrm{H}}$$

Therefore,

$$W_{3\rightarrow 4} = -Q_\mathrm{C} = -\frac{T_\mathrm{C}}{T_\mathrm{H}}\,Q_\mathrm{H} = -\frac{300.0\ \mathrm{K}}{1000.0\ \mathrm{K}} \times -25\ \mathrm{J} = -7.5\ \mathrm{J}\ \text{(per cycle)}$$

Discussion We will not prove it here, but the total work done during the two adiabatic processes is zero. Then the net work done by the engine per cycle is

$$25\ \mathrm{J} + (-7.5\ \mathrm{J}) = 17.5\ \mathrm{J}$$

The efficiency is then

$$e = \frac{17.5\ \mathrm{J}}{25\ \mathrm{J}} = 0.70$$

This should equal

$$e_\mathrm{r} = 1 - \frac{T_\mathrm{C}}{T_\mathrm{H}} = 1 - \frac{300.0\ \mathrm{K}}{1000.0\ \mathrm{K}} = 0.7000$$

Conceptual Practice Problem 15.10 Adiabatic Process in Carnot Cycle

Since there is no heat flow during the adiabatic processes and the work done during them adds to zero, why do we need adiabatic processes in the Carnot cycle? Why not just eliminate them?

Problems

79. Plot the temperature versus entropy for the four stages of the Carnot engine discussed in Example 15.10. [*Hint:* First plot the constant temperature stages and then fill in the adiabatic stages.]

✦ 80. Imagine that a car engine could be replaced by a Carnot engine with an ideal gas as the working substance. When the car is traveling at 65 mi/h, the Carnot engine goes through its cycle 900.0 times per minute. The engine's hot reservoir is at 1000.0°C (the temperature of the exploding gas in a real car engine) and the cold reservoir is at 20.0°C (the outside temperature). During the isothermal expansion part of each cycle, the volume of the ideal gas increases by a factor of 10.0. The cylinders contain 0.223 mol of gas. What is the power output of the engine?

Answers to Practice Problems

15.10 The adiabatic processes are needed to change the temperature of the working substance in the engine back and forth between T_C and T_H.