

## Lesson 11-7

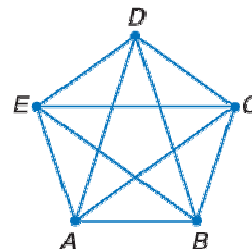
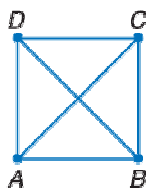
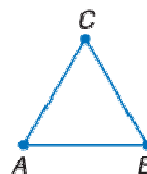
## Example 1

**EDUCATION** A school district holds orientation sessions of all the new teachers during the week before classes start for the school year. At the first orientation session, all 28 of the new teachers hired by the district were present. In order for them to get acquainted with each other, each of the new teachers shook hands with each other new teacher exactly once. How many handshakes took place?

## Solution

Each handshake involves two people. For example, when Ms. Alpert and Mr. Boswell shake hands with each other, this counts as one handshake, not two.

A diagram using dots to represent the teachers and all the segments that can be drawn to connect the dots as the handshakes can be used to model this problem. Here are diagrams showing the situation with 2, 3, 4, and 5 teachers.



Now make a table and look for a pattern.

For 2 teachers, the number of handshakes is  $\frac{2(1)}{2} = 1$ .

For 3 teachers, the number of handshakes is  $\frac{3(2)}{2} = 3$ .

For 4 teachers, the number of handshakes is  $\frac{4(3)}{2} = 6$ .

For 5 teachers, the number of handshakes is  $\frac{5(4)}{2} = 10$ .

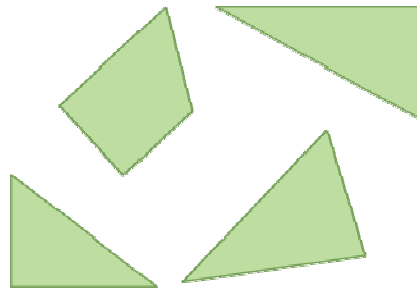
Generalizing this pattern, for  $n$  teachers, there will be  $\frac{n(n-1)}{2}$  handshakes.

This rule makes sense because each of the  $n$  teachers must shake hands with each of the  $(n - 1)$  other teachers. That would give  $n(n - 1)$  handshakes, but then each handshake is counted twice, so you must divide by 2.

Applying this formula to the case where  $n = 28$  shows that for 28 teachers, the number of handshakes will be  $\frac{28(27)}{2} = 378$ .

### Example 2

Trace the four shapes. Cut them out and rearrange the pieces to form a square.



### Solution

Note that of three triangles, two are right triangles, and also that the quadrilateral is a trapezoid with two right angles. First place the taller right triangle so that its right angle forms one of the angles of the square. Next place the trapezoid next to this triangle so that one of its right angles forms another angle of the square and the slanted side of the trapezoid fits along part of the hypotenuse of the right triangle. Next, place the shorter right triangle directly above the trapezoid. Finally, place the third triangle to complete the square.

