The IS-LM Model

Chapter 25 introduced the Keynesian model and defined short-run equilibrium as a position in which the level of output *Y* equals planned aggregate expenditure *PAE.* Chapter 27 takes the analysis a step further by introducing the money market and allowing planned expenditures to vary with the rate of interest. Chapter 27 also established that for a given level of output the equilibrium rate of interest is determined by equality between the demand for and supply of money. Hence, in this expanded model the economy consists of two markets, the market for goods and services and the market for money.

These markets are inter-linked. For a given rate of interest, as determined in the money market, we can use the model describing the market for goods and services to determine the equilibrium level of output. Conversely, for a given level of output, as determined in the market for goods and services, we can use the model describing the market for money to determine the equilibrium value of the rate of interest. Furthermore, a change in the equilibrium in one market will lead to a change in the equilibrium in the other market. For example, as we saw in Chapter 26 an increase in autonomous expenditure leads, via the income-expenditure multiplier, to an increase in the level of output. However, as explained in Chapter 27 an increase in aggregate income increases the demand for money and, given the supply of money, leads to a higher equilibrium interest rate. Likewise, an increase in the money supply reduces the equilibrium rate of interest which stimulates consumption and investment and, via the multiplier, increases the equilibrium level of output.

In short, we cannot determine equilibrium in one market independently from equilibrium in the other. Rather, we must determine the equilibrium values of income and the interest rate simultaneously. This task is achieved by the IS-LM model which uses the equilibrium conditions in each market to simultaneously determine the equilibrium values for the level of output and the rate of interest.

The IS Curve.

The IS curve plots combinations of the rate of interest and the level of output for which the market for goods and services is in equilibrium. Recall that in the appendix to Chapter 27 we saw that the short-run equilibrium in the market for goods and services can be described by the equation:

$$Y = \left(\frac{1}{1-c}\right)\left[\overline{A} - (a+b)r\right] \qquad (1)$$

Where $\overline{\boldsymbol{A}} = \overline{\boldsymbol{C}} - \boldsymbol{c}\overline{\boldsymbol{T}} + \overline{\boldsymbol{I}} + \overline{\boldsymbol{G}} + \overline{\boldsymbol{NX}}$ and the term *–(a + b)r* captures the idea that when the *real rate of interest r* rises or falls planned expenditures fall or rise by *(a + b)* times the change in *r*. We have also seen in Chapter 19 that the real rate of interest equals the difference between the nominal rate of interest (*i*) and the rate of inflation (*π*). That is, $r = i - \pi$. However in Chapter 27 we assumed that because inflation changes slowly the central bank can control the real rate of interest over the short-run. To simplify we shall assume that inflation is constant and equal to zero so that $r = i$ in the short-run. For convenience we shall also assume that the economy is closed to international trade and capital flows ($NX = KI = 0$) and that the term ($a + b$) equals a value denote by f. Given these assumptions we can write Equation 1 as:

$$Y = \left(\frac{1}{1-c}\right)\left[\overline{A} - fi\right] \qquad (2)$$

Equation (2) defines (Y, i) combinations which give equilibrium in the market for goods and services and is known as the IS equation. Figure 1 presents a graphical illustration of this equation. The top part of Figure 1 illustrates the determination of equilibrium Y and is similar to the Keynesian Cross diagram, Figure 27.6 in the text. With a rate of interest equal to i_1 the expenditure line is PAE_1 and equilibrium output is determined at the point A on the 45 degree line ($Y = PAE$) with $Y = Y_1$. The bottom part of Figure 1 has the interest rate on the vertical axis and output on the horizontal. The point C defines the interest rate, output combination (i_1, Y_1) which gives equilibrium in the market for goods and services. If the interest rate were to fall to a lower level i_2 then, as illustrated in Chapter 27, the expenditure line shifts up to PAE_2 and equilibrium output is determined at the point B on the 45 degree line ($Y = PAE$) with $Y = Y_2$. In the bottom part of Figure 1 the point D defines a second interest rate, output combination (i_2, Y_2) which also gives equilibrium in the market for goods and services. The points C and D are (i, Y) combinations which determine equilibrium in the market for goods and services. The line joining these points is known as the *IS curve.*[1]

EXAMPLE 1: The IS Curve

In a certain economy, $c = 0.8$, $f = 1{,}000$ and $\overline{A} = 1{,}010$. Derive the IS curve when $i = 0.05$, or 5% and when $i = 0.01$, or 1% .

Using Equation (1) for:

$$i = 0.05 : Y = \left(\frac{1}{0.2}\right)\left[1{,}010 - 1{,}000(0.05)\right] = 4{,}800$$
$$i = 0.01 : Y = \left(\frac{1}{0.2}\right)\left[1{,}010 - 1{,}000(0.01)\right] = 5{,}000$$

Hence in Figure 1 the point C corresponds to an (i, Y) combination (0.05, 4,800) and the point D to a combination (0.01, 5,000). As both combinations give equilibrium in the market for goods and services both lie on the IS curve.

[1] The IS, or Investment-Saving, curve gets its name from the fact that in a closed economy without international trade and capital flows, the equilibrium level of Y also corresponds to equality between domestic saving and investment. See Chapter 24.

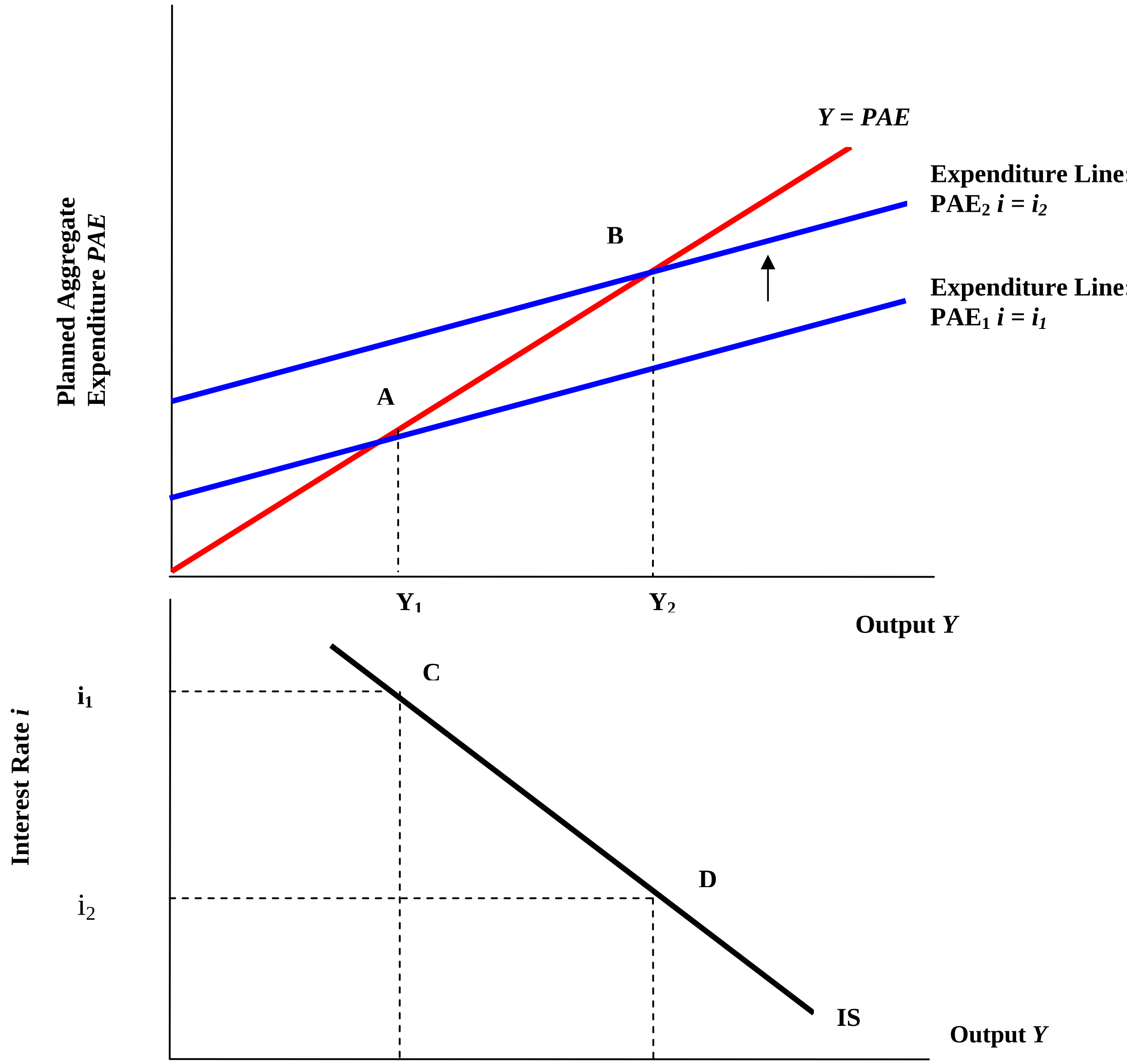

Figure 1. The IS Curve.
The IS curve traces interest rate and output combinations which give equilibrium in the market for goods and services.

The Slope of the IS Curve: The IS curve has a negative slope because a reduction in the interest rate stimulates planned aggregate expenditure and via the multiplier leads to a higher equilibrium level of output. That is, to maintain equilibrium in the market for goods and services *i* and *Y* must change in opposite directions. To derive the slope of the IS curve we can rewrite Equation (2) as:

$$\boldsymbol{i = \left(\frac{1}{f}\right)\overline{A} - \left(\frac{1-c}{f}\right)Y} \qquad \textbf{(3)}$$

Letting the Greek letter delta or Δ denote the phrase “change in” then for a constant level of autonomous expenditure $\overline{A}$:

$$\Delta i = -\left(\frac{1-c}{f}\right)\Delta Y$$

And the slope of the IS curve is:

$$\left(\frac{\Delta i}{\Delta Y}\right)_{IS} = -\left(\frac{1-c}{f}\right)$$

Hence given the value of the marginal propensity to consume *c* the slope of the IS curve depends on the parameter *f* which measures the response of consumption and investment to the rate of interest. Other things equal, the greater is *f*, or the greater the responsiveness of consumption and investment to interest rate changes, the smaller the slope and the flatter the IS curve.

The Position of the IS Curve: At a given rate of interest the position of the IS curve will shift when autonomous expenditures change. Recall that the IS equation (2) is:

$$\boldsymbol{Y = \left(\frac{1}{1-c}\right)\left[\overline{A} - fi\right]}$$

At any given rate of interest an increase in autonomous expenditure $\overline{A}$ will, via the multiplier, lead to a higher level of output. For example suppose that $\overline{A}$= 1,010, *c* = 0.8, *f* = 1,000. Then for:

$$i = 0.01 : Y = \left(\frac{1}{0.2}\right)\left[1{,}010 - 1{,}000(0.05)\right] = 5{,}000$$

$$i = 0.05 : Y = \left(\frac{1}{0.2}\right)\left[1{,}010 - 1{,}000(0.05)\right] = 4{,}800$$

If $\overline{A}$ increases by 100 then for:

$$i = 0.01 : Y = \left(\frac{1}{0.2}\right)\left[1{,}110 - 1{,}000(0.01)\right] = 5{,}500$$

$$i = 0.05 : Y = \left(\frac{1}{0.2}\right)\left[1{,}110 - 1{,}000(0.05)\right] = 5{,}300$$

Hence at each rate of interest a 100 unit increase in autonomous expenditure requires a 500 unit increase in *Y* to maintain equilibrium in the market for goods and services. This effect is illustrated by Figure 2. Starting from the point A with *i* = 0.05 a 100 unit increase in $\bar{A}$ requires 500 unit increase in *Y* (the increase in autonomous expenditures times the multiplier) to maintain equilibrium in the market for goods and services. As this result will hold at any given level of the interest rate it follows that an increase in autonomous expenditures shifts the IS curve to the right. Conversely, a reduction in autonomous expenditures shifts the IS curve to the left.

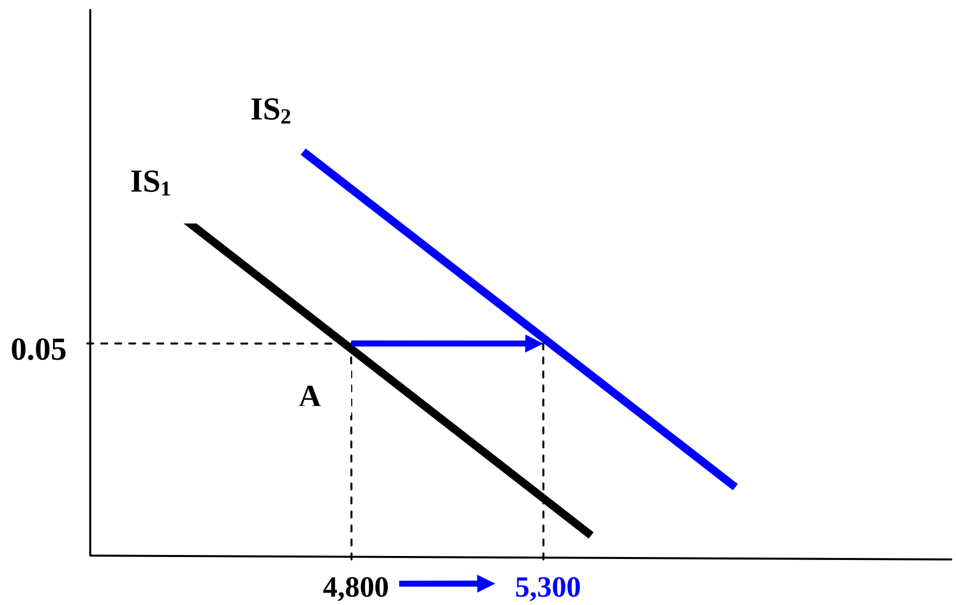

Figure 2. The Position of the IS Curve.
At each rate of interest an increase in autonomous expenditure increases *Y* and shifts the IS curve to the right from IS_1 to IS_2. Conversely a fall in autonomous expenditure reduces *Y* and shifts the IS curve to the left.

EXERCISE 1
In Euroland the components of planned aggregate expenditure are given by

$$C = \bar{C} + c(Y - T) - ai$$
$$I^P = \bar{I} - bi$$
$$\bar{G} = 200 \quad T = 320 \quad NX = 0$$

Where $\bar{C} = 1{,}245$, $\bar{I} = 310$, ***a* = 1,000, *b* = 500 and *c* = 0.75. Derive the equation for Euroland's IS curve and find the equilibrium values of *Y* when *i* = 0.01 and when *i* = 0.03.**

The LM Curve.

The LM curve plots combinations of the rate of interest and the level of output which give equilibrium for which the money market is in equilibrium. In that Chapter 27 we saw that the demand for money depends on a number of variables including the nominal rate of interest i and the level of real output Y and that the money supply is determined exogenously by the central bank. To illustrate the derivation of the LM curve suppose that the market for money can be described as:

$$\begin{aligned} &\textit{Demand} \qquad M^D = kY - hi \\ &\textit{Supply} \qquad\quad M^S = \overline{M} \\ &\textit{Equilibrium} \quad M^D = M^S \end{aligned}$$

The first equation specifies the demand for money as a function of the level of real output Y and the nominal rate of interest i. The parameter k models the transactions demand for money. For example, if k = 0.2 then a €1,000 increase in income increases the demand for money by €200. Likewise the parameter h models the idea that the interest rate is the opportunity cost of money. Hence, if k = 1,000 then a 1% increases in the interest rate, or a rise equal to 0.01, reduces the demand for money by €10. The second equation assumes that the money supply is autonomous and set by the central bank and the third equation defines equilibrium in the money market. Substituting the first two equations into the equilibrium condition gives:

$$kY - hi = \overline{M}$$

Re-arranging gives:

$$i = \frac{1}{h}\left[kY - \overline{M}\right] \qquad (4)$$

Equation (4) defines (Y, i) combinations which give equilibrium in the market for money and is known as the LM equation. Figure 3 presents a graphical illustration of this equation. The left hand panel of Figure 3 illustrates the market for money. The money supply is fixed by the central bank at MM and the demand for money is negatively related to the interest rate. At a given level of income $Y = Y_1$ the demand for money curve is L_1 and the money market equilibrium is at the point A. The right hand panel of Figure 1 has the interest rate on the vertical axis and output on the horizontal and the point C defines an interest rate, output combination (i_1, Y_1) which gives equilibrium in the money market. If income were to rise to a higher level Y_2 then the demand for money curve shifts up to L_2 and equilibrium in the money market is determined at the point B with $i = i_2$. In the right hand panel the point D defines a second interest rate, output combination (i_2, Y_2) which also gives equilibrium in the money market. Hence points such as C and D are (i, Y) combinations which determine equilibrium in the market for money. The line joining these points is known as the *LM curve.*[2]

[2] The LM curve gets its name from the money market equilibrium condition in which the demand for money, or in Keynesian terminology Liquidity Preference (L), equals the money supply (M).

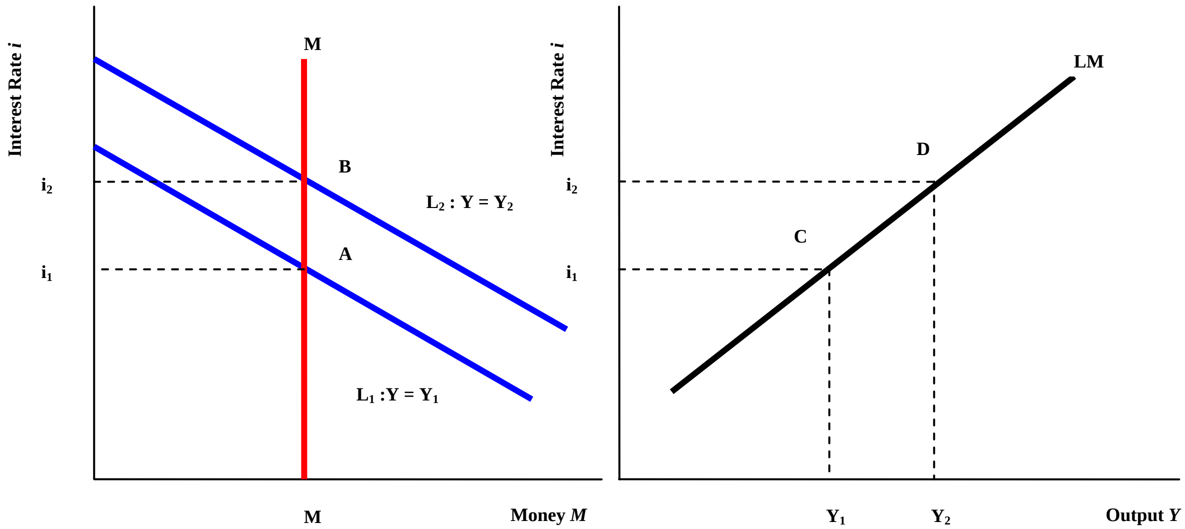

Figure 3. The LM Curve.
The LM curve traces interest rate and output combinations which give equilibrium in the money market.

EXAMPLE 2: The LM Curve

In a certain economy, *k* = 0.2, *h* = 1,000 and $\overline{M}$ = 910. Derive the LM curve when *Y* = 4,800 and when *Y* = 5,000.

Using Equation (4) for:

$$Y = 4{,}800 : i = \frac{1}{1{,}000}\left[960 - 910\right] = 0.05$$

$$Y = 5.000 : i = \frac{1}{1{,}000}\left[1{,}000 - 910\right] = 0.09$$

Hence in Figure 3 the point C corresponds to an (*i, Y*) combination (0.05, 4,800) and the point D to a combination (0.09, 5,000). As both combinations give equilibrium in the market for goods and services both lie on the LM curve.

The Slope of the LM Curve: The LM curve has a positive slope because, as explained in Chapter 27, an increase in *Y* increases the demand for money which given the money supply shifts the demand for money curve to the right resulting in a higher interest rate. That is, to maintain equilibrium in the money market *i* and *Y* must change in the same direction. We can use Equation (3) to derive the slope of the LM curve. Letting the Greek letter delta or Δ denote the phrase “change in” then for a constant money supply $\overline{M}$:

$$\Delta i = \frac{k}{h}\Delta Y$$

And the slope of the LM curve is:

$$\left(\frac{\Delta i}{\Delta Y}\right)_{LM} = \frac{k}{h}$$

Hence given the value of the parameter *k* the slope of the LM curve depends on the parameter *h* which measures the response of demand for money to the rate of interest. Other things equal, the greater is *h*, or the greater the responsiveness of demand for money to interest rate changes, the smaller the slope and the flatter the LM curve.

The Position of the LM Curve: At a given rate of interest the position of the LM curve will shift when the money supply changes. Recall that the LM equation is:

$$i = \frac{1}{h}\left[kY - \overline{M}\right]$$

At any given level of income an increase in the money supply $\overline{\boldsymbol{M}}$ will generate an excess supply of money requiring a fall in the interest rate to restore equilibrium in the money market. For example with *k* = 0.2, *h* = 1,000 and $\overline{\boldsymbol{M}}$ = 910 then for:

$$Y = 4{,}800 : i = \frac{1}{1{,}000}\left[960 - 910\right] = 0.05$$

$$Y = 5.000 : i = \frac{1}{1{,}000}\left[1{,}000 - 910\right] = 0.09$$

If $\overline{\boldsymbol{M}}$ increases by 10 to 910 then for:

$$Y = 4{,}800 : i = \frac{1}{1{,}000}\left[960 - 920\right] = 0.04$$

$$Y = 5.000 : i = \frac{1}{1{,}000}\left[1{,}000 - 920\right] = 0.08$$

Hence at each level of *Y* a 20 unit increase in the money supply requires a 1%, or 0.01, fall in *i* to maintain equilibrium in the money market. This effect is illustrated by Figure 4. Starting from the point A with *Y* = 4,800 a 20 unit increase in $\overline{\boldsymbol{M}}$ requires a 0.01 fall in *i* to maintain equilibrium in the money market. As this result will hold at any given level of income it follows that an increase in the money supply shifts the LM curve down to the right. Conversely, a reduction in the money supply shifts the LM curve to the left.

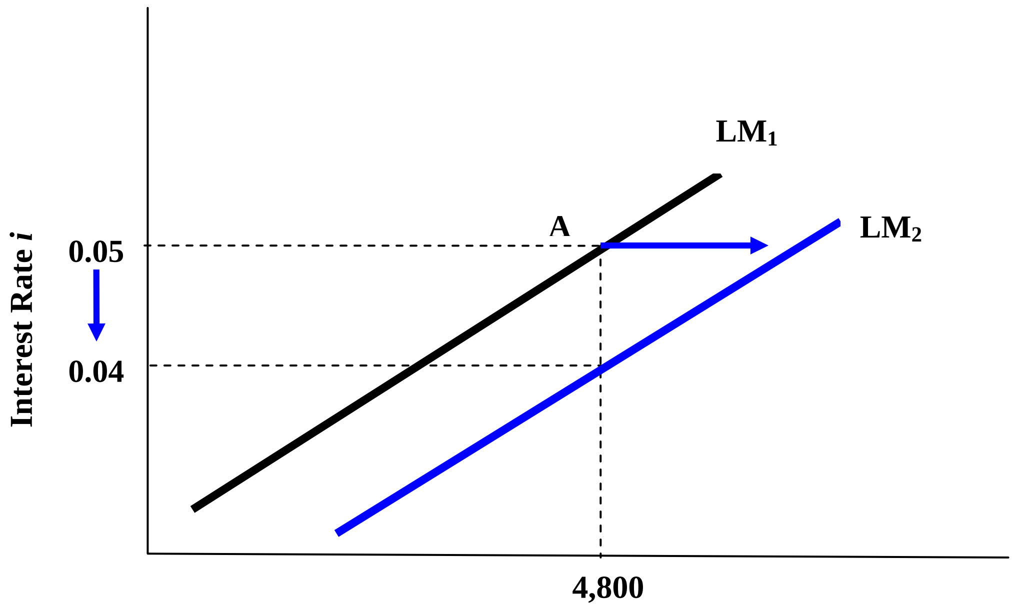

Figure 4. The Position of the LM Curve.
At each level of *Y* an increase in the money supply reduces *i* and shifts the LM curve to the right from LM_1 to LM_2. Conversely a reduction in the money supply shifts the LM curve to the left.

EXERCISE 2
Suppose the money market in Euroland can be described by:

$$\begin{array}{ll} \textit{Demand} & M^D = kY - hi \\ \textit{Supply} & M^S = \overline{M} \\ \textit{Equilibrium} & M^D = M^S \end{array}$$

Where k = 0.2, h = 2,800 and $\overline{M} = 972$

Derive the equation for Euroland's LM curve and find the equilibrium values of *i* when *Y* = 5,140 and when *Y* = 5,420.

General Equilibrium.

The IS and LM curves describe equilibrium two inter-linked markets. The IS curve traces (*i, Y*) combinations at which *Y* = *PAE*, the equilibrium condition in the market for goods and services while the LM curve traces (*i, Y*) combinations at which $M^D = M^S$ the equilibrium condition in the money market. Hence together they determine the *general equilibrium* condition for the economy. That is, the (*i, Y*) combination which gives simultaneous equilibrium in all markets. Figure 5 illustrates this overall or general equilibrium position. The intersection of the IS and LM curves at point E defines the (*i, Y*) combination which gives simultaneous equilibrium in the market for goods and services (*Y* = *PAE*) and in the money market ($M^D = M^S$).

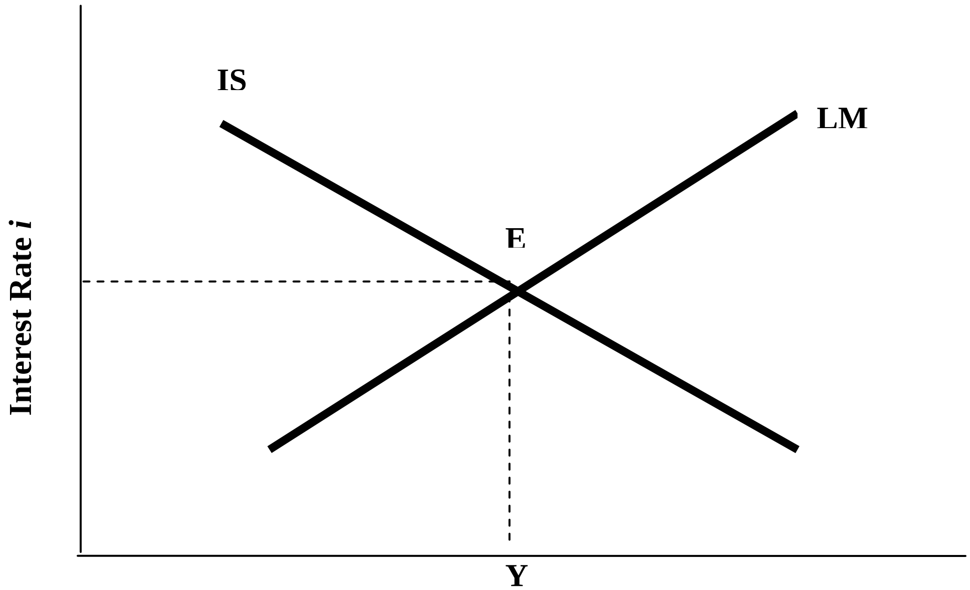

Figure 5. General Equilibrium.
.The intersection of the IS and LM curves at point E defines the (*i, Y*) combination which gives simultaneous equilibrium in the market for goods and services (*Y* = *PAE*) and in the money market ($M^D = M^S$).

EXAMPLE 3: General Equilibrium.

Suppose that in a given economy $\overline{A}$= 1,010, $\overline{M}$ = 910, *c* = 0.8 *f* = 1,000, *h* = 1,000 and *k* = 0.2. Use the IS-LM model to find the equilibrium values for *Y* and *i*.

Recall that we can write the IS and LM equations as:

$$IS : i = \left(\frac{1}{f}\right)\overline{A} - \left(\frac{1-c}{f}\right)Y$$

$$LM : i = \frac{1}{h}\left[kY - \overline{M}\right]$$

Equating the right hand side of each equation gives:

$$\left(\frac{1}{f}\right)\overline{A} - \left(\frac{1-c}{f}\right)Y = \left(\frac{k}{h}\right)Y - \left(\frac{1}{h}\right)\overline{M}$$

Collecting the terms in *Y* gives:

$$\left[\frac{k}{h}+\frac{1-c}{f}\right]Y=\left(\frac{1}{f}\right)\overline{A}+\left(\frac{1}{h}\right)\overline{M}$$

Or:

$$\left[\frac{fk+h(1-c)}{hf}\right]Y=\left(\frac{1}{f}\right)\overline{A}+\left(\frac{1}{h}\right)\overline{M}$$

Hence equilibrium *Y* is given by:

$$\begin{aligned}
Y &= \frac{hf}{fk+h(1-c)}\left[\left(\frac{1}{f}\right)\overline{A}+\left(\frac{1}{h}\right)\overline{M}\right] \\
&= \frac{hf}{fk+h(1-c))}\left[\frac{1}{hf}\left(h\overline{A}+f\overline{M}\right)\right] \\
&= \frac{1}{fk+h(1-c)}\left[h\overline{A}+f\overline{M}\right] \qquad (5)
\end{aligned}$$

Using the assumed values $\overline{A}$ **=** 1,010, $\overline{M}$ = 910, *c* = 0.8, *f* = 1,000, *h* = 1,000 and *k* = 0.2 gives the equilibrium value for *Y*:

$$Y=\frac{1}{400}\left[(1,000)1,010+(1,000)910\right]=4,800$$

Substituting the equilibrium value for *Y* into the LM equation gives the equilibrium rate of interest:

$$i=\frac{1}{1,000}\left[0.2(4,800)-910\right]=0.05$$

Hence in this economy equilibrium *Y* = 4,800 and equilibrium *i* = 0.05 or 5.0%.

We will now use the IS-LM model to illustrate the effects of stabilisation policies on the equilibrium values of *Y* and *i*.

EXERCISE 3
Using the data in Exercises 1 and 2 find the equilibrium values for *Y* and *i* in the Euroland economy.

Stabilisation Policy: Closing a Recessionary Gap.

Suppose that as in Example 3 $\overline{A}$ = 1,010, $\overline{M}$ = 910, *c* = 0.8, *f* = 1,000, *h* = 1,000 and *k* = 0.2. We have seen that in this economy equilibrium *Y* = 4,800 and equilibrium *i* = 0.05 or 5%. Now suppose that the full employment or natural level of output *Y** is 5,000 implying a recessionary gap equal to 200. We shall now illustrate how fiscal and monetary policies can be used to close the recessionary gap.

EXAMPLE 4. Using Fiscal Policy to close a Recessionary Gap.

At any given rate of interest an increase in government expenditures $\overline{G}$ will lead to an increase in autonomous expenditure $\overline{A}$ and, as we have seen, shifts the IS curve to the right closing the recessionary gap. By how much does government have to increase $\overline{G}$? Letting the Greek Δ denote "change in" and using Equation (5) the change in output is:

$$\begin{aligned} \Delta Y &= \frac{h}{fk + h(1-c)} \Delta \overline{G} \\ &= \frac{1{,}000}{400} \Delta \overline{G} \end{aligned} \qquad (6)$$

Hence at a given money supply, $\Delta \overline{M} = 0$, ΔY will equal 2.5, or 1,000/400, times the change in autonomous expenditure and for Y to increase by 80 $\overline{G}$ must be increased by 80. That is:

$$\Delta Y = 2.5[80] = 200$$

However following the analysis of Chapter 27, an increase in Y will increase the demand for money and, at a fixed money supply, lead to a higher rate of interest. From Equation (4) the change in the interest rate needed to maintain equilibrium in the money market is:

$$\begin{aligned} \Delta i &= \frac{k}{h}[\Delta Y] \\ &= \frac{0.2}{1{,}000}[200] = 0.04 \end{aligned}$$

Hence the equilibrium interest rate will increase from 0.05 to 0.09, or 9%. This effect is illustrated by Figure 6. Starting from the point E_1 with i = 0.05 and Y = 4,800 an increase in $\overline{\boldsymbol{G}}$ closes the recessionary gap by shifting the IS curve to the right and establishing a new equilibrium at the point E_2 with Y = 5,000 and i = 0.09.

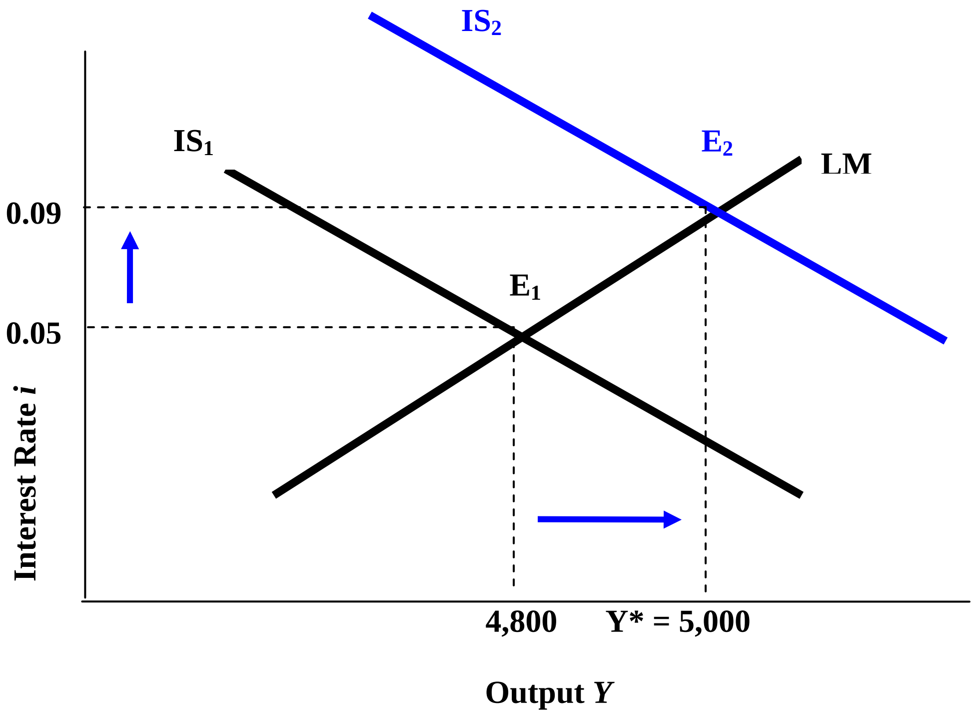

Figure 6. Closing a Recessionary Gap: Fiscal Policy.
Starting from a recessionary gap equal to 200, an 80 unit increase in government expenditure shifts the IS curve from IS_1 to IS_2 and closes the recessionary gap. Income increases by 200 but the higher level of Y leads to an excess demand for money and a higher equilibrium rate of interest.

Note that the IS-LM analysis of fiscal policy differs from that in Chapter 26. In Chapter 26 the impact of an increase in government expenditure on Y is given by:

$$\Delta Y = \frac{1}{1-c}\Delta\overline{G}$$

Hence with the marginal propensity to consume c = 0.8 the multiplier 1/(1- c) is equal to 5 and an 80 unit increase in $\overline{G}$ leads to a 400 unit increase in Y as compared to a 200 increase in the IS-LM model. The key difference is that in Chapter 26 we used a 'partial equilibrium' Keynesian model which assumes that the rate of interest is constant at all levels of Y. In contrast, the IS-LM general equilibrium model permits the market for goods and services to interact with the money market. Hence as Y increases the demand for money increases forcing a rise in the rate of interest which in turn lowers consumption and investment spending and partially offsets the effect of the fiscal stimulus. This offsetting effect is equivalent to the *crowding out* effect discussed in Chapter 22. Using arrows to denote the direction of change this sequence can be illustrated as:

$$\underbrace{\uparrow \overline{G} \rightarrow \uparrow Y}_{\textit{Multiplier}} \rightarrow \underbrace{\uparrow M^{D} \rightarrow \uparrow i}_{\textit{Money Market}} \rightarrow \underbrace{\downarrow C, I \rightarrow \downarrow Y}_{\textit{Crowding Out}}$$

To assess the strength of this crowding out effect recall that in the IS-LM model equilibrium *Y* is given by Equation (5):

$$Y = \frac{1}{fk + h(1-c)}\left[h\overline{A} + f\overline{M}\right]$$

And the income-expenditure multiplier is:

$$\frac{\Delta Y}{\Delta \overline{G}} = \frac{h}{fk + h(1-c)} = \frac{1}{\frac{fk}{h} + (1-c)}$$

For given values of *c, k* and *f* the value of the multiplier depends on the parameter *h* which measures the responsiveness of the demand for money with respect to the rate of interest rate. The higher is the parameter *h* the smaller is *fk/*h and the greater the value of the multiplier. For example, with *c* = 0.8, *f* = 1,000, *k* = 0.2 and, *h* = 1,000 the multiplier is 2.5. But with *h* = 4,000 the multiplier increases to 4. Hence the greater the responsiveness of the demand for money with respect to the rate of interest rate, the weaker the crowding out effect and the greater the effectiveness of fiscal policy.

To explain the reasoning behind this result recall from Equation (4) that for a given change in *Y* the change in *i* required to maintain equilibrium in the market for money is given by:

$$\Delta i = \frac{k}{h}\Delta Y$$

Hence the greater is *h* the smaller change in *i* required to restore equilibrium in the market for money and the smaller the rise in *i* the lower the crowding out impact on consumption and investment.

EXERCISE 4

Suppose the natural level of income in Euroland is *Y = 5,840. (a) By how much would Euroland's government have to change government expenditure for equilibrium *Y* to equal *Y**? (b) What is the resulting change in the equilibrium rate of interest?**

EXAMPLE 5. Crowding Out.

Suppose that $\overline{A}$ = 1,010, $\overline{M}$ = 910, *c* = 0.8, *f* = 1,000, *k* = 0.2, equilibrium *Y* = 4,800 and equilibrium *i* = 0.05. If government expenditure is increased by 80 find the change in the equilibrium values of *Y* and *i* when *h* = 1,000 and when *h* = 4,000.

In Example 4 we saw that when h = 1,000 the multiplier is 2.5 and an 80 unit increase in $\overline{G}$ leads to a 200 unit increase in Y and a 4% increase in i. With h = 4,000:

$$\begin{aligned}\Delta Y &= \frac{h}{fk + h(1-c)}\Delta\overline{G} \\ &= \frac{4{,}000}{1{,}000}(80) = 320 \qquad (7)\end{aligned}$$

In this case the multiplier equals 4 and an 80 unit increase in government expenditure will increase Y by 320. The change in i required to maintain equilibrium in the market for money is given by:

$$\Delta i = \frac{k}{h}\Delta Y = \frac{0.2}{4{,}000}(320) = 0.016$$

And the new equilibrium interest rate is 0.066, or 6.6%. Hence the greater is h the greater the multiplier and the smaller the increase in the interest rate required to restore equilibrium in the money market. Other things equal, the smaller the increase in i the smaller the decline in consumption and investment and the weaker the crowding out effect. This result is illustrated by Figure 7 which shows that the flatter the LM curve the smaller the impact on i and the greater the impact on Y.

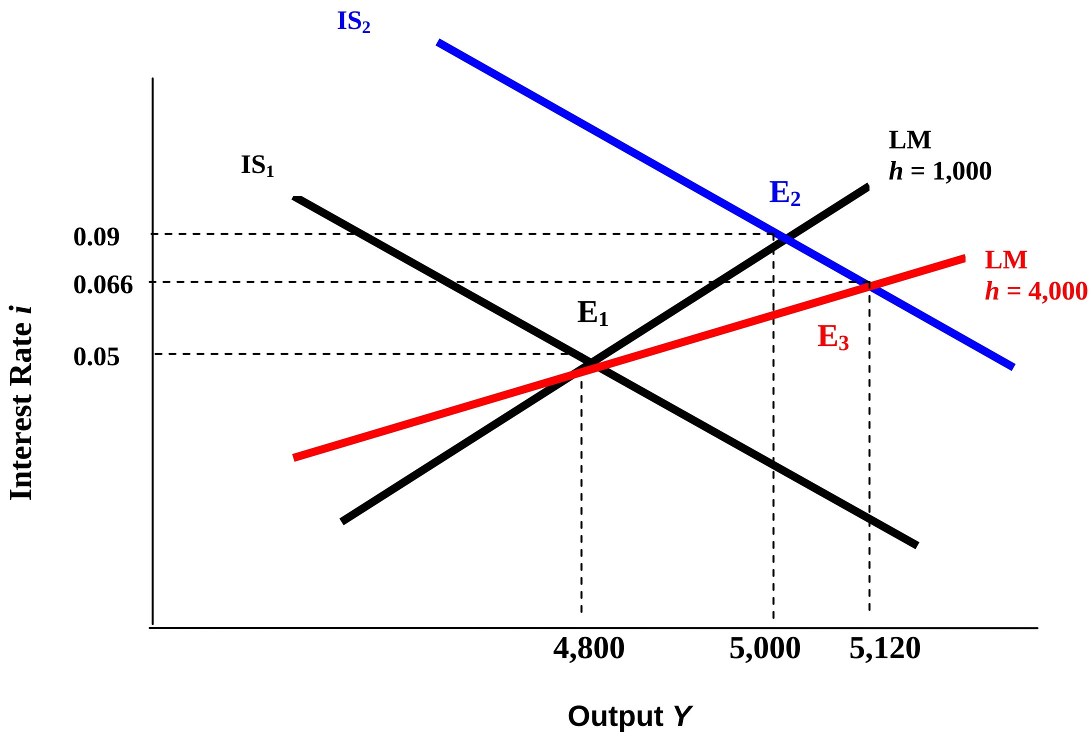

Figure 7 Crowding Out.
When h = 1,000 an 80 unit increase in government expenditure increases Y from 4,800 to 5,000 and i from 0.05 to 0.09. With h = 4,000 the LM curve is flatter and the same increase in government expenditure increases Y by 320 to 5,120 and i by 0.016 to 0.066

EXERCISE 5

What is the value of Euroland's income-expenditure multiplier? What is the value of Euroland's multiplier if the parameter *h* = 3,600 rather than 2,800. What does this imply for the effectiveness of fiscal policy in Euroland?

EXAMPLE 6. Using Monetary Policy to close a Recessionary Gap.

Recall that we are assuming that $\overline{A}$ = 1,010, $\overline{M}$ = 910, *c* = 0.8, *f* = 1,000, *h* = 1,000, *k* = 0.2, *Y* = 4,800, *i* = 0.05 and that the economy faces a recessionary gap equal to 200. At any given level of *Y* an increase in the money supply $\overline{M}$ will lead to a fall in the interest rate and, as we have seen, shifts the LM curve to the right closing the recessionary gap. By how much does the central bank have to increase $\overline{M}$? Letting the Greek Δ denote "change in" and using Equation (5) the change in output is:

$$\begin{aligned}\Delta Y &= \frac{f}{fk + h(1-c)}\Delta\overline{M} \\ &= \frac{1{,}000}{400}\Delta\overline{M} \qquad\qquad (8)\end{aligned}$$

Hence with $\Delta\overline{A} = 0$, Δ*Y* will equal 2.5, or 1,000/400, times the change in the money supply and for *Y* to increase by 200 $\overline{M}$ must be increased by 80. That is:

$$\Delta Y = 2.5[80] = 200$$

To find the change in the rate of interest we can use the LM equation:

$$i = \frac{1}{h}\left[kY - \overline{M}\right]$$

Hence the change in *i* is:

$$\begin{aligned}\Delta i &= \frac{k}{h}\Delta Y - \frac{1}{h}\Delta\overline{M} \\ &= \frac{0.2}{1{,}000}(200) - \frac{1}{1{,}000}(80) = -0.04\end{aligned}$$

Hence the equilibrium interest rate will fall from 0.05 to 0.01, or 1%. This effect is illustrated by Figure 8. Starting from the point E_1 with *i* = 0.05 and *Y* = 4,800 an increase in $\overline{M}$ shifts the LM curve to the right. The resulting fall in the rate of interest closes the recessionary gap by stimulating consumption and investment establishing a new equilibrium at the point E_2 with *Y* = 5,000 and *i* = 0.01.

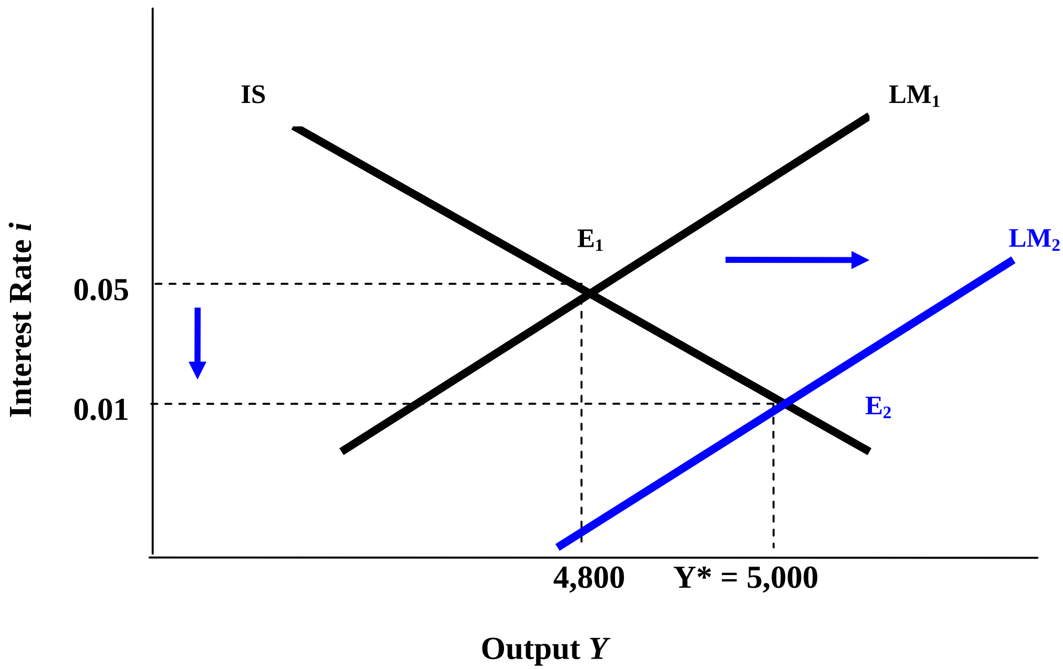

Figure 8. Closing a Recessionary Gap: Monetary Policy.
The increase in $\overline{M}$ shifts the LM curve from LM_1 to LM_2 resulting in a lower rate of interest and closes the recessionary gap by stimulating consumption and investment leading to an increase in *Y.*

To assess the effectiveness of monetary policy we use Equation (8):

$$\Delta Y = \frac{f}{fk + h(1-c)}\Delta\overline{M}$$

The term:

$$\frac{f}{fk + h(1-c)} = \frac{1}{k + h(1-c)/f}$$

measures the change in equilibrium *Y* resulting from each unit change in $\overline{M}$ and can be thought of as the 'money multiplier'. For given values of *c, k* and *h* the value of the multiplier depends on the parameter *f* which measures the responsiveness of consumption and investment to changes in the rate of interest rate. The higher is the parameter *f* the smaller is *h(1 – c)/f* and the greater the value of the money multiplier. For example, with *c* = 0.8, *h* = 1,000, *k* = 0.2 and, *f* = 1,000 the money multiplier is 2.5. But with *f* = 4,000 value of the money multiplier increases to 4. Hence the greater the responsiveness of consumption and investment to changes in the rate of interest rate greater the effectiveness of monetary policy. However if f = 4,000 the money multiplier will be 1/0.25 = 4 and an 80 unit increase in the money supply will increase equilibrium Y by 320. The corresponding change in the interest rate is given by:

$$\Delta i = \frac{k}{h}\Delta Y - \frac{1}{h}\Delta\overline{M}$$
$$= \frac{0.2}{1{,}000}(320) - \frac{1}{1{,}000}(80) = -0.016$$

Hence the equilibrium interest rate will fall from 0.05 to 0.034. This result is illustrated by Figure 9 which shows that the flatter the IS curve, or the greater is f, the smaller the impact on i and the greater the impact on Y.

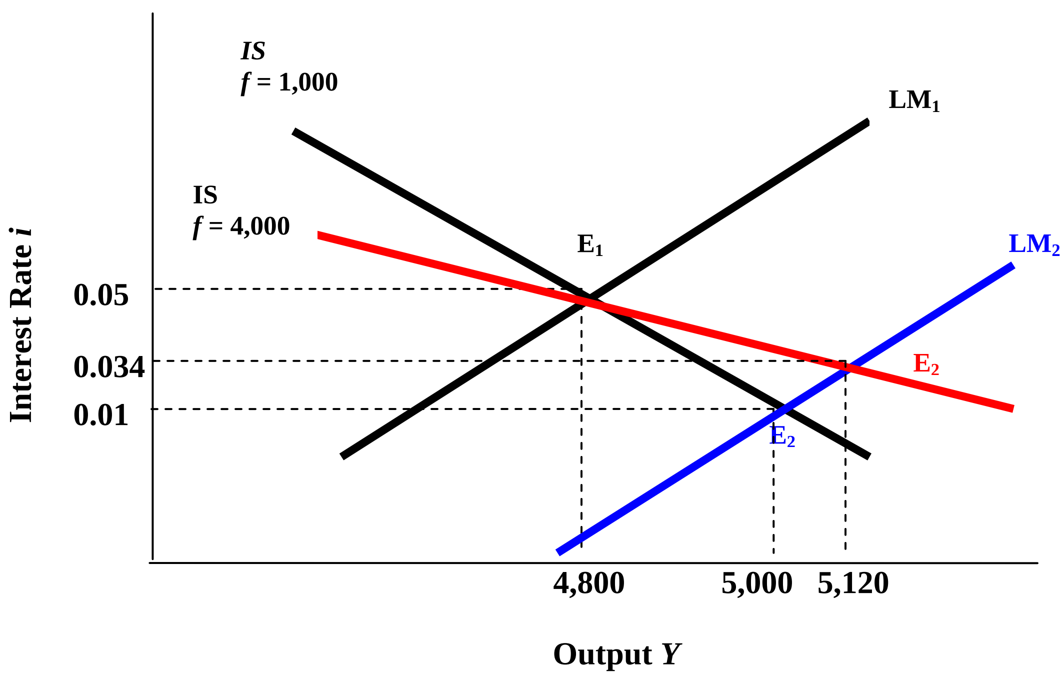

Figure 9 The Strength of Monetary Policy.
When f = 1,000 an 80 unit increase in $\overline{M}$ increases Y from 4,800 to 5,000 and reduces i from 0.05 to 0.01. With f = 4,000 the IS curve is flatter and the same increase in $\overline{M}$ increases Y by 320 to 5,120 and i by 0.016 to 0.034

EXERCISE 6
Suppose the natural level of income in Euroland is Y^* = 5,625. (a) By how much would Euroland's central bank have to change the money supply for equilibrium Y to equal Y^*? (b) What is the resulting change in the equilibrium rate of interest?

Summary

- The IS-LM model is a general equilibrium model which simultaneously determines the equilibrium values of income and the rate of interest

- The IS curve plots income and interest rate combinations which give equilibrium in the market for goods and services while the IS curve plots similar combinations for the money market.

- The slope of the IS curve depends on the responsiveness of consumption and investment to changes in the rate of interest. The more responsive are consumption and investment the lower the slope and the flatter the IS curve.

- An increase in autonomous expenditure shifts the IS curve to the right and a decrease shifts the curve to the left.

- The slope of the LM curve depends on the responsiveness of the demand for money to changes in the rate of interest. The more responsive is the demand for money the lower the slope and the flatter the LM curve

- An increase in the money supply shifts the Lm curve to the right and a decrease shifts the curve to the left.

- The intersection of the IS and LM curves determine the equilibrium values of output and the rate of interest.

- In the IS-LM model the effectiveness of fiscal policy is partially offset by higher interest rates – the crowding out effect. This crowding out effect is weaker the greater the responsiveness of the demand for money to changes in the rate of interest and the flatter the LM curve.

- In the IS-LM model the effectiveness of monetary policy increases with the responsiveness of consumption and investment to changes in the rate of interest.

Review Questions.

1. Explain how the market for goods and services and the market for money are inter-linked. Why does this inter-dependency between the two markets require a general equilibrium model of income determination rather that the simple Keynesian model introduced in Chapter 25?

2. Explain why the slope of the IS curve depends on the responsiveness of consumption and investment to changes in the rate of interest. If consumption and investment become more responsive to changes in the rate of interest does this make the IS curve flatter or steeper?

3. Explain why the slope of the LM curve depends on the responsiveness of the demand for money to changes in the rate of interest. If the demand for money becomes less responsive to changes in the rate of interest does this make the LM curve flatter or steeper?

4. Explain how an increase in government expenditure shifts the position of the IS curve.

5. Explain how a reduction in the money supply shifts the position of the LM curve.

6. Using the IS-LM model explain how (a) a increase in government expenditure and (b) a reduction in the money supply affect the equilibrium values for the level of income and the rate of interest.

7. Explain the concept of crowding-out. Why does the strength of fiscal policy depend on the responsiveness of the demand for moneys to changes in the rate of interest?

8. Would the effectiveness of monetary policy be strengthened or weakened if consumption and investment became more responsive to changes in the rate of interest.

Answers to Exercises

1. Along the IS curve the market for goods and services is in equilibrium and *PAE* = *Y*. *PAE* is given by: $PAE = C + I^P + G + NX$. Substituting for the components of *PAE* gives:

$$\begin{aligned} PAE &= \left[\overline{C} + c(Y - T) - ai\right] + \left[\overline{I} - bi\right] + \overline{G} \\ &= \left[\overline{C} + \overline{I} + \overline{G} - cT\right] - fi + cY \\ &= \overline{A} - fi + cY \end{aligned}$$

Where $\overline{A} = \overline{C} + \overline{I} + \overline{G} - cT$ and *f* = (*a* + *b*). Using the assumed values $\overline{A} = 1{,}515$ and *f* = 1,500. At each value of *i* equilibrium *Y* is given by:

$$\begin{aligned} Y &= \overline{A} - fi + cY \\ &= \left(\frac{1}{1-c}\right)\left[\overline{A} - fi\right] \end{aligned}$$

Which is the equation for Euroland's IS curve. At each value of *i* equilibrium *Y* is given by:

$$Y = \left(\frac{1}{0.25}\right)[1{,}515 - 1{,}500i]$$

Hence:

For *i* = 0.01 $\quad Y = \left(\frac{1}{0.25}\right)\left[1{,}515 - 1{,}500(0.01)\right] = 6{,}000$

For *i* = 0.03 $\quad Y = \left(\frac{1}{0.25}\right)\left[1{,}515 - 1{,}500(0.03)\right] = 5{,}880$

As the (*i, Y*) combinations (0.01, 6,000) and (0.03, 5,880) give equilibrium in the market for goods and services they are points on Euroland's IS curve.

2. Along the LM curve the market for money is in equilibrium and $M^D = M^S$. Hence in equilibrium

$$kY - hi = \overline{M}$$

or:

$$i = \left(\frac{1}{h}\right)\left[kY - \overline{M}\right]$$

Which is the equation for Euroland's LM curve? At each value of *Y* the equilibrium interest rate is given by:

$$i = \left(\frac{1}{2{,}800}\right)\left[0.2Y - 972\right]$$

Hence:

$$\text{For } Y = 5{,}140 \quad i = \left(\frac{1}{2{,}800}\right)\left[0.2x5{,}140 - 972\right] = 0.02$$

$$\text{For } Y = 5{,}420 \quad i = \left(\frac{1}{2{,}800}\right)\left[0.2x5{,}420 - 972\right] = 0.04$$

As the (*i, Y*) combinations (0.02, 5,140) and (0.04, 5,420) give equilibrium in the market for money they are points on Euroland's IS curve.

3. In Exercise 1 we saw that the equation for Euroland's IS curve is:

$$Y = \left(\frac{1}{1-c}\right)\left[\overline{A} - fi\right]$$

Re-arranging gives:

$$i = \left(\frac{1}{f}\right)\overline{A} - \left(\frac{1-c}{f}\right)Y$$

Likewise in Exercise 2 we saw that the equation for Euroland's LM curve is:

$$i = \left(\frac{1}{h}\right)\left[kY - \overline{M}\right]$$

Equating these two expressions gives:

$$\left(\frac{1}{f}\right)\overline{A} - \left(\frac{1-c}{f}\right)Y = \left(\frac{1}{h}\right)\left[kY - \overline{M}\right]$$

Re-arranging gives the equilibrium value for *Y.*

$$Y = \left(\frac{1}{fk + h(1-c)}\right)\left[h\overline{A} + f\overline{M}\right]$$

$$= \left(\frac{1}{1{,}000}\right)\left[2{,}800x1{,}515 + 1{,}500x972\right] = 5{,}700$$

Substituting Y= 5,700 into the LM equation gives the equilibrium interest rate:

$$i = \left(\frac{1}{2{,}800}\right)\left[0.2x5{,}700 - 972\right] = 0.06$$

Hence in Euroland equilibrium Y = 5,700 and equilibrium i = 0.06 or 6%.

4. As equilibrium Y = 5,700 and Y^* = 5,840 Euroland faces a recession gap equal to 140.

(a) In Exercise 3 we saw that equilibrium *Y* is given by:

$$Y = \left(\frac{1}{1{,}000}\right)\left[2{,}800\overline{A} + 1{,}500\overline{M}\right]$$

Where $\overline{A} = 1{,}515$ and $\overline{M} = 972$. As autonomous government expenditure $\overline{G}$ is a component of $\overline{A}$ the change in *Y* resulting from a change in $\overline{G}$ is:

$$\Delta Y = 2.8\Delta\overline{G}$$

For $\Delta Y = 140$

$$\Delta\overline{G} = 140/2.8 = 50$$

Hence the multiplier is 2.8 and to close a recessionary gap equal to 140 the government must increase autonomous expenditure by 50.

(b) Exercise 2 we saw that the equation for Euroland's LM curve is:

$$i = \left(\frac{1}{h}\right)\left[kY - \overline{M}\right]$$

At a constant money supply the change in the rate of interest is:

$$\Delta i = \left(\frac{1}{h}\right)\left[k\Delta Y\right]$$

$$= \left(\frac{0.2}{2{,}800}\right)140 = 0.01$$

Hence the equilibrium interest rate increases by 0.01 or 1%.

5. In Exercise 3 we saw that equilibrium *Y* is given by:

$$Y = \left(\frac{1}{fk + h(1-c)}\right)\left[h\overline{A} + f\overline{M}\right]$$

The income expenditure multiplier is:

$$\frac{\Delta Y}{\Delta\overline{A}} = \frac{h}{fk + h(1-c)}$$

With *f* = 1,500, *k* = 0.2, *c* = 0.75 and *h* = 2,800 the multiplier is:

$$\frac{\Delta Y}{\Delta\overline{A}} = \frac{2800}{300 + 700} = 2.8$$

Hence increasing government expenditure by €1 million results in a €2.8 million increase in *Y*. However, if *h* = 3,600 the multiplier is:

$$\frac{\Delta Y}{\Delta \overline{A}} = \frac{3{,}600}{300 + 900} = 3.0$$

And increasing government expenditure by €1 million results in a €3 million increase in *Y*. It follows that other things equal the greater the value of the parameter *h* the greater the multiplier and the greater the impact of fiscal policy on income.

$$Y = \left(\frac{1}{1{,}000}\right)\left[2{,}800\overline{A} + 1{,}500\overline{M}\right]$$

Where $\overline{A} = 1{,}515$ and $\overline{M} = 972$. As autonomous government expenditure $\overline{G}$ is a component of $\overline{A}$ the change in *Y* resulting from a change in $\overline{G}$ is:

$$\Delta Y = 2.8\Delta\overline{G}$$

6. As equilibrium *Y* = 5,700 and *Y** = 5,625 Euroland faces an expansionary gap equal to 75.
(a) In Exercise 3 we saw that equilibrium *Y* is given by:

$$Y = \left(\frac{1}{fk + h(1-c)}\right)\left[h\overline{A} + f\overline{M}\right]$$
$$= \left(\frac{1}{1{,}000}\right)\left[2{,}800\overline{A} + 1{,}500\overline{M}\right]$$

Where $\overline{A} = 1{,}515$ and $\overline{M} = 972$. At a given level of autonomous expenditure $\overline{A}$ the change in *Y* resulting from a change in $\overline{M}$ is:

$$\Delta Y = 1.5\Delta\overline{M}$$

For $\Delta Y = -75$

$$\Delta\overline{M} = -75/1.5 = -50$$

Hence to close an expansionary gap equal to 75 the central bank must reduce the money supply by 50.

(b) Exercise 2 we saw that the equation for Euroland's LM curve is:

$$i = \left(\frac{1}{h}\right)\left[kY - \overline{M}\right]$$

For $k = 0.2,\ \Delta Y = -75\ and\ \Delta\overline{M} = 50$ The resulting change in the rate of interest is:

$$\Delta i = \left(\frac{1}{h}\right)\left[k\Delta Y - \Delta\overline{M}\right]$$

$$= \left(\frac{1}{2{,}800}\right)\left[-15 + 50\right] = 0.0125$$

Hence the equilibrium interest rate increases by 0.0125 or 1.25%.