

Lesson 8-5

Example 1

Evaluate the determinant of matrix A. $A = \begin{bmatrix} 2 & 1 \\ -4 & 5 \end{bmatrix}$

Solution

$$\begin{aligned} \det A &= \begin{vmatrix} 2 & 1 \\ -4 & 5 \end{vmatrix} \\ &= 2(5) - 1(-4) \quad a = 2, b = 1, c = -4, d = 5 \\ &= 10 + 4 \\ &= 14 \end{aligned}$$

Example 2

Solve the system of equations using the method of determinants.

$$\begin{aligned} 3x - 3y &= 9 \\ -2x + 4y &= 18 \end{aligned}$$

Solution

Write the coefficients of x and y in a determinant A .

$$A = \begin{vmatrix} 3 & -3 \\ -2 & 4 \end{vmatrix}$$

↓ y -coefficients
↑ x -coefficients

Write another determinant. Use A and replace the x -column with the constants from the equations. Label it A_x .

$$A_x = \begin{vmatrix} 9 & -3 \\ 18 & 4 \end{vmatrix}$$

↑ Replace the x -coefficients with constants.

Write a third determinant. Use A and replace the y -column with the constants from the equations. Label it A_y .

$$A_y = \begin{vmatrix} 3 & 9 \\ -2 & 18 \end{vmatrix}$$

↑ Replace the y -coefficients with constants.

If $A \neq 0$, the solution of the system is (x, y) , where $x = \frac{A_x}{A}$ and $y = \frac{A_y}{A}$.

Solve for x and y .

$$x = \frac{A_x}{A} = \frac{\begin{vmatrix} 9 & -3 \\ 18 & 4 \end{vmatrix}}{\begin{vmatrix} 3 & -3 \\ -2 & 4 \end{vmatrix}} = \frac{9(4) - (-3)(18)}{3(4) - (-3)(-2)} = \frac{36 + 54}{12 - 6} = \frac{90}{6} = 15$$

$$y = \frac{A_y}{A} = \frac{\begin{vmatrix} 3 & 9 \\ -2 & 18 \end{vmatrix}}{\begin{vmatrix} 3 & -3 \\ -2 & 4 \end{vmatrix}} = \frac{3(18) - 9(-2)}{3(4) - (-3)(-2)} = \frac{54 + 18}{12 - 6} = \frac{72}{6} = 12$$

Check the solution.

$3x - 3y = 9$	$-2x + 4y = 18$
$3(15) - 3(12) \stackrel{?}{=} 9$	$-2(15) + 4(12) \stackrel{?}{=} 18$
$45 - 36 \stackrel{?}{=} 9$	$-30 + 48 \stackrel{?}{=} 18$
$9 = 9 \checkmark$	$18 = 18 \checkmark$

The solution is $(15, 12)$.

Example 3

Use a graphing utility to solve the system of equations. $-4x + 5y = 23$
 $-2x + 7y = -11$

Solution

Let A be a 2×2 matrix with elements of the coefficients of x and y .

Let B be a 2×1 matrix with elements of the constant terms of both equations.

Use the matrix feature to enter matrix $A = \begin{bmatrix} -4 & 5 \\ -2 & 7 \end{bmatrix}$ and matrix $B = \begin{bmatrix} 23 \\ -11 \end{bmatrix}$.

On the home screen, enter $A^{-1} \times B$. The solution matrix is $\begin{bmatrix} -12 \\ -5 \end{bmatrix}$.

So $x = -12$ and $y = -5$. Check the solution in both equations.