

Chapter

18 Inflation, unemployment and aggregate supply

Inflation and unemployment are two of the most important macroeconomic problems. Indeed, the main goals of macroeconomic stabilization policy are to fight cyclical unemployment and to avoid high inflation. In this chapter we explore the relationship between inflation and unemployment. As we shall see, understanding the link between these two variables is crucial for understanding how the supply side of the economy works and how the economy reacts to shocks. In short, studying the relationship between inflation and unemployment is fundamental for understanding business fluctuations.

18.1 Background: a brief history of the Phillips curve

For many years after the Second World War most economists and policy makers believed that there was an inescapable trade-off between inflation and unemployment: if you want less inflation, you have to live with permanently higher unemployment, and vice versa. Figures 18.1a and 18.1b, taken from a famous article published in 1958 by the New Zealand-born economist A.W. Phillips, suggest why most observers came to believe in a permanent unemployment–inflation trade-off. Figure 18.1a reproduces the curve which Phillips fitted to describe the relationship between unemployment and the rate of money wage inflation in the United Kingdom in the period 1861–1913. We see that he found a clear (although non-linear) negative correlation between the two variables. Phillips then showed that the curve fitted to the 1861–1913 data was able to explain the relationship between UK unemployment and wage inflation in the much later period 1948–1957, shown in Fig. 18.1b. Apparently Phillips had discovered a very stable and fundamental trade-off. This trade-off was therefore quickly incorporated into macroeconomic models under the name of the Phillips curve.

As illustrated in Fig. 18.2a which is based on US data on unemployment and the rate of consumer price inflation, the Phillips curve trade-off also seemed to exist throughout most of the 1960s. However, in the 1970s the relationship broke down completely (see Fig. 18.2b). Many times during the 1970s the US experienced a simultaneous rise in inflation and unemployment, much to the perplexity and frustration of economic policy

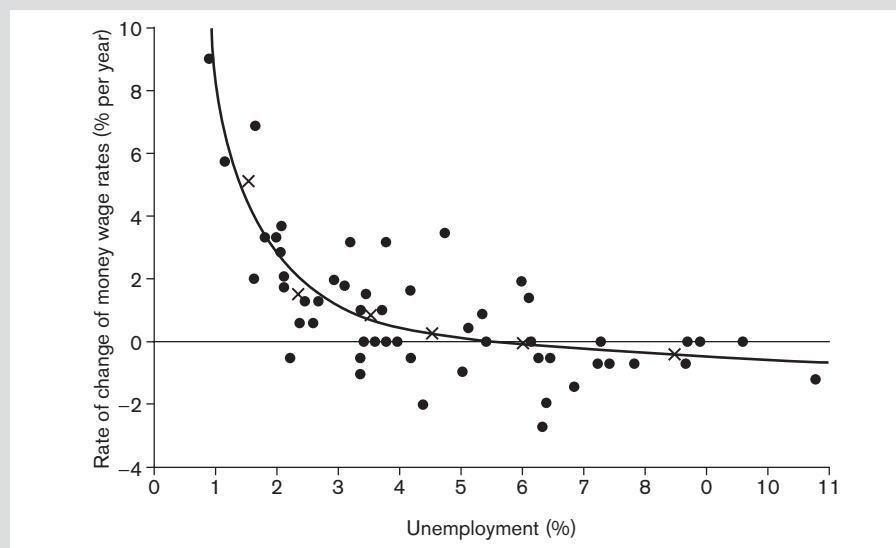


Figure 18.1a: The Phillips Curve in the United Kingdom, 1861–1913

Source: Figure 1 of A.W. Phillips, 'The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861–1957', *Economica*, New Series, **25** (100), Blackwell Publishing, (Nov., 1958), pp. 283–299.

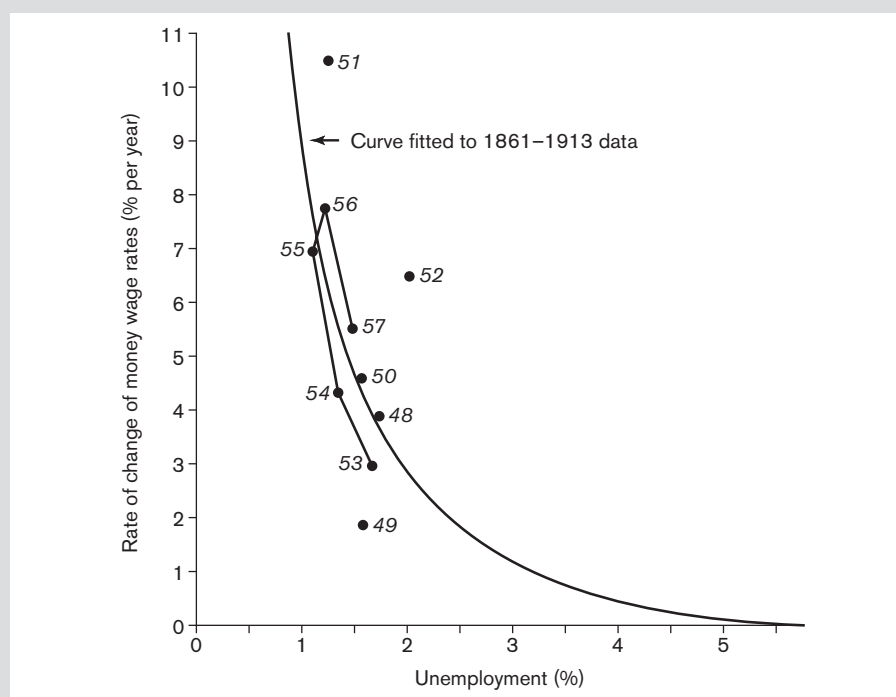


Figure 18.1b: The Phillips Curve in the United Kingdom, 1948–1957

Source: Figure 10 of A.W. Phillips, 'The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861–1957', *Economica*, New Series, **25** (100), Blackwell Publishing, (Nov., 1958), pp. 283–299.

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Figure 18.2a: The Phillips curve in the United States of the 1960s

Source: R.B. Mitchell, *International Historical Statistics*, Macmillan, 1998; and Bureau of Labor Statistics.



Figure 18.2b: The breakdown of the simple Phillips curve in the United States

Source: R.B. Mitchell, *International Historical Statistics*, Macmillan, 1998; and Bureau of Labor Statistics.

makers. The same thing happened in practically all OECD countries during that decade. What was going on?

In this chapter we develop a theory of inflation and unemployment which offers an explanation for the apparently stable Phillips curve trade-off before the 1970s as well as the relationship between unemployment and inflation in the more recent decades. Our theory of wage and price formation will be consistent with the theory of structural unemployment presented in Part 4. As we shall see, this framework can explain the

short-run link between inflation and unemployment as well as the factors determining the long-run equilibrium rate of unemployment, the 'natural' rate. The relationship we shall arrive at is the so-called *expectations-augmented Phillips curve*,

$$\pi = \pi^e + \alpha(\bar{u} - u), \quad \alpha > 0, \quad (1)$$

where π is the actual rate of inflation, π^e is the expected inflation rate, u is the actual rate of unemployment, and \bar{u} is the natural unemployment rate.

Many roads lead to the expectations-augmented Phillips curve. This chapter will take you down some of these roads. In Section 2 we offer a theory of the expectations-augmented Phillips curve in line with the theory of trade union behaviour introduced in Chapter 13.¹ However, the same qualitative results may be obtained from the theory of efficiency wages presented in Chapter 12, as we shall see in Chapter 23.

The model of inflation and unemployment presented in Section 2 assumes that nominal wages are rigid in the short run. In Section 3 we show that the expectations-augmented Phillips curve may also be derived from a model of a competitive labour market with fully flexible wages and prices. By comparing the models in Sections 2 and 3, we are able to highlight how nominal rigidities exacerbate the employment fluctuations which occur when economic agents underestimate or overestimate the rate of inflation.

18.2 Nominal rigidities, expectational errors and employment fluctuations

Inflation is a continuous rise in the general price level. A theory of inflation therefore requires a theory of price formation. Since prices depend on the cost of inputs, and since labour is the most important input, our theory of price formation will build on a theory of wage formation. The theory will allow for imperfect competition in the markets for goods as well as labour. Introducing imperfect competition in output markets complicates the analysis, but in return it enables us to illustrate how structural changes in product markets affect inflation and the natural rate of unemployment.

In Book One where we focused on the long run, we assumed that agents had correct expectations about the general level of wages and prices, as must be the case in any long-run equilibrium. By contrast, in the present short-run context we assume that people do not have perfect information about the current general price level. As we shall see, this means that employment and output may deviate from their long-run equilibrium levels.

This section assumes that wages are 'sticky' in the short run, being set by trade unions. We will therefore start with a description of trade union behaviour and wage formation.

The trade union's objective

We consider an economy which is divided into a number of different sectors each producing a differentiated product. Workers in each sector are organized in a trade union which monopolizes the supply of labour to all firms in the sector. Because of its monopoly

1. The exposition in this chapter does not assume that you have already studied Chapter 13, so you should still be able to understand all parts of the present chapter even if you have not had the opportunity to go through Book One.

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position, the trade union in each sector may dictate the nominal wage rate to be paid by employers in that sector, but employers have the 'right to manage', that is, they can freely choose the level of employment. For simplicity, we assume that the number of working hours for the individual worker is fixed, so total labour input is proportional to the number of workers employed.

Workers in sector i are educated and trained to work in that particular sector, so they cannot move to another sector to look for a job. If a worker fails to find a job in his sector, he therefore becomes unemployed. His real income will then be equal to the real rate of unemployment benefit b .² An employed worker in sector i earns the real wage $w_i \equiv W_i/P$, where W_i is the sectoral money wage and P is (an index of) the general price level, so his *net* income gain from being employed is $w_i - b$. The trade union for sector i cares about this real income gain for its employed members, but it also cares about the total number of jobs L_i secured for the membership. We formalize this by assuming that the union sets the nominal wage rate with the purpose of maximizing a utility function Ω of the form:

$$\Omega = (w_i - b)L_i^\eta, \quad \eta > 0. \quad (2)$$

The parameter η reflects the weight which the union attaches to high employment relative to the goal of a high real wage for employed union members. The more the union is concerned about employment relative to wages, the higher is the value of η . In the benchmark case where $\eta = 1$ (corresponding to the cases analysed in Chapters 1 and 13), the union is simply interested in the aggregate net income gain obtained by employed members.

When setting the wage rate, the union must account for the fact that a higher real wage will lower the employer's demand for labour. Our next step is to derive this constraint on the union's optimization problem.

Price setting and labour demand

The representative employer in sector i uses a technology described by the production function:

$$Y_i = BL_i^{1-\alpha}, \quad 0 < \alpha < 1, \quad (3)$$

where Y_i is the volume of real output produced and sold in sector i , and B is a productivity parameter. Since we are concentrating on the short run where the capital stock is fixed, we have not included capital explicitly in the production function.³ According to (3), the marginal product of labour, MPL_i , is:

$$MPL_i \equiv dY_i/dL_i = (1 - \alpha)BL_i^{-\alpha}, \quad (4)$$

which is seen to diminish as labour input increases, due to the fixity of the capital stock.

2. In Chapters 12 and 13 and in Section 4 of Chapter 1, where we focused on the long run, we assumed that workers who fail to find a job in their initial sector have time to retrain so that they can move into other sectors to look for alternative employment opportunities. In that case the expected real income obtainable by a sector i worker who loses his initial job is $v = (1 - u)w + ub$, where u is the general unemployment rate, w is the average real wage outside sector i , and where the employment rate $1 - u$ represents the probability of finding a job outside sector i . In Exercise 1 you are asked to consider such a case with intersectoral labour mobility and to show that this case leads to the expectations-augmented Phillips curve as well.

3. In Book One we worked with the Cobb–Douglas production function $Y = BK^\alpha L^{1-\alpha}$. Equation (3) is just a version of this production function where we have fixed the capital stock K at unity.

The employer representing industry i produces a differentiated product and therefore has some monopoly power, so we assume that he faces a downward-sloping demand curve of the form:

$$Y_i = \left(\frac{P_i}{P}\right)^{-\sigma} \frac{Y}{n}, \quad \sigma > 1, \quad (5)$$

This demand curve has a constant numerical price elasticity of demand equal to $\sigma = -(dY_i/dP_i)(P_i/Y_i)$, where P_i is the price charged per unit of Y_i . The variable Y is total GDP, and n is the number of different sectors in the economy. Aggregate output Y is a measure of the total size of the national market, and Y/n is the market share captured by each industry if they all charge the same prices (so that $P_i = P$).⁴ The total revenue of firm i is $TR_i \equiv P_i Y_i$, so according to (5) its marginal revenue (the increase in total revenue from selling an extra unit of output) will be:

$$MR_i \equiv \frac{dTR_i}{dY_i} = P_i + Y_i \left(\frac{dP_i}{dY_i}\right) = P_i \left(1 + \frac{dP_i}{dY_i} \frac{Y_i}{P_i}\right) = P_i \left(1 - \frac{1}{\sigma}\right). \quad (6)$$

From microeconomic theory we know that a profit-maximizing firm will expand output to the point where marginal revenue equals marginal cost, $MR_i = MC_i$. Because labour is the only variable factor of production, marginal cost is equal to the price of an extra unit of labour – the nominal wage rate, W_i – divided by labour's marginal product, MPL_i , since MPL_i measures the additional units of output produced by an extra unit of labour. Thus $MC_i = W_i/MPL_i$. From (4) and (6), the necessary condition for maximization of profits, $MR_i = MC_i$, therefore becomes:

$$P_i \left(\frac{\sigma - 1}{\sigma}\right) = \frac{W_i}{(1 - \alpha)BL_i^{-\alpha}},$$

which is equivalent to:

$$P_i = m^p \cdot \overbrace{\left(\frac{W_i}{(1 - \alpha)BL_i^{-\alpha}}\right)}^{MC_i}, \quad m^p \equiv \frac{\sigma}{\sigma - 1} > 1. \quad (7)$$

Equation (7) shows that the profit-maximizing representative firm in sector i will set its price as a mark-up over its marginal cost. Our previous assumption $\sigma > 1$ guarantees that the mark-up factor m^p is positive and greater than one. The price elasticity, σ , is a measure of the strength of product market competition. The higher the elasticity, the greater is the fall in demand induced by a higher price (the flatter is the demand curve), and the lower is the mark-up of price over marginal cost. In the limiting case where the price elasticity tends to infinity, the price is driven down to the level of marginal cost ($\sigma \rightarrow \infty \Rightarrow m^p \rightarrow 1$), corresponding to perfect competition.

We can now derive the labour demand curve of sector i , showing the relationship between the real wage W_i/P claimed by the union in sector i and the level of employment in that sector. Dividing by P on both sides of (7) gives the relative price, P_i/P , of sector i 's

4. As we mentioned in Chapter 12, the demand curve (5) may be derived from the solution to the consumer's problem of utility maximization if utility functions are of the CES form. In that case the parameter σ is the representative consumer's elasticity of substitution between good i and any other good. See Exercise 3 in Chapter 12.

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product. Inserting this P_i/P into (5) then gives production, Y_i , in sector i . Finally, we can use (3) to compute how much employment, L_i , is needed to produce that level of output. Performing these operations, we end up with:

$$L_i = \left(\frac{Y}{nB} \right)^{\epsilon/\sigma} \left(\frac{B(1-\alpha)}{m^p} \right)^{\epsilon} \left(\frac{W_i}{P} \right)^{-\epsilon}, \quad \epsilon \equiv \frac{\sigma}{1 + \alpha(\sigma - 1)} > 0. \quad (8)$$

The numerical real wage elasticity of labour demand at the sectoral level (defined as $-(dL_i/d(W_i/P))((W_i/P)/L_i)$) is equal to the constant ϵ . From the expression for ϵ you may verify that a higher price elasticity of product demand (tougher competition in product markets) increases the wage elasticity of sectoral labour demand. This is intuitive: a rise in the wage rate will drive up the output price by raising the firm's marginal cost. The higher the price elasticity of output demand, the greater is the fall in sales and output, so the greater is the resulting fall in labour demand.

Wage setting

The labour demand curve (8) implies that employment in sector i is a declining function, $L_i(w_i)$, of the real wage, $w_i \equiv W_i/P$. The union's utility function (2) may therefore be written as:

$$\Omega(w_i) = (w_i - b)[L_i(w_i)]^\eta. \quad (9)$$

Suppose for the moment that the union has perfect information about the current price level so that it may perfectly control the real wage $w_i \equiv W_i/P$ via its control of the money wage W_i . The union will then choose w_i so as to maximize $\Omega(w_i)$. The necessary condition for a maximum, $d\Omega(w_i)/dw_i = 0$, is $L_i^\eta + (w_i - b)\eta L_i^{\eta-1}(dL_i/dw_i) = 0$, which is equivalent to:

$$1 + \frac{\eta(w_i - b)}{w_i} \left(\frac{dL_i}{dw_i} \frac{w_i}{L_i} \right) = 0.$$

Using the fact that $(dL_i/dw_i)(w_i/L_i) = -\epsilon$, we may rewrite this expression as:

$$w_i = m^w \cdot b, \quad m^w \equiv \frac{\eta\epsilon}{\eta\epsilon - 1}. \quad (10)$$

According to (10) the union's target real wage is a mark-up over the opportunity cost of employment. The opportunity cost of employment is the rate of unemployment benefit b , since this is the income a worker forgoes by being employed rather than unemployed. To secure that (10) actually implies a positive real wage, we assume that $\eta\epsilon > 1$. It then follows that the wage mark-up factor, m^w , is greater than 1.

Equation (10) implies that the union's real wage claim will be lower the greater the weight it attaches to the goal of high employment, i.e., the higher the value of η . It also follows from (10) that the target real wage will be lower the higher the elasticity of labour demand, ϵ . The reason is that a higher labour demand elasticity increases the loss of jobs resulting from any given increase in the real wage. Finally, we see from (10) that a higher rate of unemployment benefit drives up the target real wage because it reduces the income loss incurred by those union members who lose their jobs when the union charges a higher wage rate.

We have so far assumed that the union has perfect information about the current price level and therefore perfectly controls the real wage W_i/P through its control of the money wage rate, W_i . However, in practice, nominal wage rates are almost always *pre-set* for a certain period of time, that is, in the short run the nominal wage rate is *rigid*. Moreover, at the start of the period when wages are set, trade union leaders cannot perfectly foresee the price level which will prevail over the period during which the nominal wage rate will be fixed by the wage contract. A trade union setting the wage rate at the start of the current period must therefore base its money wage claim on its *expectation* of the price level which will prevail over the coming period. Given that the union strives to obtain the real wage specified in (10), it will then set the *money* wage rate so as to achieve an *expected* real wage equal to the target real wage m^wb .⁵ If the expected price level for the current period is P^e , the nominal wage rate set by the union at the start of the period will thus be:

$$W_i = P^e \cdot m^wb. \quad (11)$$

Having developed a theory of wage and price setting as well as a theory of labour demand, we are now ready to derive the link between inflation and unemployment.

The expectations-augmented Phillips curve

Equation (11) implies that the *actual* real wage may be written as $W_i/P = (P^e/P)m^wb$. Inserting this expression into the labour demand curve (8) and rearranging, we obtain the level of employment in sector i :

$$L_i = \left(\frac{Y}{nB} \right)^{\epsilon/\sigma} \left(\frac{B(1-\alpha)}{m^p m^w b} \cdot \frac{P}{P^e} \right)^{\epsilon}. \quad (12)$$

The higher the actual price level relative to the expected price level, P/P^e , that is, the more the trade union underestimates the price level, the lower is its nominal wage claim relative to the actual price level, so the lower is the real wage and the higher is the level of sectoral employment, as we see from (12).

We will now show that a similar qualitative relationship between employment and the ratio of actual to expected prices will prevail at the aggregate level. In doing so we will assume that all sectors in the economy are *symmetric* so that output and employment in each sector are given by Eqs (3) and (12), respectively, where all the parameters as well as the ratio P/P^e are the same across sectors. Total employment (L) will then be $L = nL_i$, and total GDP will be $Y = nY_i = nBL_i^{1-\alpha}$. Substituting the latter expression into (12) and computing $L = nL_i$, we get

$$L = nL_i = n \cdot \left(\frac{B(1-\alpha)}{m^p m^w b} \cdot \frac{P}{P^e} \right)^{1/\alpha}, \quad (13)$$

where we have used the definition of ϵ given in (8) according to which $1 - \epsilon(1-\alpha)/\sigma = \alpha\epsilon$.

5. We assume for simplicity that the union has a correct estimate of the level of b . For example, we may assume that the nominal rate of unemployment benefit is automatically indexed to the current price level so as to protect its real value. The union will then be able to forecast the level of the real rate of unemployment benefit even if it cannot perfectly foresee the price level. In Exercise 1 you are asked to consider the alternative case where the union does not have perfect information about the real value of the nominal rate of unemployment benefit.

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Note that since the real wage $w \equiv W/P$ is the same in all sectors and equal to $(P^e/P)m^wb$, we may also write (13) as:

$$L = n \left(\frac{B(1-\alpha)}{m^p} \right)^{1/\alpha} \left(\frac{W}{P} \right)^{-1/\alpha}. \quad (14)$$

This expression shows that at the *aggregate* level the numerical real wage elasticity of labour demand is $1/\alpha$, whereas at the level of the individual sector we found it to be equal to $\epsilon = \sigma/[1 + \alpha(\sigma - 1)]$. In Exercise 2 we ask you to provide an intuitive explanation for this difference between the labour demand elasticities at the macro and the micro levels.

In a long-run equilibrium expectations must be fulfilled. Inserting $P^e = P$ into (13), we therefore obtain the long-run equilibrium level of aggregate employment, \bar{L} , also called the 'natural' level of employment:

$$\bar{L} = n \left(\frac{B(1-\alpha)}{m^p m^w b} \right)^{1/\alpha}. \quad (15)$$

Equation (15) gives the level of employment which will prevail when price expectations are correct so that trade unions actually obtain their target real wage. Dividing (13) by (15), we get a simple relationship between the actual and the natural level of employment:

$$\frac{L}{\bar{L}} = \left(\frac{P}{P^e} \right)^{1/\alpha}. \quad (16)$$

If the aggregate labour force is N and the unemployment rate is u , it follows by definition that $L \equiv (1-u)N$. Similarly, the 'natural' unemployment rate, \bar{u} , is defined by the relationship $\bar{L} \equiv (1-\bar{u})N$. Substitution of these identities into (16) gives $(1-u)/(1-\bar{u}) = (P/P^e)^{1/\alpha}$. Taking natural logarithms on both sides and using the approximations $\ln(1-u) \approx -u$ and $\ln(1-\bar{u}) \approx -\bar{u}$, we get:

$$p = p^e + \alpha(\bar{u} - u), \quad p \equiv \ln P, \quad p^e \equiv \ln P^e.$$

Subtracting $p_{-1} \equiv \ln P_{-1}$ on both sides finally gives:

$$\pi = \pi^e + \alpha(\bar{u} - u), \quad \pi \equiv p - p_{-1}, \quad \pi^e \equiv p^e - p_{-1}, \quad (17)$$

where the subscript '-1' indicates that the variable in question refers to the previous time period. Recalling that the change in the log of some variable roughly equals the relative change in that variable, it follows that π is the *actual* rate of inflation whereas π^e is the *expected* rate of inflation, assuming that agents know the previous period's price level p_{-1} when they form their expectation about the current price level.

Equation (17) is a key macroeconomic relationship called the *expectations-augmented Phillips curve*,⁶ and it provides the link between inflation and unemployment we have been looking for. It shows that for any given *expected* rate of inflation, a lower level of unemployment is associated with a higher *actual* rate of inflation, and vice versa. More precisely, we see from (17) that *unanticipated* inflation ($\pi > \pi^e$) will drive unemployment

6. The theory of the expectations-augmented Phillips curve was developed almost simultaneously by the US economists Milton Friedman and Edmund Phelps. See Milton Friedman, 'The Role of Monetary Policy', *American Economic Review*, **58**, 1968, pp. 1-17, and Edmund S. Phelps, 'Money-Wage Dynamics and Labor Market Equilibrium', *Journal of Political Economy*, **76**, 1968, pp. 678-711.

below its natural rate. The reason is that an unexpected rise in the rate of inflation causes the real value of the pre-set money wage rate to fall below the target real wage of trade unions, thereby inducing firms to expand employment beyond the natural level.

The simple versus the expectations-augmented Phillips curve

If we set the expected inflation rate in (17) equal to 0, we obtain a version of the *simple Phillips curve* presented in Section 1, describing the unemployment–inflation trade-off discovered by Phillips:

$$\pi = \alpha(\bar{u} - u). \quad (18)$$

We may now offer an explanation why the simple unemployment–inflation trade-off estimated by Phillips broke down in the US and elsewhere in the OECD from around 1970. Over the long historical period considered by Phillips – from around 1860 to the 1950s – there was no systematic tendency for prices to rise for extended periods of time, as you can see from Fig. 18.3. Because of this long experience of approximate price stability, it was natural for economic agents to expect prices to be roughly constant. In such circumstances where $\pi^e = 0$, Eq. (17) does indeed predict that a lower unemployment rate will always be associated with a higher inflation rate, and vice versa.

However, towards the end of the 1960s inflation had been systematically positive and gradually rising for several years, so people started to consider a positive inflation rate as a normal state of affairs. As a consequence, the expected inflation rate started to increase. According to (17) this tended to drive up the actual rate of inflation associated with any given level of unemployment, just as portrayed in Fig. 18.2b which showed that many years during the 1970s were characterized by simultaneous increases in inflation and unemployment. There were also other reasons for these developments, such as dramatic

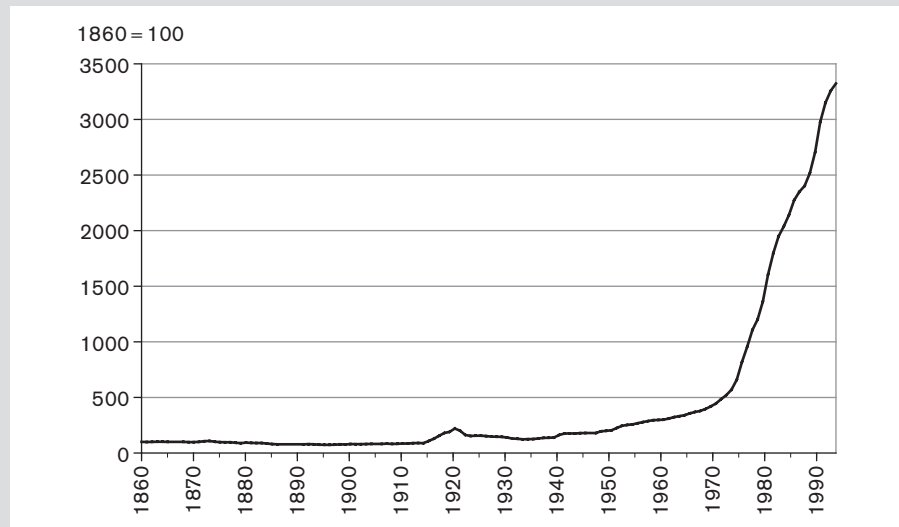


Figure 18.3: The consumer price index in the United Kingdom, 1860–1993

Source: B.R. Mitchell, *International Historical Statistics: Europe 1750–1993*, Macmillan Press, 1998.

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increases in the price of oil due to turmoil in the Middle East, but rising inflation expectations probably played an important role in the breakdown of the simple Phillips curve from the end of the 1960s.

The implication of all this is that the simple negative Phillips curve relationship between inflation and unemployment is a *short-run* trade-off which will hold only as long as the expected rate of inflation stays constant. For this reason the simple downward-sloping Phillips curve (defined for a given expected rate of inflation) may also be called the *short-run* Phillips curve. Whenever the expected inflation rate π^e increases, the short-run Phillips curve will shift upwards, as illustrated in Fig. 18.4 which shows three different short-run Phillips curves, each corresponding to different levels of expected inflation. In a long-run equilibrium the expected inflation rate equals the actual inflation rate, $\pi^e = \pi$. According to (17) this means that only one unemployment rate – the natural rate \bar{u} – is compatible with long-run equilibrium. We may say that *the long-run Phillips curve is vertical*, passing through $u = \bar{u}$, as indicated in Fig. 18.4. Hence there is no *permanent* trade-off between inflation and unemployment.

Outside long-run equilibrium, the expected inflation rate differs from the actual inflation rate. If such expectational errors persist, it is natural to assume that economic agents will gradually revise their expectations as they observe that their inflation forecasts turn out to be wrong. One simple hypothesis encountered in the previous chapter is that people have *static expectations*, expecting that this period's inflation rate will correspond to the rate of inflation observed during the previous period:

$$\pi^e = \pi_{-1}. \quad (19)$$

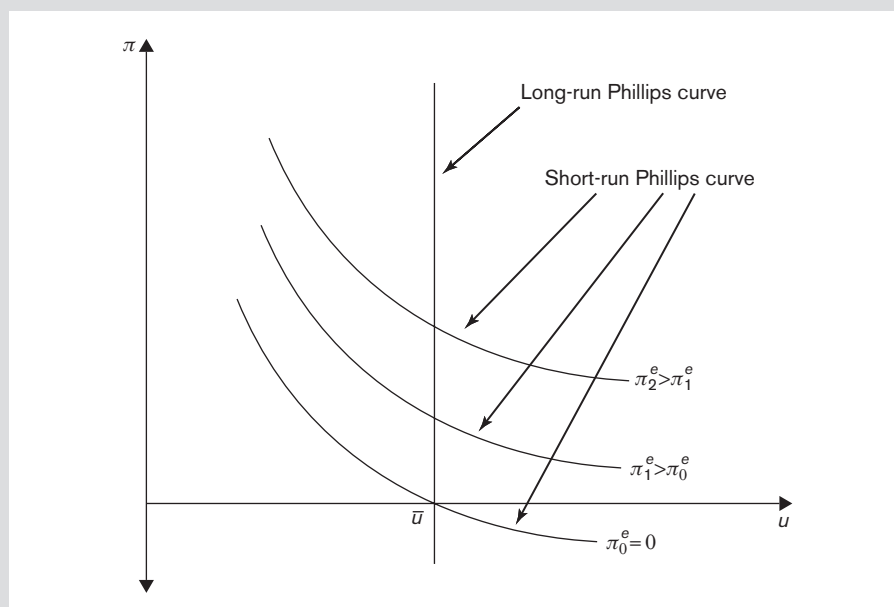


Figure 18.4: The expectations-augmented Phillips curve

This hypothesis means that agents will change their inflation forecasts whenever they observe a change in last period's inflation rate. Substitution of (19) into (17) gives:

$$\Delta\pi \equiv \pi - \pi_{-1} = \alpha(\bar{u} - u), \quad (20)$$

which shows that inflation will *accelerate* when unemployment is *below* its natural rate, and *decelerate* when unemployment is *above* the natural level. To prevent inflation from accelerating (or decelerating), unemployment will thus have to be kept at its natural rate. For this reason the natural rate is sometimes called the 'Non-Accelerating-Inflation-Rate-of-Unemployment', or just the NAIRU, for short.

Another important implication of the expectations-augmented Phillips curve (17) is that there is *nominal inertia*: if unemployment is at its natural rate, the inflation prevailing today will automatically continue tomorrow because it is built into expectations. To bring down inflation, it is necessary to push the actual unemployment rate above the NAIRU for a while.

What determines the natural rate of unemployment?

It is obvious that the natural rate of unemployment plays an important role in our theory of inflation, given that inflation will tend to rise if unemployment is pushed below that level. But what determines the natural rate? Our expression (15) for the natural level of employment provides the key to an answer. Recall that $\bar{L} \equiv (1 - \bar{u})N$. For simplicity, let us set the number of workers in each sector equal to 1 so that the total labour force becomes $N = n$, implying $\bar{L} \equiv (1 - \bar{u})n$. Inserting (15) into this expression and rearranging, we get:

$$\bar{u} = 1 - \left(\frac{B(1 - \alpha)}{m^p m^w b} \right)^{1/\alpha}. \quad (21)$$

Equation (21) shows that the natural unemployment rate depends on the level of the real unemployment benefit, b , among other things. It is reasonable to assume that the government allows the rate of unemployment benefit to grow in line with real income per capita, at least over the longer run. From Book One we know that the long-run growth rate of per-capita income will equal the growth rate of total factor productivity B . We will therefore assume that the level of unemployment benefits is tied to the level of productivity so that $b = cB$, where $c > 0$ is a parameter reflecting the generosity of the system of unemployment compensation. Substituting cB for b in (21), we get the following expression for the natural rate of unemployment, where we assume that the combination of parameter values ensures a positive value of \bar{u} :

$$\bar{u} = 1 - \left(\frac{1 - \alpha}{m^p m^w c} \right)^{1/\alpha}. \quad (22)$$

According to (22) the natural unemployment rate is higher the higher the mark-ups in wage and price setting, and the more generous the level of unemployment benefits (the higher the value of c). A rise in $m^p = \sigma/(\sigma - 1)$ reflects a fall in the representative firm's price elasticity of demand (σ) which means that it takes a larger cut in the firm's relative price P_i/P to obtain a given increase in sales. To sell the extra output produced by an extra

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worker, the firm must therefore accept a larger price cut the lower the value of σ . For any given wage level, the profit-maximizing level of employment will thus be lower the lower the value of σ . This is why the natural unemployment rate will be higher the higher the mark-up factor m^p .

A fall in σ will also increase the wage mark-up, since the sectoral labour demand elasticity $\epsilon \equiv \sigma/[1 + \alpha(\sigma - 1)]$ is increasing in σ , and since $m^w = \eta\epsilon/(\eta\epsilon - 1)$ is decreasing in ϵ . The intuition for this rise in the wage mark-up is that a lower price elasticity of demand for the output of the representative firm reduces the drop in sales and employment occurring when a higher union wage claim drives up the firm's marginal cost and price. Hence it becomes less costly (in terms of jobs lost) for the union to push up the wage rate, and this invites more aggressive wage claims.

The representative firm's price elasticity of demand reflects the degree of competition in product markets. The greater the number of competing firms in each market, and the greater the substitutability of the products of different firms, the tougher competition will be, and the greater will be the price elasticity of demand faced by the individual firm or industry. Thus our analysis shows that a lower degree of competition in product markets (a lower σ) will spill over to the labour market and raise the natural rate of unemployment, partly because it lowers labour demand, and partly because it induces more aggressive wage claims. This is an interesting example of how imperfections in some markets may exacerbate imperfections in other markets.

It is worth noting two more points from (22). First, a greater union concern about employment, reflected in a higher value of the parameter η , will reduce the natural rate of unemployment by lowering the wage mark-up $m^w = \eta\epsilon/(\eta\epsilon - 1)$. Second, the level of productivity B does not affect the natural rate of unemployment. This prediction is in line with empirical observations. As illustrated by Fig. 11.1 in Chapter 11, the unemployment rate tends to fluctuate around a constant level over the very long run despite the fact that productivity is steadily growing over time. However, as we shall see later in this chapter, short-run fluctuations in productivity growth do affect the short-run unemployment-inflation trade-off.

Nominal price rigidity

For simplicity, our model of wage and price setting assumes that while nominal wages are rigid in the short run, output prices adjust immediately to changes in demand and marginal costs. This is a way of capturing the stylized fact that nominal wages tend to be fixed for longer periods of time than most goods prices. But in reality many output prices are also held constant for considerable periods, as we noted in Chapter 1. In that chapter we also saw that small 'menu costs' of price adjustment – such as the costs of printing new price catalogues and communicating new prices to customers – may make it suboptimal for firms to adjust prices too frequently. When you interpret our theory of inflation and unemployment, you should therefore keep in mind that a sluggish adjustment of inflation – and hence a sluggish adjustment of unemployment to its natural level – may not only be due to a slow adjustment of nominal wage rates. It may also reflect that it takes time for changes in expectations to feed into the actual price level because it is costly for firms to change prices too frequently.

18.3 The expectations-augmented Phillips curve in a competitive labour market

The theory of inflation and unemployment presented above included two elements which are typically used to explain how it is possible for economic activity to deviate from its long-run equilibrium level: expectational errors (erroneous price expectations) and nominal rigidity. We allowed for nominal rigidity by assuming that nominal wage rates are *pre-set* at the start of each period and do not adjust within the period even if labour demand changes. In other words, *within* each period the nominal wage rate is fixed, although it adjusts *between* periods as price expectations change.

In this section we will try to deepen your understanding of the importance of expectational errors and nominal rigidity for explaining fluctuations in employment. We will show that while expectational errors are both necessary *and* sufficient to generate deviations of unemployment from its natural rate, nominal rigidity is not necessary but will *amplify* the fluctuations in employment caused by expectational errors. To demonstrate this, we will analyse the response of employment to unanticipated inflation in a model with fully flexible nominal wages and compare this model with the one developed above where nominal wages are 'sticky' in the short run.

The link between inflation and employment in a competitive labour market: the worker-misperception model

To highlight the role of nominal rigidity, it is instructive to consider the link between inflation and employment which would prevail if the labour market were *competitive*, that is, if nominal and real wages were fully flexible, adjusting instantaneously to balance labour supply and labour demand.

In a competitive labour market there are no trade unions. Our wage setting equation (11) specifying union wage claims is therefore replaced by a labour supply curve showing how workers adjust their labour supply in response to changes in the expected real wage. To facilitate comparison with the trade union model considered above, we continue to assume that each employed worker works a fixed number of hours which we may denote by H . Changes in aggregate labour supply will then take the form of more workers entering the labour market or some workers exiting the market. Suppose that worker j requires a minimum real wage w_j to be willing to sacrifice H hours of leisure by taking a job. In that case he will only enter the labour market if the expected real wage, W/P^e , is at least equal to w_j . Suppose further that different workers have different valuations of leisure, with some requiring only a low real wage to be willing to accept a job, while others require a high real wage to be willing to enter the labour market. If there is a continuum of required minimum real wages, the number of workers entering the labour market, L^s , will rise continuously as the expected real wage increases, implying an aggregate labour supply function of the form $L^s = f(W/P^e)$, $f' > 0$. For simplicity, let us assume that the distribution of the taste for leisure across workers (the distribution of required real wages) is such that the function $f(W/P^e)$ has a constant elasticity ϕ with respect to the expected real wage. We then get the aggregate labour supply function:

$$L^s = Z \left(\frac{W}{P^e} \right)^\phi = Z \left(w \cdot \frac{P}{P^e} \right)^\phi, \quad Z > 0, \quad (23)$$

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where Z is a constant reflecting the size of the population, and where, you recall, that $w \equiv W/P$ is the *actual* real wage. Equation (23) makes the reasonable assumption that the worker knows his nominal wage rate W when he accepts a job, but he does not have perfect information on the current price level when he makes his labour supply decision, so he must base his decision on his *expectation* of the current price level.

Aggregate labour demand L^d is still given by Eq. (14) which may be written as:

$$L^d = Xw^{-1/\alpha}, \quad X \equiv n \left(\frac{B(1-\alpha)}{m^p} \right)^{1/\alpha}. \quad (24)$$

In a competitive labour market, the real wage w will adjust to balance supply and demand, $L^s = L^d$, implying:

$$w = \left(\frac{X}{Z} \right)^{1/(\phi+1/\alpha)} \left(\frac{P}{P^e} \right)^{-\phi/(\phi+1/\alpha)}. \quad (25)$$

The equilibrium real wage found in Eq. (25) may be inserted into (24) to give the level of employment in the competitive labour market:

$$L = X^{\phi/(\phi+1/\alpha)} Z^{(1/\alpha)/(\phi+1/\alpha)} \left(\frac{P}{P^e} \right)^{(\phi/\alpha)/(\phi+1/\alpha)}. \quad (26)$$

Figure 18.5 illustrates how the equilibrium levels of w and L are determined by the intersection of the aggregate labour supply curve (23) and the aggregate labour demand curve (24). The natural employment level \bar{L} is found at the equilibrium point \bar{E} where

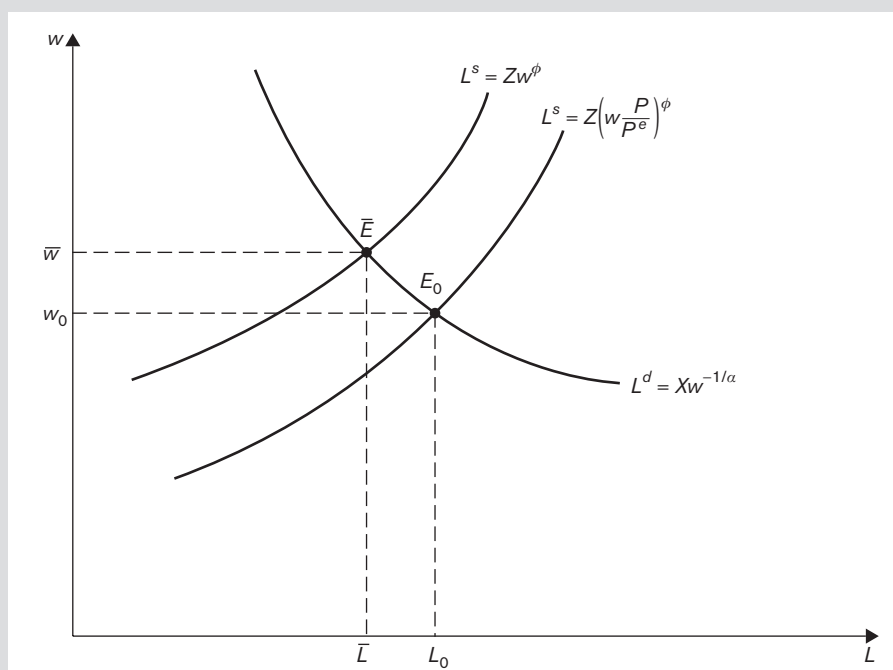


Figure 18.5: Labour market equilibrium in the worker-misperception model

price expectations are correct ($P^e = P$) so that the labour supply curve (23) collapses to $L^s = Zw^{e^s}$.

In the equilibrium E_0 employment is above the natural level because workers underestimate the price level ($P > P^e$). Whenever there is a change in the ratio of the actual to the expected price level, P/P^e , the labour supply curve will shift, generating new short-run equilibrium levels of the real wage and employment. This model of the labour market is sometimes called 'the worker-misperception model' because it postulates that employment fluctuations are driven by workers' misperceptions of the price level, that is, by fluctuations in P/P^e .

Natural employment may be found from (26) by setting $P^e = P$. We then get:

$$\bar{L} = X^{\phi/(\phi+1/\alpha)} Z^{(1/\alpha)/(\phi+1/\alpha)}. \quad (27)$$

Dividing (26) by (27) gives:

$$\frac{L}{\bar{L}} = \left(\frac{P}{P^e} \right)^{(\phi/\alpha)/(\phi+1/\alpha)}. \quad (28)$$

We see that (28) has exactly the same form as our earlier (16) which was derived from the model with union wage setting. The only difference is that the coefficient $1/\alpha$ has now been replaced by $(\phi/\alpha)/(\phi+1/\alpha)$. By taking logs on both sides of (28) and using the approximations $\ln(1-u) \approx -u$ and $\ln(1-\bar{u}) \approx -\bar{u}$, we can still derive an expectations-augmented Phillips curve of the form $\pi = \pi^e + \hat{\alpha}(\bar{u} - u)$, where $\hat{\alpha}$ is a constant. This shows that the expectations-augmented Phillips curve is quite a general relationship which does not assume a particular market structure. But as we shall demonstrate below, it is quite important for the *quantitative* relationship between employment and unanticipated inflation whether the labour market is competitive or not.

The competitive versus the unionized labour market

To see how unanticipated inflation affects employment in the competitive and in the unionized labour market, we take natural logs on both sides of (28) and (16) and use the definitions of the actual and the expected inflation rate, $\pi \equiv \ln P - \ln P_{-1}$ and $\pi^e \equiv \ln P^e - \ln P_{-1}$. We then obtain the following results:

$$\text{Competitive labour market: } \ln L - \ln \bar{L} = \left(\frac{1/\alpha}{1 + 1/\alpha\phi} \right) (\pi - \pi^e), \quad (29)$$

$$\text{Unionized labour market: } \ln L - \ln \bar{L} = (1/\alpha)(\pi - \pi^e). \quad (30)$$

Since $(1/\alpha)/(1 + 1/\alpha\phi) < 1/\alpha$, these equations show that for any given amount of unanticipated inflation, $\pi - \pi^e$, the percentage deviation of employment from its natural level, $\ln L - \ln \bar{L}$, will be larger in the unionized than in the competitive labour market. This may be explained as follows. Suppose that, due to some unanticipated positive shock to aggregate demand for goods, the actual price level rises unexpectedly, driving π above π^e . This increase in the price level will erode the real wage, W/P , thereby stimulating the demand for labour. In the competitive labour market the increase in labour demand immediately drives up the flexible money wage, generating an increase in the expected real wage, W/P^e , which induces workers to supply more labour. Because of the rise in W ,

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the fall in the actual real wage generated by the rise in P will be dampened, which in turn dampens the increase in labour demand and employment. However, in the unionized labour market the nominal wage is fixed at the start of the period, and consequently there is no dampening effect on labour demand stemming from an increase in W . Hence the short-run increase in employment generated by any given amount of unanticipated inflation is greater in the unionized than in the competitive labour market.⁷

We may also say that individual labour supply in the unionized labour market is infinitely elastic. Because there is involuntary unemployment, workers are willing to increase their labour supply in response to an increase in labour demand even if the expected real wage stays constant or falls (as long as the expected real wage does not fall too much). By contrast, in the competitive labour market there is no involuntary unemployment, so labour supply will only increase if the expected real wage goes up. This difference in the short-run flexibility of labour supply explains why employment fluctuates more strongly in the competitive than in the unionized labour market. To convince yourself of this, note from (29) and (30) that if individual labour supply in the competitive labour market were infinitely elastic so that $\phi \rightarrow \infty$, the two equations would be equivalent, implying the same employment response to any given amount of unanticipated inflation.

Equations (29) and (30) also demonstrate the basic point that expectational errors ($\pi \neq \pi^e$) are both necessary *and* sufficient to cause deviations between the actual and the natural level of employment. In other words, it is not really necessary to assume nominal rigidities in order to explain why employment sometimes deviates from its trend level. However, our analysis shows that once expectational errors occur, nominal rigidities will *amplify* the resulting fluctuations in employment. In the real world money wage rates are in fact typically fixed by wage contracts for a certain period of time even in non-unionized labour markets. It is therefore realistic to assume some amount of short-run nominal wage rigidity, so we will continue to work with our model with a pre-set nominal wage rate.

18.4 Supply shocks

Our expectations-augmented Phillips curve (20) postulates a strict deterministic relation between the unemployment rate and the change in the rate of inflation. In this section we shall see that the link between these two variables is not really that tight. The reason is that the labour market is frequently hit by shocks which generate ‘noise’ in the relationship between unemployment and inflation. These so-called supply shocks may take the form of short-run fluctuations in our parameters m^p , m^w and B around their long-run trend levels. Below we will extend our model of unemployment and inflation to account for supply shocks.

In Section 2 we assumed that the rate of unemployment benefit is tied to the level of B (since productivity determines long-run income per capita). In that case we should observe substantial short-run fluctuations in unemployment benefits as B oscillates

7. To put it another way, the short-run aggregate supply curve to be derived in Section 6 is *flatter* when nominal wages are rigid than when they are flexible.

around its long-run growth trend. However, in practice benefits do not move up and down in this way. It is reasonable to assume that benefits are instead linked to the underlying *trend* level of productivity, denoted by \bar{B} , which evolves gradually and smoothly over time. This is equivalent to assuming that benefits are allowed to rise in line with the underlying trend growth in per-capita income.

The magnitude $m^w b$ in the denominator on the right-hand side of (13) is the representative trade union's target real wage. Given our new assumption that $b = c\bar{B}$, the target real wage becomes $m^w c\bar{B}$. Inserting this in (13), we find that the actual level of employment is given by:

$$L = n \cdot \left(\frac{B(1-\alpha)}{m^p m^w c\bar{B}} \cdot \frac{P}{P^e} \right)^{1/\alpha}. \quad (31)$$

Our next step is to redefine the natural level of employment. Specifically, we now define natural employment as the level of employment which will prevail when expectations are fulfilled *and* when productivity as well as the wage and price mark-ups are all at their 'normal' long-run trend levels. Denoting the normal mark-ups by \bar{m}^p and \bar{m}^w and remembering that $b = c\bar{B}$, it follows that the natural employment level previously stated in (15) now modifies to:

$$\bar{L} = n \cdot \left(\frac{1-\alpha}{\bar{m}^p \cdot \bar{m}^w \cdot c} \right)^{1/\alpha}. \quad (32)$$

Dividing (31) by (32) and using the facts that $L \equiv (1-u)N$ and $\bar{L} \equiv (1-\bar{u})N$, we get:⁸

$$\frac{1-u}{1-\bar{u}} = \left(\frac{B \cdot \bar{m}^p \cdot \bar{m}^w}{\bar{B} m^p m^w} \cdot \frac{P}{P^e} \right)^{1/\alpha}. \quad (33)$$

Taking logs on both sides of (33) and using the approximations $\ln(1-u) \approx -u$ and $\ln(1-\bar{u}) \approx -\bar{u}$ plus the definitions $\pi \equiv \ln P - \ln P_{-1}$ and $\pi^e \equiv \ln P^e - \ln P_{-1}$, we end up with:

$$\pi = \pi^e + \alpha(\bar{u} - u) + \tilde{s}, \quad \tilde{s} \equiv \ln\left(\frac{m^p}{\bar{m}^p}\right) + \ln\left(\frac{m^w}{\bar{m}^w}\right) - \ln\left(\frac{B}{\bar{B}}\right). \quad (34)$$

Equation (34) is an expectations-augmented Phillips curve, extended to allow for supply shocks. The specification of the supply shock variable, \tilde{s} , shows that a positive shock to inflation occurs if the wage mark-up or the price mark-up rises above its normal level, whereas a negative shock to inflation occurs if productivity rises above its trend level. By construction, \tilde{s} will fluctuate around a mean value of 0, since \bar{m}^p and \bar{m}^w are the average values of m^p and m^w , respectively, and since B is on average on its trend growth path \bar{B} .

We are now ready to confront our theory of inflation and unemployment with some data.

8. For simplicity, we do not distinguish between the current labour force N and its trend level \bar{N} , implicitly assuming $N = \bar{N}$. In Chapter 14 we saw that the cyclical fluctuations in the labour force tend to be quite modest.

18.5 Testing the Phillips curve theory

Does the Phillips curve theory fit the data?

If we insert our earlier assumption of static inflation expectations, $\pi^e = \pi_{-1}$, into (34), we obtain an equation of the form

$$\Delta\pi = a_0 - a_1 u + \tilde{s}, \quad E[\tilde{s}] = 0, \quad (35)$$

where a_0 and a_1 are positive constants, and $E[\cdot]$ is the expectations operator. Thus our theory implies that the *change* in the rate of inflation should be negatively related to the rate of unemployment. If we have data for inflation and unemployment, we can use econometric techniques (regression analysis) to estimate the magnitude of the parameters a_0 and a_1 . In this way we can check whether the estimated parameter values have the 'correct' positive sign expected from theory, and we can test whether they are significantly different from zero in a statistical sense.

Figure 18.6 shows observations of the unemployment rate and the annual change in the rate of consumer price inflation in the US in the period 1962–1995. The downward-sloping straight line in the figure is a regression line indicating the 'average' relationship between unemployment and the change in inflation. The regression line has the following quantitative properties, where the figures in brackets indicate the standard errors of the estimated coefficients, and where R^2 is the so-called coefficient of determination measuring the share of the variation in $\Delta\pi$ which is explained by our estimated regression line:

$$\Delta\pi = \underset{(se=1.081)}{4.467} - \underset{(se=0.172)}{0.723} \cdot u, \quad R^2 = 0.355. \quad (36)$$

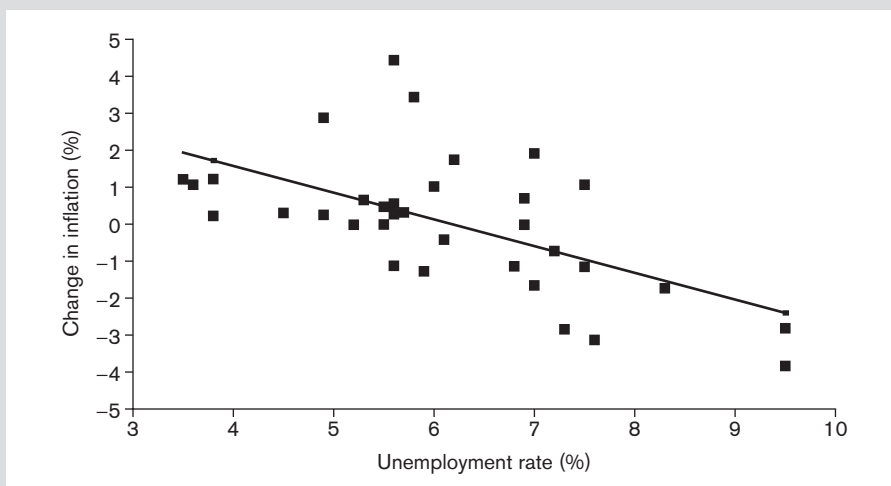


Figure 18.6: Relation between unemployment and the change in inflation in the United States, 1962–1995

Source: R.B. Mitchell, *International Historical Statistics*, Macmillan 1998; and Bureau of Labor Statistics.

The coefficients in (36) do indeed have the signs we would expect from theory. They are also significantly different from 0 in a statistical sense.⁹ Figure 18.6 shows a fairly clear tendency for inflation to fall as unemployment goes up.

Note that the estimated coefficients in (36) enable us to offer an estimate of the natural rate of unemployment in the US. According to (34) and (35) we have $\Delta\pi = \alpha(\bar{u} - u) = a_0 - a_1 u$ if we set \tilde{s} equal to its mean value of 0. Since $\Delta\pi = 0$ when $\bar{u} = u$, it follows that $\bar{u} = a_0/a_1$. Inserting the estimated parameter values from (36), we find that $\bar{u} = 4.467/0.723 \approx 6.2$. This implies that the natural unemployment rate in the US averaged around 6.2 per cent in the estimation period 1962–1995.

In summary, the theory of the expectations-augmented Phillips curve seems roughly consistent with the US data. At the same time we also see that the observed change in inflation has often deviated quite a lot from the estimated regression line. Indeed, the value of R^2 suggests that variations in unemployment can only explain a little over one-third of the variation of inflation. According to our theory, the rest of the variation must be accounted for by the exogenous shocks incorporated in our supply shock variable \tilde{s} .

Given the strong simplifying assumptions we have made, it is not really surprising that our regression equation leaves a lot of the variation in inflation unexplained. Our assumption that inflation expectations are static is rather mechanical. For example, in periods where the fiscal or monetary authorities announce a significant change in economic policies, the private sector may have good reasons to believe that tomorrow's inflation rate will not simply equal the current rate of inflation.¹⁰ As another example, our simple production function (3) abstracts from the fact that production requires inputs of raw materials as well as labour input. Hence our Phillips curve does not capture changes in inflation which are driven by changes in the international price of important raw materials such as oil.

Productivity growth, the Phillips curve and the 'New Economy'

Despite these weaknesses, the important message from Eq. (36) is that there seems to be a systematic and statistically highly significant negative relationship between the *level* of unemployment and the *change* in the rate of inflation. However, in the second half of the 1990s many observers began to question this relationship. The reason was the remarkable performance of the US economy during that period. As you can see from Fig. 18.7, having been located to the far northeast of the unemployment–inflation scatter diagram, during the 1990s the short-run Phillips curve seemed to shift all the way back to the favourable position it had occupied in the 1960s. Apparently this shift was not a simple consequence of a fall in expected inflation generated by the observed fall in actual inflation since the early 1980s. This point is illustrated in Fig. 18.8. The figure compares the actual inflation rate with the rate of inflation predicted from our Phillips curve, (36), which was estimated from data for 1962–1995. We see that from 1996 and onwards, the rate of inflation predicted from the historical link between unemployment and inflation

9. As a rule of thumb, the estimated coefficient should be numerically at least twice as big as its standard deviation to be statistically significant. This condition is easily met in our estimated equation (36). For the benefit of readers who are familiar with regression analysis, the value of the Durbin–Watson statistic is 1.515 which indicates that there are no serious problems of autocorrelation in our regression.

10. In Chapter 21 we shall discuss the formation of private sector expectations in more detail.

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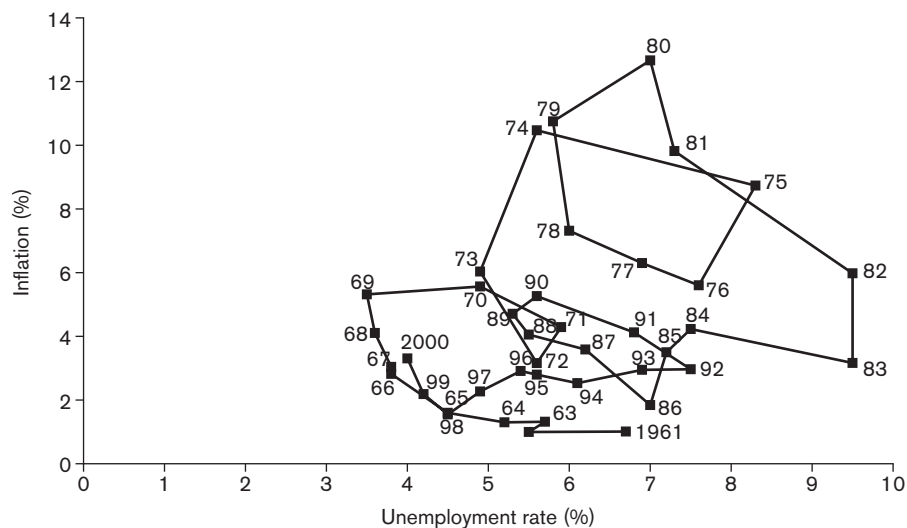


Figure 18.7: The shifting short-run Phillips curve in the United States

Source: R.B. Mitchell, *International Historical Statistics*, Macmillan, 1998; and Bureau of Labor Statistics.

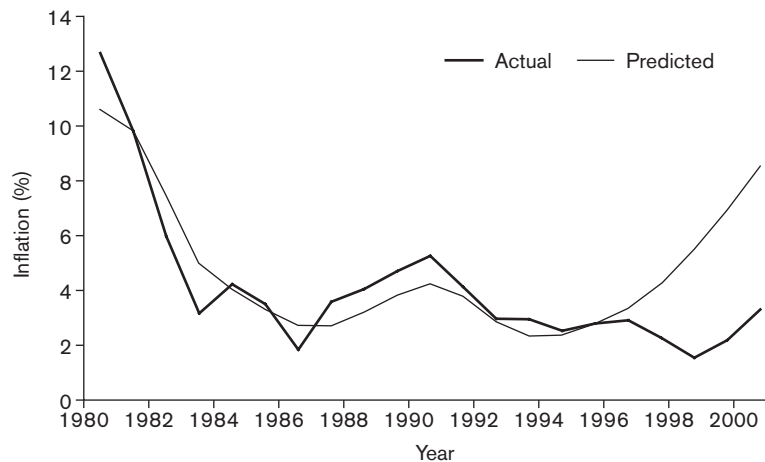


Figure 18.8: Actual and predicted inflation in the United States

Note: The predicted inflation rate was found from Eq. (39).

Source: R.B. Mitchell, *International Historical Statistics*, Macmillan, 1998; and Bureau of Labor Statistics.

systematically overshoots the actual inflation rate. For example, based on the behaviour observed during the 1962–1995 period, one would have expected to see a US inflation rate of 8.5 per cent in 2000, but the actual inflation rate remained subdued at a level of 3.3 per cent, despite the low rate of unemployment. In other words, it seemed that a *structural shift* took place in the US economy around the mid 1990s, causing a breakdown of

the expectations-augmented Phillips curve which had fitted the data reasonably well up until 1995.

At the same time as unemployment fell without driving up the rate of inflation, the growth rate of US labour productivity started to pick up. This is shown in Fig. 18.9. As indicated by the horizontal lines, the average growth rate of labour productivity during the period of the prolonged productivity slowdown from 1974 to 1995 was only 1.35 per cent per year, whereas the average productivity growth rate rose to 2.42 per cent per year during 1996–2001.

Impressed by these developments, many commentators argued that a ‘New Economy’ had arrived which did not obey the ‘old rules of the game’. Some participants in the economic policy debate even claimed that it was time to scrap the established macro-economic theory which apparently could no longer explain what was going on. These proponents of the ‘New Economy’ paradigm argued that it was now possible to maintain a much lower unemployment rate in the US without provoking high and accelerating inflation.

However, with a slight reinterpretation our theory of wage formation actually offers an explanation for recent US developments. Recall that the ‘normal’ productivity level \bar{B} enters our supply shock variable, \tilde{s} , because the target real wage of workers (which we may denote by w^*) is tied to normal productivity:

$$w^* = m^w c \bar{B}. \quad (37)$$

We may interpret the target real wage as a *wage norm* reflecting worker perceptions of what a ‘fair’ or ‘normal’ real wage ought to be. Worker aspirations regarding the growth rate of real wages are presumably influenced by the rate of real wage growth experienced in the past, which is determined by the historical growth in productivity. In (37) the growth over time in the variable \bar{B} captures the speed with which workers believe that their real wages ‘ought’ to grow. During a period like the late 1990s when productivity

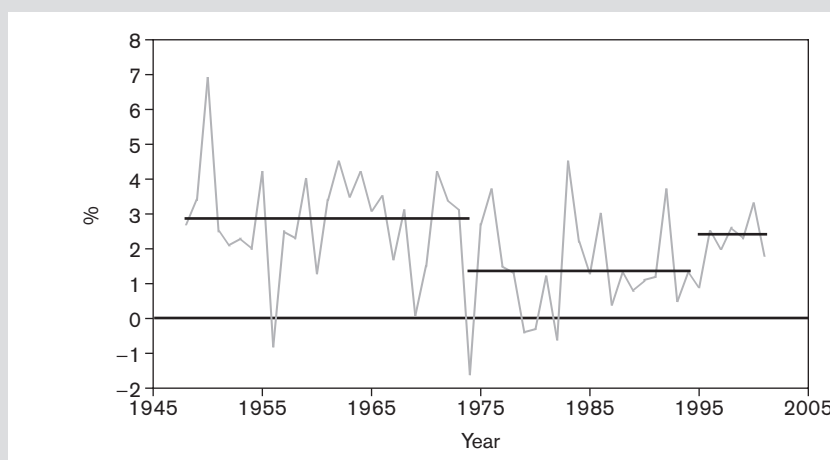


Figure 18.9: Annual growth rate of labour productivity in the United States

Note: Growth rate of output per hour in non-farm business sector. The horizontal lines are average growth rates over the subperiods indicated.

Source: Bureau of Labor Statistics.

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growth was accelerating after a long period of slow growth, the growth in the target real wage (the growth in \bar{B}) will probably lag behind the growth in actual productivity B , since workers emerging from a long period of relatively low productivity growth are still accustomed to a relatively low growth rate in their real earnings. In a time of accelerating productivity growth, the magnitude $\ln(B/\bar{B}) = \ln B - \ln \bar{B}$ included in our supply shock variable \tilde{s} will thus tend to be positive. According to (34) this will reduce the rate of inflation associated with any given level of unemployment and expected inflation, in line with US experience since 1995.

The idea that accelerating productivity growth may explain the favourable shift in the US Philips curve has received empirical support in a recent study by American economists Laurence Ball and Robert Moffitt (2001).¹¹ They show that a Phillips curve which assumes that the real wage growth demanded by workers is a geometrically weighted average of realized past rates of real wage growth (which depend on past productivity growth) can fully account for the favourable US unemployment–inflation trade-off in the late 1990s.

According to the theory of Ball and Moffitt, an *acceleration* of productivity growth can temporarily improve the short-run unemployment–inflation trade-off. However, the theory also implies that once real wage expectations catch up with the higher rate of productivity growth, the rate of inflation will pick up again, restoring the old relationship between inflation and unemployment.

18.6 The aggregate supply curve

Our theory of aggregate demand presented in Chapter 17 implied a systematic link between the output gap (the percentage deviation of output from trend) and the rate of inflation. We shall now show that our theory of inflation and unemployment implies another systematic link between these two variables.

Recall that in a symmetric general equilibrium, total GDP is $Y = nY_i$, and total employment is $L = nL_i$. From (3) we then have:

$$Y = nB \left(\frac{L}{n} \right)^{1-\alpha} = n^\alpha B L^{1-\alpha}. \quad (38)$$

Taking logs on both sides of (38) and using $L \equiv (1-u)N$ plus $\ln(1-u) \approx -u$, we get:

$$\begin{aligned} y &\equiv \ln y = \ln n^\alpha + \ln B + (1-\alpha)\ln[(1-u)N] \\ &\approx \ln n^\alpha + \ln B + (1-\alpha)\ln N - (1-\alpha)u \quad \Leftrightarrow \\ u &= \ln N + \frac{\ln n^\alpha + \ln B - y}{1-\alpha}. \end{aligned} \quad (39)$$

Let us now define ‘natural’ output, \bar{Y} , as the volume of output produced when employment is at its natural level *and* productivity is at its trend level:

$$\bar{Y} = n^\alpha \bar{B} \cdot \bar{L}^{1-\alpha}. \quad (40)$$

11. Laurence Ball and Robert Moffitt, ‘Productivity Growth and the Phillips Curve’, National Bureau of Economic Research, NBER Working Paper 8421, August 2001.

In other words, natural output – sometimes also referred to as potential output – is the level of production prevailing when the economy is on its long-run growth trend. Defining $\bar{y} \equiv \ln \bar{Y}$ and using $\bar{L} \equiv (1 - \bar{u})N$ plus $\ln(1 - \bar{u}) \approx -\bar{u}$, we take logs in (40) and find an expression analogous to (39):

$$\bar{u} = \ln N + \frac{\ln n^\alpha + \ln \bar{B} - \bar{y}}{1 - \alpha}. \quad (41)$$

Substituting (39) and (41) into our expectations-augmented Phillips curve (17) and collecting terms, we end up with the economy's *short-run aggregate supply (SRAS) curve*:

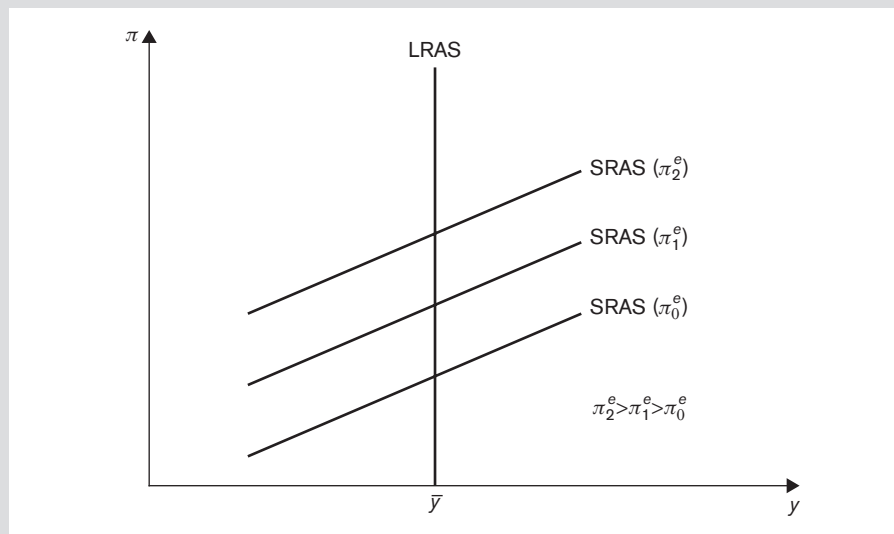
$$\pi = \pi^e + \gamma(y - \bar{y}) + s, \quad (42)$$

$$\gamma \equiv \frac{\alpha}{1 - \alpha} > 0, \quad s \equiv \ln\left(\frac{m^p}{\bar{m}^p}\right) + \ln\left(\frac{m^w}{\bar{m}^w}\right) - \frac{\ln(B/\bar{B})}{1 - \alpha}.$$

The magnitude $y - \bar{y}$ is the percentage deviation of output from trend, referred to as the *output gap*. From (42) we see that, *ceteris paribus*, the rate of inflation varies positively with the output gap. The reason is that a rise in output requires a rise in employment, and because of diminishing marginal productivity of labour, higher employment generates an increase in marginal cost which is translated into an increase in prices via the mark-up pricing behaviour of firms. Equation (42) also implies that the actual rate of inflation varies positively with the expected rate of inflation and with the supply shock variable, s , capturing shocks to mark-ups and to productivity.

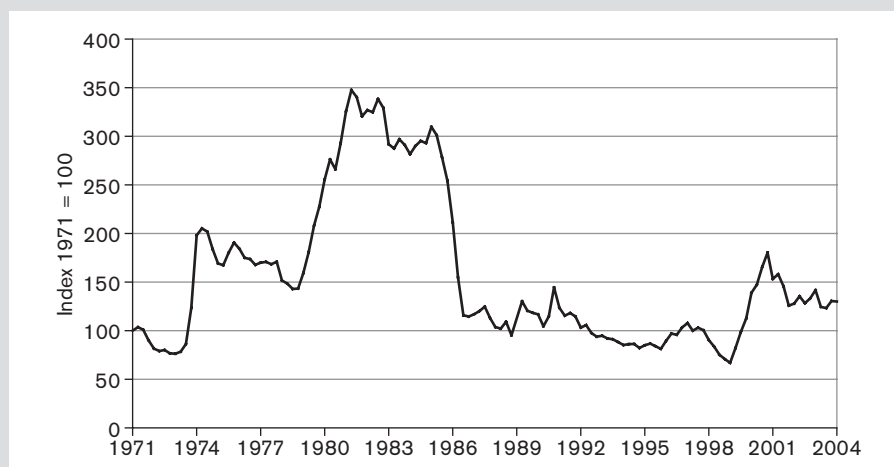
The short-run aggregate supply curve in (42) summarizes the supply side of the economy. Because the expected inflation rate is here taken as given, the curve is a *short-run* relationship. Over time, the expected inflation rate will gradually adjust in reaction to previous inflation forecast errors. When π^e changes, it follows from (42) that the short-run aggregate supply curve will shift upwards or downwards. This is illustrated in Fig. 18.10 which shows three different SRAS curves corresponding to three different levels of expected inflation rate. In long-run equilibrium, when expected inflation equals actual inflation and there are no shocks ($s = 0$), we see from (42) that output must be equal to its 'natural' level, \bar{y} . The natural rate of output is independent of the rate of inflation, since the natural unemployment rate \bar{u} is independent of π . The *long-run aggregate supply (LRAS) curve* is therefore vertical, as shown in Fig. 18.10.

Apart from depending on the expected rate of inflation, the position of the short-run aggregate supply curve also depends on the supply shock variable, s . From (42) we see that the SRAS curve will shift upwards in the case of a positive shock to one of the mark-ups m^w or m^p , or in the case of a negative shock to productivity ($B < \bar{B}$). Note that several types of supply shocks may be modelled as productivity shocks. For example, a loss of output due to industrial conflict may be interpreted as a temporary fall in labour productivity. An unusually bad harvest due to bad weather conditions may likewise be seen as a temporary drop in productivity. An exogenous increase in the real price of imported raw materials such as oil will also work very much like a negative productivity shock. If the price of oil increases relative to the general price level, an economy dependent on imported oil will have to reserve a greater fraction of domestic output for exports to maintain a

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Figure 18.10: Aggregate supply in the short run (SRAS) and in the long run (LRAS)

given volume of oil imports. Thus, for given inputs of domestic labour and capital, a lower amount of domestic output will be available for domestic consumption, just as if factor productivity had declined. More generally, any exogenous change in the economy's international terms of trade (a shift in import prices relative to export prices) may be modelled as a productivity shock in our AS-AD model.

Over the last three decades, the real price of energy inputs has fluctuated considerably, as illustrated in Fig. 18.11. For example, following political upheaval in the Middle


Figure 18.11: The real price of fuel imports in Denmark

Source: MONA database.

East, the OPEC cartel of oil-exporting countries was able to raise the real price of oil quite dramatically in 1973–74 and again in 1979–80. Because most OECD economies were large net importers of oil at the time, these oil price shocks worked like a significant negative productivity shock for the OECD area. On the other hand, the collapse of oil prices from around 1985 tended to boost real incomes in the OECD, just like a positive productivity shock.

This completes our theory of the aggregate supply side. In the next chapter we shall see how aggregate supply interacts with aggregate demand to determine total GDP as well as the rate of inflation.

18.7 Summary

1. The link between inflation and unemployment determines how the supply side of the economy works. Some decades ago most economists and policy makers believed in the simple Phillips curve which postulates a permanent trade-off between inflation and unemployment: a permanent reduction in the rate of unemployment can be only achieved by accepting a permanent increase in the rate of inflation, and vice versa.
2. Empirically the simple Phillips curve broke down in the stagflation of the 1970s. This led to the theory of the expectations-augmented Phillips curve which says that the simple Phillips curve is just a short-run trade-off between inflation and unemployment, existing only as long as the expected rate of inflation is constant. When the expected inflation rate goes up, the actual inflation rate increases by a corresponding amount, other things equal.
3. The expectations-augmented Phillips curve implies the existence of a 'natural' rate of unemployment, defined as the level of unemployment which will prevail in a long-run equilibrium where the expected inflation rate equals the actual inflation rate. Since any fully anticipated rate of inflation is compatible with long-run equilibrium, the long-run Phillips curve is vertical. When the actual inflation rate exceeds the expected inflation rate, the actual unemployment rate falls below the natural unemployment rate, and vice versa.
4. Several different theories of wage and price formation lead to the expectations-augmented Phillips curve. One such theory is the 'sticky-wage model' in which nominal wage rates are pre-set at the start of each period. In the sticky wage model presented in this chapter, money wages are dictated by trade unions seeking to achieve a certain target real wage, given their expectations of the price level which will prevail over the next period. Given the wage rate set by unions, profit-maximizing monopolistically competitive firms set their prices as a mark-up over marginal costs and choose a level of employment which is declining in the actual real wage. According to this model, employment increases above its natural rate when the actual price level exceeds the expected price level, and vice versa. The model also implies that, in general, there is some amount of involuntary unemployment.
5. In the sticky-wage model the target real wage is a mark-up over the opportunity cost of employment which is given by the rate of unemployment benefit. The wage mark-up factor – and hence the target real wage – is higher the lower the wage elasticity of labour demand,

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and the lower the weight the union attaches to the goal of high employment relative to the goal of a high real wage. The mark-up of prices over marginal costs is higher the lower the price elasticity of demand for the output of the representative firm. The natural rate of unemployment is higher the higher the wage and price mark-ups and the more generous the level of unemployment benefits.

6. Another theory leading to the expectations-augmented Phillips curve is the 'worker-misperception model' which assumes a competitive clearing labour market with fully flexible wages. Labour demand is a declining function of the actual real wage, while labour supply is an increasing function of the expected real wage, since workers are imperfectly informed about the current general price level. This model also implies that employment rises above the natural level when the actual price level exceeds the expected price level. However, for any given amount of unanticipated inflation, the increase in employment is smaller in the worker-misperception model than in the sticky wage model where nominal wage rates are fixed in the short run. Even in the absence of nominal rigidities, unanticipated inflation will thus generate deviations of employment from the natural rate, but nominal rigidities will amplify the fluctuations in employment.
7. According to the hypothesis of static expectations, the expected inflation rate for the current period equals the actual inflation rate observed during the previous period. Combined with the expectations-augmented Phillips curve, the assumption of static expectations implies that the rate of inflation will keep on accelerating (decelerating) when actual unemployment is below (above) the natural unemployment rate.
8. So-called supply shocks in the form of fluctuations in productivity and in the wage and price mark-ups create 'noise' in the relationship between inflation and unemployment. An unfavourable supply shock implies an increase in the actual rate of inflation for any given levels of unemployment and expected inflation. In the presence of supply shocks the natural unemployment rate is defined as the rate of unemployment prevailing when inflation expectations are fulfilled and productivity as well as the wage and price mark-ups are at their trend levels.
9. An expectations-augmented Phillips curve with static inflation expectations is consistent with US data on inflation and unemployment in the period from the early 1960s to the mid 1990s. In the 'New Economy' of the late 1990s inflation was surprisingly low, given the low rate of unemployment prevailing during that period. This experience may be seen as a result of a favourable supply shock arising from the fact that target real wages were lagging behind the accelerating rate of productivity growth.
10. The economy's short-run aggregate supply curve (the SRAS curve) implies a positive link between the output gap and the actual rate of inflation, given the expected rate of inflation. The SRAS curve may be derived from the expectations-augmented Phillips curve, using the production function which links the unemployment rate to the level of output. The SRAS curve shifts upwards when the expected inflation rate goes up, or when the economy is hit by an unfavourable supply shock. When there are no supply shocks and expected inflation equals actual inflation, the economy is on its long-run aggregate supply curve (the LRAS curve) which is vertical at the natural level of output corresponding to the natural rate of unemployment.

18.8 Exercises

Exercise 1. Intersectoral labour mobility and the expectations-augmented Phillips curve

In Section 2 we abstracted from intersectoral labour mobility by assuming that individual workers are educated and trained to work in a particular sector. In that case a worker's outside option (the income he could expect to earn if he were not employed by his current employer) is simply equal to the rate of unemployment benefit. In this exercise we ask you to show that one can still derive the expectations-augmented Phillips curve from a trade union model of wage setting even if unemployed workers can move between sectors in their search for jobs, as we assumed in our analysis of the labour market in Chapters 1, 12 and 13.

To simplify matters a bit, we now set our productivity parameter $B = 1$ and work with a linear production function with constant marginal productivity of labour, corresponding to $\alpha = 0$ in Eq. (3) in the main text. For $\alpha = 0$ and $B = 1$, it follows from (7) in Section 2 that the representative firm will set its product price as:

$$P_i = m^p W_i, \quad m^p > 1. \quad (43)$$

As in Section 2, an optimizing monopoly union for workers in firm (sector) i will set the nominal wage rate W_i to attain an expected real wage which is a mark-up over the expected real value of its members' outside option, denoted by v^e . The union expects the current price level to be P^e . Hence it will set the wage rate:

$$\frac{W_i}{P^e} = m^w \cdot v^e, \quad m^w > 1. \quad (44)$$

This equation is a parallel to Eq. (11) in Section 2 where we assumed that v^e is simply equal to the real rate of unemployment benefit, b . Here we assume instead that workers who are initially members of the trade union for sector i are qualified to apply for a job in other sectors if they fail to find one in sector i . For an average job seeker, the probability of finding work is equal to the rate of employment, $1 - u$, where u is the unemployment rate which gives the probability that an average job seeker will remain unemployed. If the ratio of unemployment benefits to the average wage level is equal to the constant c , we thus have:

$$v^e = (1 - u)w^e + ub^e = (1 - u + uc)w^e, \quad 0 < c < 1, \quad (45)$$

where w^e is the expected average level of real wages, and $b^e = cw^e$ is the expected real rate of unemployment benefit.

Since the outside option v^e is the same across all sectors and all unions are assumed to hold the same price expectations, it follows from (44) and (43) that all unions will charge the same nominal wage rate, W , and that all firms will charge the same output price, P :

$$W_i = W, \quad P_i = P. \quad (46)$$

According to (43) the average real wage will then be $W/P = 1/m^p$. Let us suppose that union wage setters have a realistic estimate of the average real wage so that:

$$w^e = 1/m^p. \quad (47)$$

1. Show that the relationship between the actual and the expected price level may be written as:

$$P = m^w(1 - \gamma u)P^e, \quad \gamma \equiv 1 - c > 0. \quad (48)$$

2. Use (48) to derive an expectations-augmented Phillips curve of the same form as equation (17) in Section 2. (Hint: use the approximation $\ln(1 - \gamma u) \approx -\gamma u$ and define $\bar{u} \equiv \ln m^w/\gamma$.) Explain intuitively what determines the natural unemployment rate, \bar{u} . Explain intuitively what determines the slope (γ) of the expectations-augmented Phillips curve.
3. Does the theory embodied in (43)–(48) assume short-run nominal wage rigidity?
4. The specification of the outside option (45) assumes that unions know the current aggregate unemployment rate when they set the wage rate. Suppose instead that unemployment statistics are published with a lag so that unions base their estimate of the outside option on last period's recorded unemployment rate, u_{-1} :

$$v^e = (1 - u_{-1})w^e + u_{-1}b^e = (1 - u_{-1} + u_{-1}c)w^e. \quad (49)$$

Derive the expectations-augmented Phillips curve on the assumption that the perceived outside option is given by (49) rather than (45). Do we now have nominal wage rigidity in the short run?

Exercise 2. Wage setting, labour demand and unemployment

1. Explain why union wage claims are moderated by a higher price elasticity in the representative firm's product demand curve.
2. In the text we found that the wage elasticity of labour demand is higher at the sectoral level than at the aggregate level. Explain why this is so.
3. 'Tougher product market competition will reduce structural unemployment'. Explain this statement. Discuss what the government could do in practice to promote fiercer product market competition.

Exercise 3. The Phillips curve with endogenous price mark-ups

In the main text we assumed for simplicity that the representative firm's mark-up m^p of price over marginal cost was an exogenous constant. However, empirical research for the United States suggests that price mark-up factors in that country tend to move in a countercyclical fashion. In other words, the mark-up tends to fall during business cycle expansions and to rise during recessions. There are several potential reasons for this countercyclical behaviour of mark-ups, including the possibility that during booms when the demand pressure is high, more new firms find it profitable to enter the market, thereby increasing the degree of competition and forcing existing firms to reduce their profit margins.

Since the rate of unemployment moves countercyclically, the countercyclical variation of the price mark-up means that m^p will tend to move in the same direction as the unemployment rate. For concreteness, suppose this relationship takes the form:

$$m^p = \tilde{m} \cdot e^{\varphi u}, \quad \tilde{m} > 1, \quad \varphi \geq 0, \quad (50)$$

where e is the exponential function, and φ is a parameter measuring the sensitivity of the mark-up to changes in the unemployment rate, u . In the main text of the chapter we focused on the case of $\varphi = 0$, corresponding to a constant mark-up. In this exercise we ask you to study the implications of assuming $\varphi > 0$ which is more in line with the empirical evidence for the US.

As in Exercise 1, we consider an economy with intersectoral labour mobility, union wage setting and mark-up price setting. Hence we have (see Exercise 1 in case you need further explanation):

$$P_i = m^p W_i \quad (51)$$

$$\frac{W_i}{P^e} = m^w \cdot v^e, \quad m^w > 1. \quad (52)$$

$$v^e = (1 - u)w^e + ub^e = (1 - u + uc)w^e, \quad 0 < c < 1, \quad (53)$$

where w^e is the expected average level of real wages, $b^e = cw^e$ is the expected real rate of unemployment benefit, and v^e is the individual worker's expected outside option. The outside option is the same across all sectors, so all unions charge the same nominal wage rate and all firms charge the same price:

$$W_i = W, \quad P_i = P. \quad (54)$$

Union wage setters assume that the average real wage is:

$$w^e = 1/\bar{m}, \quad \bar{m} > 1, \quad (55)$$

where the constant \bar{m} is the expected 'normal' price mark-up.

1. Show that the model consisting of (50)–(55) implies an expectations-augmented Phillips curve of the form:

$$\pi = \pi^e + (1 - c - \varphi)(\bar{u} - u), \quad \bar{u} \equiv \frac{\ln \tilde{m} + \ln m^w - \ln \bar{m}}{1 - c - \varphi}, \quad (56)$$

where $\pi \equiv p - p_{-1}$, $\pi^e \equiv p^e - p_{-1}^e$, and $p \equiv \ln P$, and where you may assume that $1 - c - \varphi > 0$ to ensure a positive solution for the natural unemployment rate, \bar{u} . Explain intuitively how the parameter φ affects the sensitivity of inflation to unemployment. Explain and discuss the various factors determining the natural unemployment rate.

2. Suppose that union wage setters expect the 'normal' price mark-up \bar{m} appearing in (55) to be

$$\bar{m} = \tilde{m} \cdot e^{\varphi \bar{u}}. \quad (57)$$

Discuss whether the assumption made in (57) is reasonable. Derive a new expression for the natural unemployment rate \bar{u} on the assumption that (57) holds. Compare the new expression to the expression for \bar{u} given in (56) and comment on the differences.

Exercise 4. The Phillips curve and active labour market policy

The model with intersectoral labour mobility presented in Exercise 1 assumes that all workers are competing on equal terms and with equal intensity for the available jobs. In that case it seems reasonable that any individual worker's probability of finding a job is simply equal to the overall rate of employment, $1 - u$, as we assumed when specifying a worker's outside option.

In the present exercise we assume instead that some of the workers recorded in the unemployment statistics are not fully 'effective' in competing for jobs, perhaps because their skills do not fully match the qualifications demanded by employers, or perhaps because they are not actively searching for a job all the time. We may model this in a simple way by assuming that only a fraction, s , of the registered unemployed workers contribute fully to the

available labour supply in the sense of being immediately ready and able to accept the jobs available. Thus we may specify the 'effective' labour supply from the pool of unemployed workers as su . In the following, we will refer to s as the Job-Search-and-Matching-Efficiency parameter (the JSME parameter). For the moment, we will assume that s is an exogenous constant.

If we normalize the total labour force to equal 1, there are thus $(1-s)u$ unemployed workers who are not really competing with their fellow workers for a job. Hence we may measure the 'effective' labour force as $1 - (1-s)u$. For an average member of the effective labour force (a qualified person who is ready to accept the available jobs), the probability p^u of ending up in the unemployment pool is the ratio of the 'effective' number of unemployed to the 'effective' labour force, $p^u = su/[1 - (1-s)u]$. By implication, a qualified person's probability of finding a job in the labour market is $1 - p^u$. If the expected average real wage is w^e , and if the ratio of the unemployment benefit to the average wage level is c , we may therefore specify the real value of a qualified worker's expected outside option as:

$$v^e = \left(1 - \frac{su}{1 - (1-s)u}\right)w^e + \left(\frac{su}{1 - (1-s)u}\right) \overbrace{cw^e}^{\text{unemployment benefit}} = \left(\frac{1 - (1-cs)u}{1 - (1-s)u}\right)w^e, \quad 0 < c < 1. \quad (58)$$

The trade union for sector i expects the current price level to be P^e and sets its nominal wage rate W_i to attain an expected real wage W_i/P^e which is a mark-up over the outside option of the employable union members:

$$\frac{W_i}{P^e} = m^w v^e, \quad m^w > 1. \quad (59)$$

The representative firm uses a linear production function with $\alpha = 0$ and $B = 1$, so according to Eq. (7) in the main text it sets its price P_i as:

$$P_i = m^p W_i, \quad m^p > 1. \quad (60)$$

Since v^e is the same across all sectors, it follows from (59) and (60) that all unions will set the same wage rate and that all firms will charge the same price:

$$W_i = W, \quad P_i = P. \quad (61)$$

In accordance with (60) and (61), the representative union expects the average real wage to be:

$$w^e = 1/m^p. \quad (62)$$

1. Demonstrate through a logarithmic approximation that the model (58)–(62) leads to an expectations-augmented Phillips curve of the form

$$\pi = \pi^e + s\gamma(\bar{u} - u), \quad \gamma \equiv 1 - c > 0, \quad \bar{u} \equiv \frac{\ln m^w}{s\gamma}, \quad (63)$$

where $\pi \equiv p - p_{-1}$, $\pi^e \equiv p^e - p_{-1}^e$, and $p \equiv \ln P$. (Hint: use the approximations $\ln[1 - (1-cs)u] \approx -(1-cs)u$ and $\ln[1 - (1-s)u] \approx -(1-s)u$.) Explain in economic terms how the JSME parameter s affects the sensitivity of inflation to unemployment.

Let us now analyse the effects of active labour market policy. Suppose that a fraction l of the unemployed workers is enrolled in public education and training programmes aimed at

improving their qualifications for the available jobs. Such programmes may increase the JSME parameter s partly by improving the match between the qualifications demanded by employers and the skills possessed by the unemployed, and partly by increasing workers' motivation to look for jobs (say, because it makes more attractive jobs available to them). Hence we assume that s is an increasing function of l :

$$s = \bar{s} \cdot l^\eta, \quad 0 < \bar{s} < 1, \quad \eta > 0, \quad 0 < l < 1. \quad (64)$$

The elasticity η is a parameter measuring the degree to which the labour market programmes succeed in actually upgrading the skills and motivation of the unemployed. However, (64) does not capture all of the effect of active labour market programmes. When people are enrolled in such a programme, they will often not be immediately available for a job should they receive a job offer, or they may not have the time to look for a job. For simplicity, let us assume that only that fraction $1 - l$ of the unemployed which is not currently engaged in education and training is able to take a job. Moreover, let us assume that these job seekers have benefited from previous training so that their Job-Search-and-Matching-Efficiency corresponds to the value of s specified in (64). The 'effective' labour supply coming from the pool of unemployed workers is then given by:

$$\text{'Effective' unemployment rate} = s(1 - l)u, \quad (65)$$

where s is determined by (64). The effective labour force consists of those unemployed workers who are effectively available for work, $s(1 - l)u$, plus those who are already employed, $1 - u$. Thus a qualified worker's probability of being unemployed is:

$$p^u = \frac{s(1 - l)u}{1 - u + s(1 - l)u} = \frac{s(1 - l)u}{1 - [1 - s(1 - l)]u}. \quad (66)$$

In the questions below we will assume that l is a policy instrument controlled by the makers of labour market policy. Furthermore, we assume that workers enrolled in active labour market programmes receive the same unemployment benefits and enjoy the same utility as unemployed workers who are not enrolled in programmes. This implies that active labour market policy has no effect on the outside option other than the effect working through the impact of l on p^u .

- Following the same procedure as in Question 1, demonstrate through a logarithmic approximation that when effective unemployment is given by (65) and the JSME parameter is given by (64), we obtain an expectations-augmented Phillips curve of the form:

$$\pi = \pi^e + \hat{\gamma}(\bar{u} - u), \quad \hat{\gamma} \equiv (1 - c)\bar{s}l^\eta(1 - l), \quad \bar{u} \equiv \frac{\ln m^w}{\hat{\gamma}}. \quad (67)$$

(Hints: start by using (66) to respecify v^e . Later on, when you take logs, use the approximations $\ln\{1 - [1 - cs(1 - l)]u\} \approx -[1 - cs(1 - l)]u$ and $\ln\{1 - [1 - s(1 - l)]u\} \approx -[1 - s(1 - l)]u$.)

- How does the natural unemployment rate react to an increase in the proportion of the unemployed enrolled in active labour market programmes? (Hint: derive $\partial \bar{u} / \partial l$.) Explain the offsetting effects of an increase in l .
- Suppose that the government wishes to minimize the natural rate of unemployment through its active labour market programmes. Derive the value of l which will achieve this goal.

(Hint: remember that a necessary condition for minimization of \bar{u} is $\partial \bar{u} / \partial l = 0$.) Give an intuitive interpretation of your result. Discuss briefly whether the government should necessarily push active labour market policy to the point implied by your formula. (Hint: are there any costs and benefits of active labour market policy which we have not included in our analysis?)

5. Suppose that unemployed workers prefer not to be enrolled in active labour market programmes, say, because they consider enrolment to be stigmatizing, or because it reduces their leisure time. Discuss whether this would make active labour market policy more or less effective as a means of reducing structural unemployment.

Exercise 5. The Phillips curve with a time-varying NAIRU

Empirical estimates of the natural unemployment rate (the NAIRU) typically find that the evolution of the NAIRU tends to track the evolution of the actual rate of unemployment, at least in the short and medium run. This exercise extends our theory of the Phillips curve in order to explain why the NAIRU tends to move in the same direction as the actual unemployment rate in the shorter run.

We consider the following model with intersectoral labour mobility, where we apply the same notation as in Exercise 1, and where p^u is an average worker's probability of remaining out of work if he fails to find a job in his original sector:

$$\text{Price formation: } P_i = m^p W_i, \quad m^p > 1, \quad (68)$$

$$\text{Wage claim of the representative union: } \frac{W_i}{P^e} = m^w v^e, \quad m^w > 1, \quad (69)$$

$$\text{The 'outside option' of union members: } v^e = (1 - p^u)w^e + p^u b^e, \quad (70)$$

$$\text{Expected average real wage: } w^e = 1/m^p, \quad (71)$$

$$\text{Expected real rate of unemployment benefit: } b^e = c w^e, \quad 0 < c < 1, \quad (72)$$

Probability of remaining unemployed in case of job loss:

$$p^u = u + \theta(u - u_{-1}), \quad \theta \geq 0, \quad p^u \leq 1 \quad (73)$$

The variables u and u_{-1} are the unemployment rates in the current and in the previous period, respectively. The new feature of the model above is Eq. (73) which says that, *ceteris paribus*, an unemployed worker has a smaller chance of finding a job if unemployment is rising than if unemployment is falling.

1. Discuss briefly whether the specification in (73) is plausible.
2. Use the model (68)–(73) to derive an expectations-augmented Phillips curve of the form:

$$\pi = \pi^e + \ln m^w - \gamma u - \gamma \theta(u - u_{-1}), \quad \gamma \equiv 1 - c > 0, \quad (74)$$

where $\pi \equiv p - p_{-1}$, $\pi^e \equiv p^e - p_{-1}^e$, and $p \equiv \ln P$. (You may use the usual approximation $\ln(1 - x) \approx -x$ which is valid as long as x is not too far from zero.) Comment on the expression in (74) and compare with the expectations-augmented Phillips curve derived in the main text of the chapter.

In the following we assume that inflation expectations are static so that $\pi^e = \pi_{-1}$.

3. Define the *long-run NAIRU* as the rate of unemployment \bar{u} which will be realized when the rate of inflation as well as the rate of unemployment are constant over time, that is when

$\pi = \pi_{-1}$ and $u = u_{-1}$. Derive an equation for the long-run NAIRU and use this expression to explain the factors which determine the equilibrium rate of unemployment in the long run.

4. Define the *short-run NAIRU* as the rate of unemployment \bar{u}_s which will be compatible with a constant inflation rate in the *short* run where we do not necessarily have $u = u_{-1}$. (At the short-run NAIRU we thus have $\pi = \pi_{-1}$ but not necessarily $u = u_{-1}$.) Derive an expression for the short-run NAIRU and show that \bar{u}_s may be written as a weighted average of the long-run NAIRU and last period's actual rate of unemployment u_{-1} . Which parameter determines how much the short-run NAIRU is affected by last period's actual unemployment rate?
5. Assume that in period 0 unemployment increases from the long-run NAIRU (\bar{u}) to the level $\bar{u} + \Delta u_0$. Will it be possible to return to the unemployment rate \bar{u} in period 1 without creating higher inflation? Give reasons for your answer.

Exercise 6. Estimating the time-varying NAIRU

In Section 5 we saw that the natural unemployment rate in the US seems to have varied over time. In this exercise we invite you to estimate the level and variation in the NAIRU and to investigate whether changes in the underlying rate of productivity growth may help to explain the evolution of the NAIRU.

If inflation expectations are static, the expectations-augmented Phillips curve allowing for supply shocks (\tilde{s}) takes the form:

$$\pi - \pi_{-1} = \gamma(\bar{u} - u) + \tilde{s}$$

which may be rearranged to give:

$$\bar{u} + \tilde{s}/\gamma = u + \Delta\pi/\gamma, \quad \Delta\pi \equiv \pi - \pi_{-1}. \quad (75)$$

Thus we may see the movements in the magnitude $u + \Delta\pi/\gamma$ on the right-hand side of (75) to be a result of gradual movements in the NAIRU, \bar{u} , as well as a result of the shorter-term and more erratic supply shocks captured by \tilde{s} . If we have somehow obtained an estimate of the parameter γ , we may construct an estimate of $\bar{u} + \tilde{s}/\gamma$ by calculating $u + \Delta\pi/\gamma$, using available data on unemployment and inflation. We may then use the HP filter introduced in Chapter 14 to split our estimate of $\bar{u} + \tilde{s}/\gamma$ into a smooth underlying trend, which we interpret as an estimate of \bar{u} , and a residual term which we take to reflect \tilde{s}/γ . This is the methodology we ask you to follow below.

1. The first step is to obtain an estimate of our parameter γ . At the internet address www.econ.ku.dk/pbs/courses/models&data.htm you will find annual data on the unemployment rate and the rate of inflation in the US for the period 1959–2003. Using these data, estimate a standard expectations-augmented Phillips curve of the form:

$$\Delta\pi = a_0 + a_1 u + \tilde{s}, \quad (76)$$

by performing an OLS regression analysis for the period 1960–2003. (You may assume that \tilde{s} is normally distributed with a zero mean and a constant variance.) Is your estimate of a_1 statistically significant and does it have the expected sign? Assuming you can answer in the affirmative, you may use your estimate of the numerical value of a_1 as an estimate of γ in your further analysis.

2. Armed with your estimate of γ and your data set, you can now calculate a time series for the magnitude $u + \Delta\pi/\gamma$. In this way you obtain an estimated time series for $\bar{u} + \bar{s}/\gamma$ for the period 1960–2003. At the internet address www.econ.ku.dk/pbs/courses/models&data.htm you will also find a link to a software facility enabling you to estimate a trend by means of the HP filter (plus a brief guide on how to use this program). Use this facility to estimate an HP trend in your time series for $\bar{u} + \bar{s}/\gamma$, setting the λ parameter equal to 1000 to obtain a quite smooth trend. Interpret your HP trend as an estimate of \bar{u} and construct a diagram in which you plot your estimated time series for the NAIRU. What is the range within which the NAIRU has varied?

In Section 5 we argued that the trade-off between the level of unemployment and the change in the rate of inflation will tend to improve in periods of accelerating productivity growth, and vice versa. According to (75), a longer-lasting improvement in the trade-off between u and $\Delta\pi$ (that is, a fall in $u + \Delta\pi/\gamma$) must reflect a fall in the NAIRU, \bar{u} . Thus the NAIRU should tend to fall when the underlying rate of productivity growth accelerates, whereas \bar{u} should tend to rise when underlying productivity growth slows down. The next question asks you to explore this relationship.

3. The internet address www.econ.ku.dk/pbs/courses/models&data.htm contains annual data on the growth rate in output per hour worked in the US for the period 1959–2003. Use the HP filter to estimate an underlying trend in this productivity growth rate, setting $\lambda = 1000$. Plot the resulting estimate of trend productivity growth against your estimate of the NAIRU. Do the two time series tend to move in opposite directions, as our theory predicts? Explain the theoretical reasons why accelerating productivity growth may be expected to reduce the NAIRU, at least temporarily.

(Postscript: if you would like to know more about the likely reasons for the variations in the US NAIRU, you may want to consult the following readable article from which the idea for this exercise was taken: Laurence Ball and N. Gregory Mankiw, 'The NAIRU in Theory and Practice', *Journal of Economic Perspectives*, **16** (4), Fall 2002, pp. 115–136.)