

Useful Formulas

Measures of Risk

Variance of returns: $\sigma^2 = \sum_s p(s)[r(s) - E(r)]^2$

Standard deviation: $\sigma = \sqrt{\sigma^2}$

Covariance between returns: $\text{Cov}(r_i, r_j) = \sum_s p(s)[r_i(s) - E(r_i)][r_j(s) - E(r_j)]$

Beta of security i : $\beta_i = \frac{\text{Cov}(r_i, r_M)}{\text{Var}(r_M)}$

Portfolio Theory

Expected rate of return on a portfolio with weights w_i in each security: $E(r_p) = \sum_{i=1}^n w_i E(r_i)$

Variance of portfolio rate of return: $\sigma_p^2 = \sum_{j=1}^n \sum_{i=1}^n w_j w_i \text{Cov}(r_i, r_j)$

Market Equilibrium

The security market line: $E(r_i) = r_f + \beta_i[E(r_M) - r_f]$

Multifactor security market line (in excess returns): $E(R_i) = \beta_{iM}E(R_M) + \sum_{k=1}^K E(R_k)$

Fixed-Income Analysis

Present value of \$1:

Discrete period compounding: $\text{PV} = 1/(1 + r)^T$

Continuous compounding: $\text{PV} = e^{-rT}$

Forward rate of interest for period T : $f_T = \frac{(1 + y_T)^T}{(1 + y_{T-1})^{T-1}} - 1$

Real interest rate: $rr = \frac{1 + m}{1 + i} - 1$

where m is the nominal interest rate and i is the inflation rate

Duration of a security: $D = \sum_{t=1}^T t \times \frac{CF_t}{(1 + y)^t} / \text{Price}$

Modified duration: $D^* = D/(1 + y)$

Equity Analysis

Constant growth dividend discount model: $V_0 = \frac{D_1}{k - g}$

Sustainable growth rate of dividends: $g = \text{ROE} \times b$

Price/earnings multiple: $P/E = \frac{1 - b}{k - \text{ROE} \times b}$

ROE = (1 - Tax rate) $\left[\text{ROA} + (\text{ROA} - \text{Interest rate}) \frac{\text{Debt}}{\text{Equity}} \right]$

Derivative Assets

Put-call parity: $P = C - S_0 + \text{PV}(X + \text{dividends})$

Black-Scholes formula: $C = Se^{-\delta T}N(d_1) - Xe^{-rT}N(d_2)$
(with constant dividend yield) $d_1 = \frac{\ln(S/X) + (r - \delta + \sigma^2/2)T}{\sigma\sqrt{T}}$

$d_2 = d_1 - \sigma\sqrt{T}$

Spot-futures parity: $F_0 = S_0(1 + r - d)^T$

Interest rate parity: $F_0 = E_0 \left(\frac{1 + r_{\text{US}}}{1 + r_{\text{foreign}}} \right)^T$

Performance Evaluation

Sharpe ratio: $S_p = \frac{\bar{r}_p - \bar{r}_f}{\sigma_p}$

Treynor's measure: $T_p = \frac{\bar{r}_p - \bar{r}_f}{\beta_p}$

Jensen's alpha: $\alpha_p = \bar{r}_p - [\bar{r}_f + \beta_p(\bar{r}_M - \bar{r}_f)]$

Geometric average return: $r_G = [(1 + r_1)(1 + r_2) \dots (1 + r_T)]^{1/T} - 1$