Lesson G

Compound Interest

Use after Lesson 13-8

Goals ■ Calculate compound interest

Applications Investments, Banking, History

1. Find the amount of simple interest on $1000 at an annual interest rate of 5% after one year.
2. Find the total amount of the principal and interest after one year.
3. Find the amount of simple interest on the total in Question 2 at an annual interest rate of 5% after another year.
4. Find the total of the principal in Question 2 and the interest in Question 3.
5. Find the total of a principal of $1000 and the simple interest on $1000 at an interest rate of 5% after 2 years.
6. Compare the values found in Question 4 and Question 5.

BUILD UNDERSTANDING

Simple interest, which you studied in a previous lesson, is paid only on the initial principal of a savings account or loan. **Compound interest** is paid on the initial principal and on interest earned in the past. Compound interest can be compounded over any length of time.

Example 1

INVESTMENTS **Mackenzie is planning to invest $1000 at an annual interest rate of 8% for 2 years. Determine the amount she will have if the interest is compounded over each length of time.**

a. 1 year **b.** 6 months **c.** 3 months

Solution

a. Use a table to find the total amount.

Time	Beginning Principal	Interest	New Principal
1 year	$1000.00	$1000.00 × 0.08 = $80.00	$1000.00 + $80.00 = $1080.00
2 years	$1080.00	$1080.00 × 0.08 = $86.40	$1080.00 + $86.40 = $1166.40

b. If the interest is compounded every 6 months, the interest rate for each time period is $\frac{1}{2}$ of 8% or 4%. Round to the nearest cent.

Time	Beginning Principal	Interest	New Principal
6 months	$1000.00	$1000.00 × 0.04 = $40.00	$1000.00 + $40.00 = $1040.00
1 year	$1040.00	$1040.00 × 0.04 = $41.60	$1040.00 + $41.60 = $1081.60
18 months	$1081.60	$1081.60 × 0.04 ≈ $43.26	$1081.60 + $43.26 = $1124.86
2 years	$1124.86	$1124.86 × 0.04 ≈ $44.99	$1124.86 + $44.99 = $1169.85

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c. If the interest is compounded every 3 months, the interest rate for each time period is $\frac{1}{4}$ of 8% or 2%. Round to the nearest cent.

Time	Beginning Principal	Interest	New Principal
3 months	\$1000.00	$\$1000.00 \times 0.02 = \$20.00$	$\$1000.00 + \$20.00 = \$1020.00$
6 months	\$1020.00	$\$1020.00 \times 0.02 = \$20.40$	$\$1020.00 + \$20.40 = \$1040.40$
9 months	\$1040.40	$\$1040.40 \times 0.02 \approx \$20.81$	$\$1040.40 + \$20.81 = \$1061.21$
1 year	\$1061.21	$\$1061.21 \times 0.02 \approx \$21.22$	$\$1061.21 + \$21.22 = \$1082.43$
15 months	\$1082.43	$\$1082.43 \times 0.02 \approx \$21.65$	$\$1082.43 + \$21.65 = \$1104.08$
18 months	\$1104.08	$\$1104.08 \times 0.02 \approx \$22.08$	$\$1104.08 + \$22.08 = \$1126.16$
21 months	\$1126.16	$\$1126.16 \times 0.02 \approx \$22.52$	$\$1126.16 + \$22.52 = \$1148.68$
2 years	\$1148.68	$\$1148.68 \times 0.02 \approx \$22.97$	$\$1148.68 + \$22.97 = \$1171.65$

Notice that as the length of the compounding time decreases, the amount of money Mackenzie will have increases.

The tables help you to understand how compounding works. You can also use the following formula to find the amount of the investment.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

In this formula, A = the amount of the investment,
P = the initial amount of the investment,
r = annual rate of interest expressed as a decimal,
n = the number of times that the interest is compounded each year and
t = the number of years that the money is invested.

Example 2

INVESTMENTS Use the formula to find the amount of money Mackenzie will have if she invests \$1000 for 2 years at an annual interest rate of 8% which is compounded every 3 months.

Solution

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 1000\left(1 + \frac{0.08}{4}\right)^{4 \cdot 2} \quad P = 1000,\ r = 0.08,\ n = 4, \text{ and } t = 2.$$

$$\approx 1171.66 \quad \text{Use a calculator.}$$

Mackenzie will have \$1171.66. Notice that this value is one cent more than the number found in the table. The difference is due to rounding.

Try These Exercises

1. Suppose \$5000 is invested at an annual interest rate of 6% compounded semiannually. Make a table showing the value of the account every 6 months for the next 3 years. Round to the nearest cent.

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Find the value of each account. Round to the nearest cent.

2. Amount of investment: $4000
Annual interest rate: 4%
Compounding period: 6 months
Length of investment: 5 years

3. Amount of investment: $10,000
Annual interest rate: 10%
Compounding period: 3 months
Length of investment: 4 years

4. Amount of investment: $7500
Annual interest rate: 6%
Compounding period: 1 month
Length of investment: 10 years

5. Amount of investment: $13,000
Annual interest rate: 6%
Compound period: 3 months
Length of investment: 8 years

PRACTICE EXERCISES

Find the value of each account. Round to the nearest cent.

6. Amount of investment: $5000
Annual interest rate: 5%
Compounding period: 6 months
Length of investment: 6 years

7. Amount of investment: $16,000
Annual interest rate: 8%
Compounding period: 3 months
Length of investment: 7 years

8. Amount of investment: $56,000
Annual interest rate: 6%
Compounding period: 2 months
Length of investment: 8 years

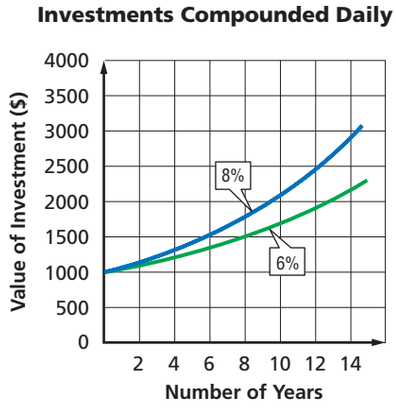
9. Amount of investment: $28,000
Annual interest rate: 10%
Compounding period: 3 months
Length of investment: 4 years

10. Amount of investment: $17,000
Annual interest rate: 5%
Compounding period: 3 months
Length of investment: 10 years

11. Amount of investment: $33,000
Annual interest rate: 8%
Compounding period: 1 week
Length of investment: 4 years

12. **INVESTMENTS** Shane is planning to invest $5000 at an annual interest rate of 8% compounded daily. How much will he have after 2 years? (Assume there are 365 days in a year.)

13. **BANKING** Jason can invest $1000 in two different banks. One bank pays 8% compounded daily and the other bank pays 6% compounded daily. Use the graph at the right to estimate how long it will take Jason to double his money under each plan.



14. **INVESTMENTS** Determine the amount of an investment if $400 is invested at an interest rate of 7.25% compounded quarterly for 7 years.

15. **BANKING** Danielle wants to invest $5000 for 3 years. Which of the banks listed at the right would give her the most interest?

Bank	Savings Plan
Central City Bank	5% compounded daily
Country Bank	5.2% compounded monthly
Best Bank	5.3% compounded semiannually

16. How much more interest is paid if $3000 is invested for one year at 6% compounded daily rather than at 6% compounded semiannually?

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17. **HISTORY** In 1626, Peter Minuit, governor of the colony of New Netherland, bought the island of Manhattan for beads, cloth, and trinkets worth $24. If the $24 were invested at 6% per year compounded monthly, how much money would there be in the year 2026?

18. **WRITING MATH** Explain the difference between simple interest and compound interest.

Extended Practice Exercises

19. **INVESTMENTS** In January, Andrea had $6000 in an account. She plans to add $1000 to her investment at the beginning of each coming year. If the account pays 5% compounded monthly, how much will she have in 5 years?

20. **BANKING** Shane has $1000 in a savings account at the bank on the last day of a month. On the 15th of the next month, he made a deposit of $400. On the 29th of that month, he withdrew $200. If the bank pays 3% interest compounded daily, how much money will he have the on 30th of the month?

21. **INVESTMENTS** Kelly plans to invest $10,000 for 4 years in an account that pays 5% compounded quarterly. How much interest will Kelly make in the fourth year?

22. **INVESTMENTS** Determine the amount of an investment if $14,000 is invested at an interest rate of 7% compounded quarterly for 3 years.

23. **INVESTMENTS** Chase is planning to invest $2000 at an annual interest rate of 6% compounded daily. How much will he have after 3 years? (Assume there are 365 days in a year.)

24. **INVESTMENTS** Find the amount of money Kaylee will have if she invests $5000 for 3 years at an annual interest rate of 7% which is compounded every 3 months.

25. **INVESTMENTS** How much interest is paid if $3000 is invested for 4 years at 8% compounded semiannually?

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