Enrichment Workbook

Glencoe

Geometry

Concepts and Applications

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Enrichment Masters

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CONTENTS

Enrichment

Mind Reading

Make four cards having the following numbers and letters.

Ask someone to think of a number from 1 to 15. Then ask the person which cards the number is on.

A	B	C	D
1 3 5	2 3 6	4 5 6	8 9 10
7 9	7 10	7 12	11 12
11 13 15	11 14 15	13 14 15	13 14 15

For example, suppose the number appears on cards A, B, and C. Add the least number on each card. This sum is 1 + 2 + 4, or 7. The person's number is 7.

Answer these questions.

1. What is the pattern of the numbers on card A?

2. The least number on card B is 2. Note that the next counting number, 3, is also on the card. The next two counting numbers, 4 and 5, are skipped. Then the next two counting numbers, 6 and 7, do appear on the card. This pattern is repeated up to 15. Can you explain the patterns on cards C and D?

3. The least numbers on card A through card D are 1, 2, 4, 8. How are these numbers related to the base-two numeration system?

Identify the number chosen when it appears on the following cards.

4. C and D

5. A, B, and D

6. A, B, C, and D

Each of the following numbers is chosen. Write the corresponding 4-digit base-two numeral. Arrange the cards in base-two place-value position from left to right, D (eights), C (fours), B (twos), A (ones). Interpret 1 as on the card and 0 as not on the card. Use the base-two numeral to determine on which cards each given number appears.

7. 6

8. 14

9. 10

Suppose you want to do this trick with five cards, A through E. Which numbers should appear on each of the following cards?

10. card A

11. card B

12. card C

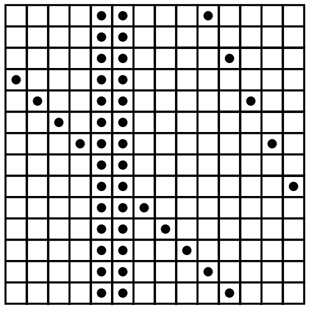
13. card D

14. card E

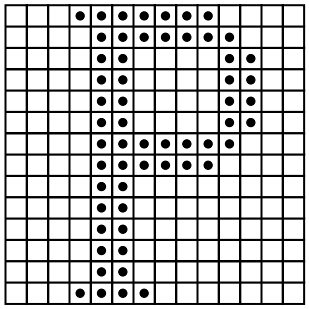
NAME _________________________________ DATE __________ PERIOD _____

Enrichment

Points and Lines on a Matrix



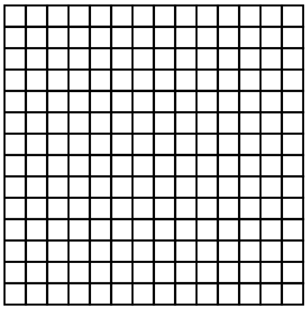
A **matrix** is a rectangular array of rows and columns. Points and lines on a matrix are not defined in the same way as in Euclidean geometry. A **point** on a matrix is a dot, which can be small or large. A **line** on a matrix is a path of dots that “line up.” Between two points on a line there may or may not be other points. Three examples of lines are shown at the upper right. The broad line can be thought of as a single line or as two narrow lines side by side.



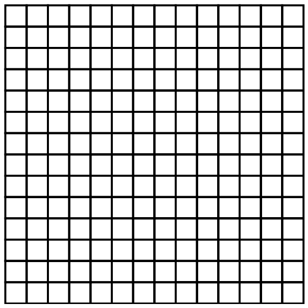
A dot-matrix printer for a computer uses dots to form characters. The dots are often called **pixels**. The matrix at the right shows how a dot-matrix printer might print the letter P.

Draw points on each matrix to create the given figures.

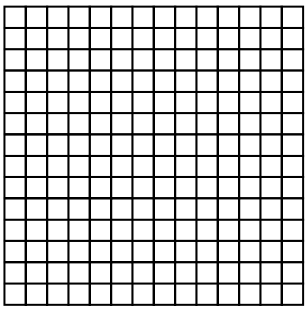
1. Draw two intersecting lines that have four points in common.



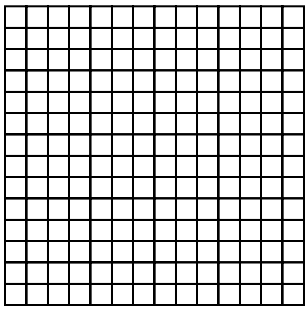
2. Draw two lines that cross but have no common points.



3. Make the number 0 (zero) so that it extends to the top and bottom sides of the matrix.



4. Make the capital letter O so that it extends to each side of the matrix.



5. Using separate grid paper, make dot designs for several other letters. Which were the easiest and which were the most difficult?

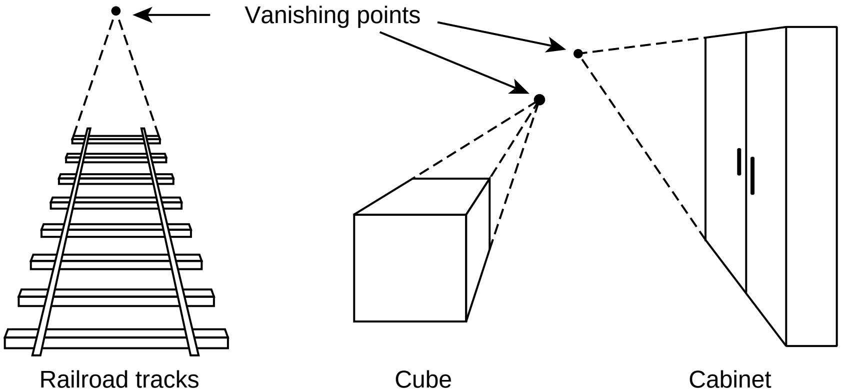
NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 18–23

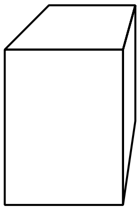
Perspective Drawings

To draw three-dimensional objects, artists make **perspective drawings** such as the ones shown below. To indicate depth in a perspective drawing, some parallel lines are drawn as converging lines. The dotted lines in the figures below each extend to a **vanishing point**, or spot where parallel lines appear to meet.

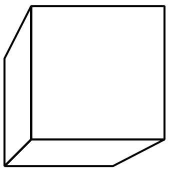


Draw lines to locate the vanishing point in each drawing of a box.

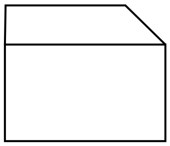
1.



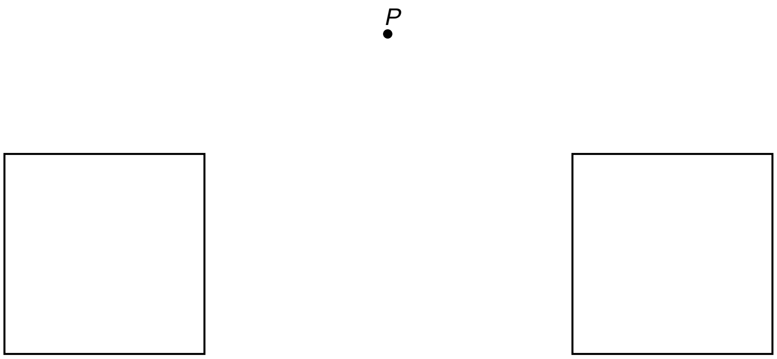
2.



3.



4. The fronts of two cubes are shown below. Using point *P* as the vanishing point for both cubes, complete the perspective drawings of the cubes.



5. Find an example of a perspective drawing in a newspaper or magazine. Trace the drawing and locate a vanishing point.

NAME ______________________________ DATE __________ PERIOD _____

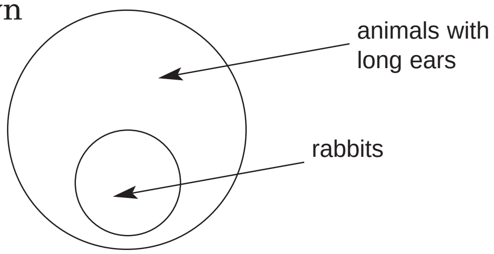
Enrichment

Student Edition
Pages 24–28

Venn Diagrams

A type of drawing called a **Venn diagram** can be useful in explaining conditional statements. A Venn diagram uses circles to represent sets of objects.

Consider the statement “All rabbits have long ears.” To make a Venn diagram for this statement, a large circle is drawn to represent all animals with long ears. Then a smaller circle is drawn inside of the first to represent all rabbits. The Venn diagram shows that every rabbit is included in the group of long-eared animals.



The set of rabbits is called a subset of the set of long-eared animals.

The Venn diagram can also explain how to write the statement, “All rabbits have long ears,” in if-then form. Every rabbit is in the group of long-eared animals, so if an animal is a rabbit, then it has long ears. This is the if-then form of the statement.

For each statement, draw a Venn diagram. Then write the sentence in if-then form.

1. Every dog has long fur.

2. All rational numbers are real numbers.

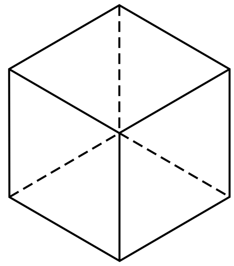
3. People who live in Iowa like corn.

4. Staff members are allowed in the faculty cafeteria.

NAME ______________________________ DATE __________ PERIOD ______

Enrichment

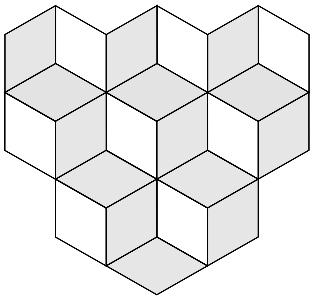
Optical Illusions



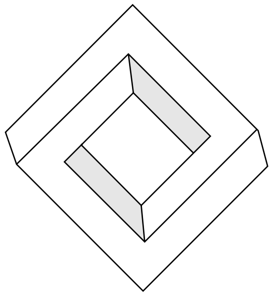
In drawings, diagonal lines may create the illusion of depth. For example, the figure at the right can be thought of as picturing a flat figure or a cube. The optical illusions on this page involve depth perception.

Answer each question.

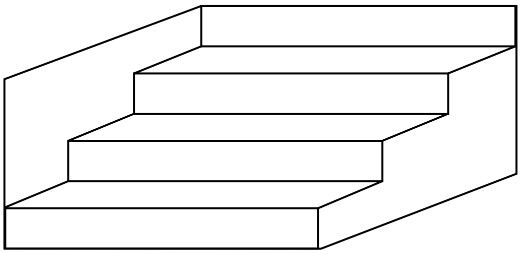
1. How many cubes do you see in the drawing?



2. Can this figure show an actual object?

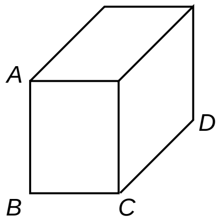


3.



Does the drawing show a view from the top or the bottom of the stairs?

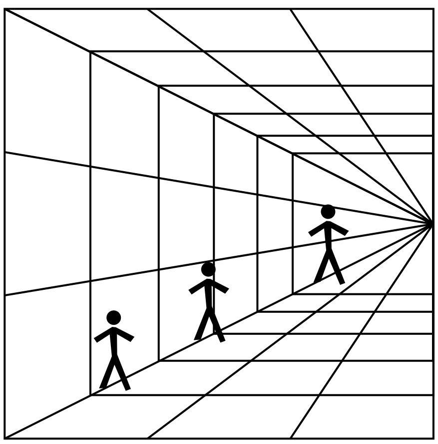
4.



Which line segment is longer, $\overline{AB}$ or $\overline{CD}$? Measure to check your answer.

5. Which person in the drawing at the right appears to be tallest? Measure to check your answer.

6. Draw two more objects the same size on the figure at the right. Does one appear larger than the other?



NAME ______________________________ DATE __________ PERIOD ______

Enrichment

Student Edition
Pages 35–41

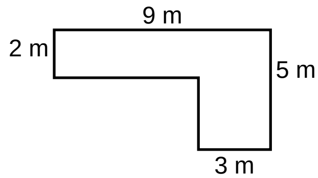
Perimeter and Area of Irregular Shapes

Two formulas that are used frequently in mathematics are perimeter and area of a rectangle.

Perimeter: $P = 2\ell + 2w$,
Area: $A = \ell w$, where ℓ is the length and w is the width

However, many figures are combinations of two or more rectangles, creating **irregular shapes**. To find the area of an irregular shape, it helps to separate the shape into rectangles, calculate the formula for each rectangle, then find the sum of the areas.

Example: Find the area of the figure at the right.



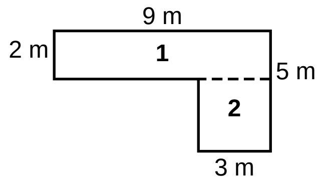
Separate the figure into two rectangles.

$A = \ell w$

$A_1 = 9 \cdot 2 = 18$ $\qquad$ $A_2 = 3 \cdot 3 = 9$

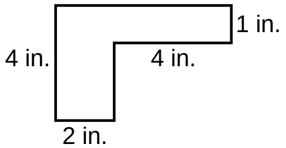
$18 + 9 = 27$

The area of the irregular shape is 27 m^2.

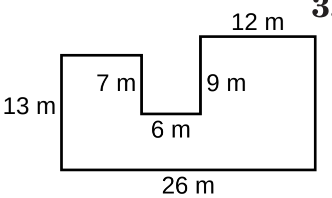


Find the area of each irregular shape.

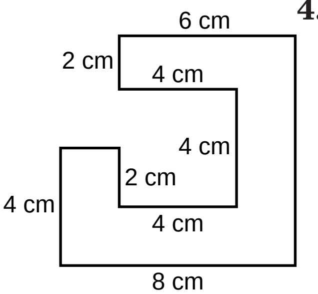
1.



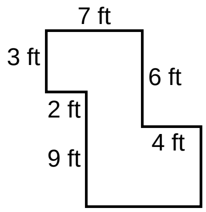
2.



3.



4.



For questions 5–8, find the perimeter of the figures in Exercises 1–4.

5. ______________ **6.** ______________ **7.** ______________ **8.** ______________

9. Describe the steps you used to find the perimeter in Exercise 1.

____

____

____

NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 50–55

Distances and Coordinates

The distance between points on a number line can be found by subtracting the smaller coordinate from the larger one, provided it is known which coordinate is greater. If the values of the coordinates of the two points are unknown, then either coordinate may be greater. Therefore, two cases must be considered.

Example: Suppose A and B are points on a number line. The coordinate of A is $3x + 1$, the coordinate of B is $x + 5$, and $AB = 4$. Find both possible values of x and their corresponding sets of coordinates for points A and B.

Case I: Suppose the coordinate of point B is greater than that of point A.

The distance between A and $B = (x + 5) - (3x + 1)$.

$$\begin{aligned} AB &= -2x + 4 \\ 4 &= -2x + 4 \quad \textit{Substitution} \\ 0 &= -2x \\ 0 &= x \end{aligned}$$

If $x = 0$, then the coordinates of points A and B are 1 and 5, respectively.

Case II: Suppose the coordinate of point A is greater than that of point B.

The distance between A and $B = (3x + 1) - (x + 5)$.

$$\begin{aligned} AB &= 2x - 4 \\ 4 &= 2x - 4 \quad \textit{Substitution} \\ 8 &= 2x \\ 4 &= x \end{aligned}$$

If $x = 4$, then the coordinates of points A and B are 13 and 9, respectively.

For each of the following, find both possible values of x and their corresponding sets of coordinates for points A and B.

1. coordinate of A is $x + 5$, coordinate of B is $2x$, $AB = 11$

2. coordinate of A is $x + 3$, coordinate of B is $2x - 1$, $AB = 6$

3. coordinate of A is $2x + 15$, coordinate of B is $-3x - 5$, $AB = 25$

4. coordinate of A is $5x - 7$, coordinate of B is $3x + 2$, $AB = 5$

NAME ________________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 56–61

Symmetric, Reflexive, and Transitive Properties

Equality has three important properties.

Reflexive	$a = a$
Symmetric	If $a = b$, then $b = a$.
Transitive	If $a = b$ and $b = c$, then $a = c$.

Other relations have some of the same properties. Consider the relation "is next to" for objects labeled X, Y, and Z. Which of the properties listed above are true for this relation?

X is next to X. *false*
If X is next to Y, then Y is next to X. *true*
If X is next to Y and Y is next to Z, then X is next to Z. *false*

Only the Symmetric Property is true for the relation "is next to."

For each relation, state which properties (Symmetric, Reflexive, Transitive) are true.

1. is the same size as

2. is a family descendant of

3. is in the same room as

4. is the identical twin of

5. is warmer than

6. is on the same line as

7. is a sister of

8. is the same weight as

9. Find two other examples of relations, and tell which properties are true for each relation.

2-3

NAME ______________________________ DATE __________ PERIOD ______

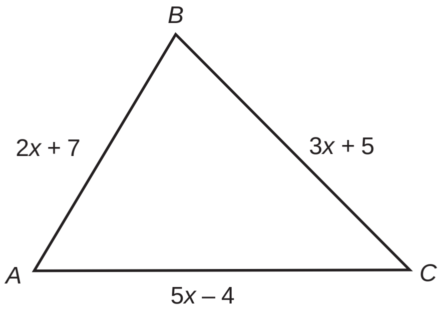
Enrichment

Student Edition
Pages 62–67

Perimeters and Unknown Values

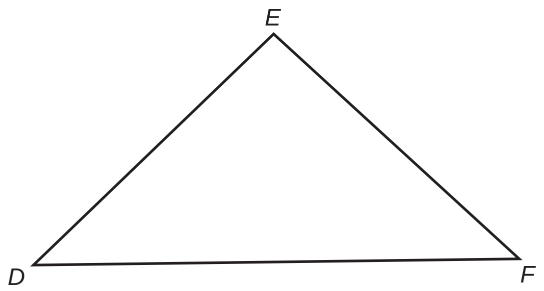
Use the information given to find the unknown values in each of the following.

1.



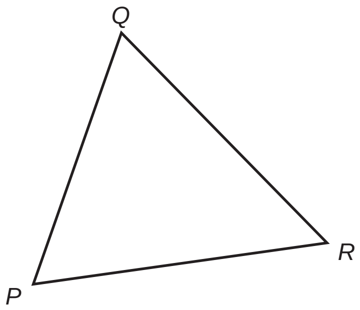
The perimeter is 68.
Find x, AB, BC, and AC.

2.



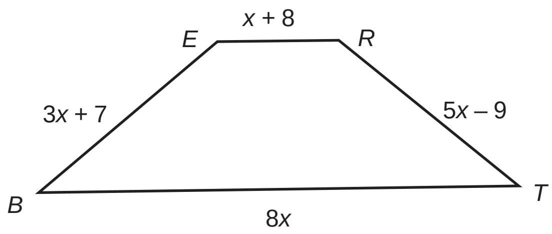
$DF = 3x + 1$, $\overline{DE} \cong \overline{EF}$, $DE = x + 5$
The perimeter is 31.
Find x, DE, and DF.

3.



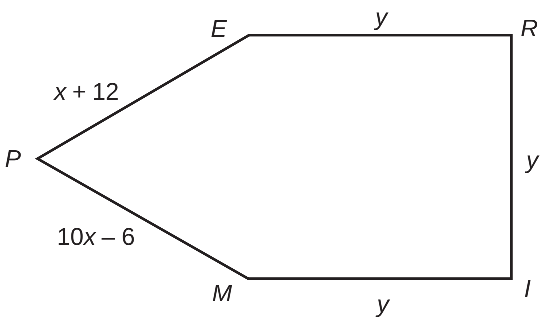
$\overline{PR} \cong \overline{QR}$, $PQ = x + 3$, $QR = x + 6$
The perimeter is 93.
Find x and PR.

4.



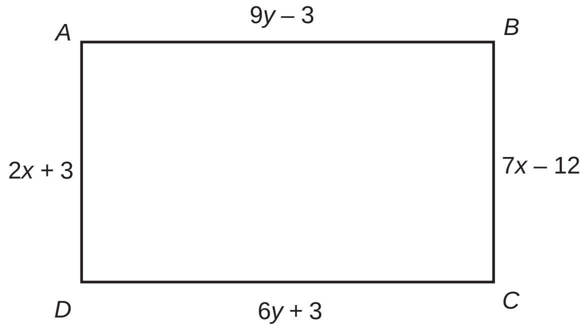
$\overline{BE} \cong \overline{RT}$
Find x, BT, RT, and the perimeter.

5.



$\overline{PE} \cong \overline{PM}$, $y = 5x + 5$
Find the perimeter of the figure.

6.



$\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$
Find the perimeter of the figure.

NAME ______________________________ DATE __________ PERIOD ______

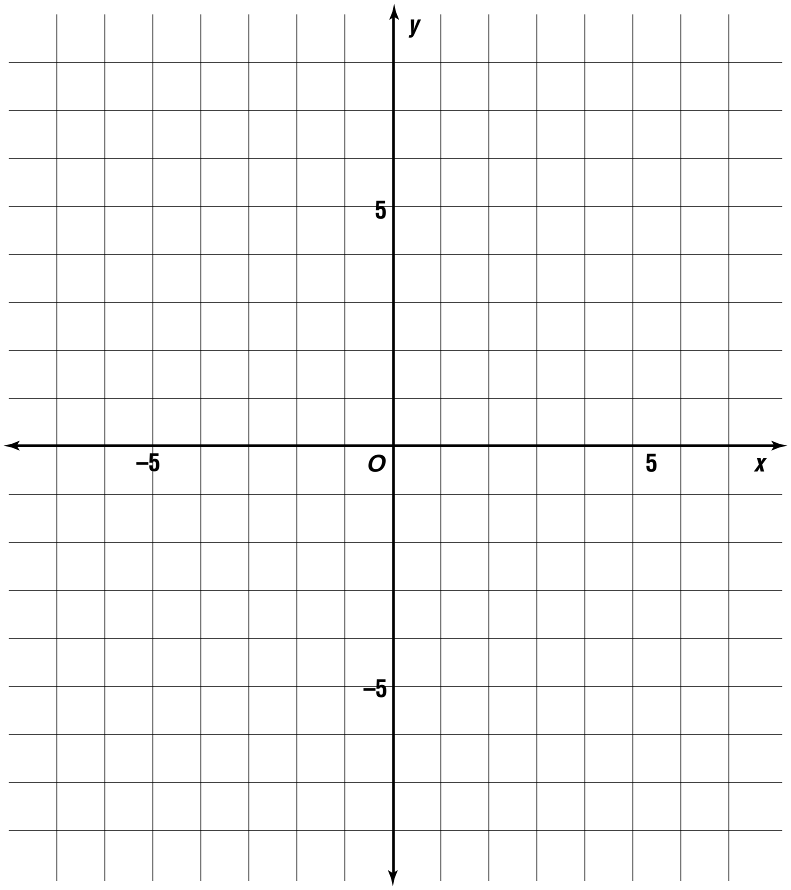
Enrichment

Student Edition
Pages 68–73

Coordinate Pictures

By connecting the points for two ordered pairs, you can make a line segment. Line segments can be combined to make pictures.

Graph the two ordered pairs in each exercise, and connect them with a line segment. After you draw each segment, identify the picture.



1. (−2, 5.5) and (5, 5.5)

2. (−3, 4) and (4, 4)

3. (−2, 3) and (3, 3)

4. (−2, −0.5) and (3, −0.5)

5. (1, −1.5) and (3, −1.5)

6. (−3, −2) and (4, −2)

7. (−3, −3) and (4, −3)

8. (−4, −4) and (5.5, −4)

9. (−3, −4.5) and (4, −4.5)

10. (−3.5, −5) and (3.5, −5)

11. (−4, −5.5) and (3, −5.5)

12. (−4.5, −6) and (2.5, −6)

13. (−6.5, −6.5) and (3.5, −6.5)

14. (−6.5, −7) and (3.5, −7)

15. (−3, 4) and (−3, −3)

16. (−2, −0.5) and (−2, 3)

17. (3, −0.5) and (3, 3)

18. (4, −3) and (4, 4)

19. (5, −1.5) and (5, 5.5)

20. (3.5, −6.5) and (3.5, −7)

21. (5.5, −4) and (5.5, −4.5)

22. (−6.5, −6.5) and (−6.5, −7)

23. (4, −2) and (5, −0.5)

24. (4, −3) and (5, −1.5)

25. (−3, 4) and (−2, 5.5)

26. (4, 4) and (5, 5.5)

27. (−6.5, −6.5) and (−4, −4)

28. (5.5, −4) and (3.5, −6.5)

29. (5.5, −4.5) and (3.5, −7)

30. What does the picture show? ______________________________

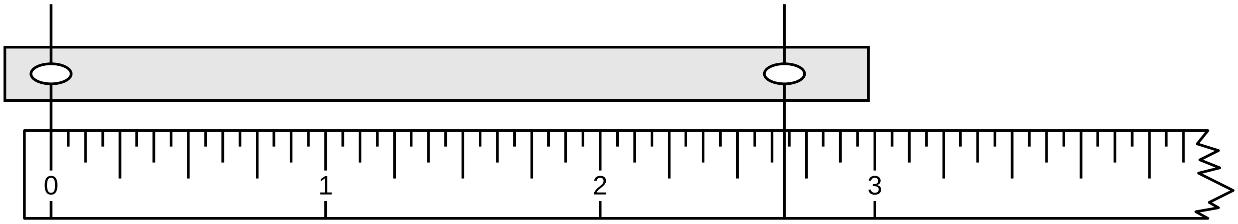
NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 76–81

Absolute Error

The **absolute error** of a measurement is defined to be one-half the smallest unit used in making the measurement. For example, this drawing shows the distance between the centers of the two holes in a piece of metal.



If the distance were measured to the nearest quarter of an inch, the absolute error would be one-eighth of an inch. The symbol ± means "plus or minus." This symbol is often used to report measurements.

$2\frac{3}{4}$ in. ± $\frac{1}{8}$ in.

This way of reporting measurements helps to show how accurate the measurement is. The actual measurement will lie somewhere in this interval.

$$2\tfrac{3}{4}\text{ in.} - \tfrac{1}{8}\text{ in.} < m < 2\tfrac{3}{4}\text{ in.} + \tfrac{1}{8}\text{ in.}$$
$$2\tfrac{5}{8}\text{ in.} < m < 2\tfrac{7}{8}\text{ in.}$$

Write each reported measurement using an interval. Use m to represent the actual measurement.

1. 25,000 ± 500 voters

2. 15 ± 0.5 kg

3. 750 ± 25 customers

4. 75 ± 5 mi

5. 14 ± $\frac{1}{2}$ gal

6. $7\frac{1}{4}$ ± $\frac{1}{4}$ in.

Name the unit of measure used to make each measurement.

7. 32 ± $\frac{1}{2}$ ft

8. 23 ± 0.5 m

9. $5\frac{1}{4}$ ± $\frac{1}{8}$ mi

10. 14 ± 0.5 cm

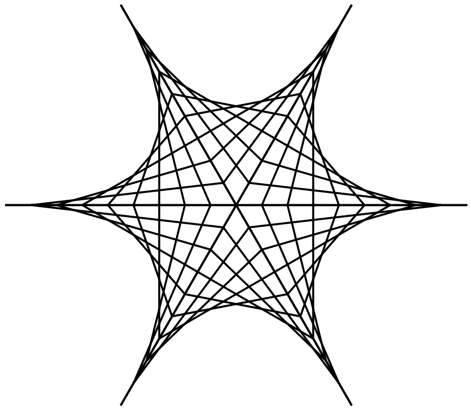
11. $2\frac{3}{8}$ ± $\frac{1}{16}$ in.

12. 8 ± $\frac{1}{2}$ yd

NAME ______________________ DATE __________ PERIOD _____

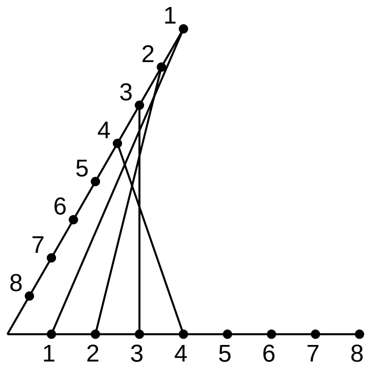
Enrichment

Curve Stitching



The star design at the right was created by a method known as **curve stitching**. Although the design appears to contain curves, it is made up entirely of line segments.

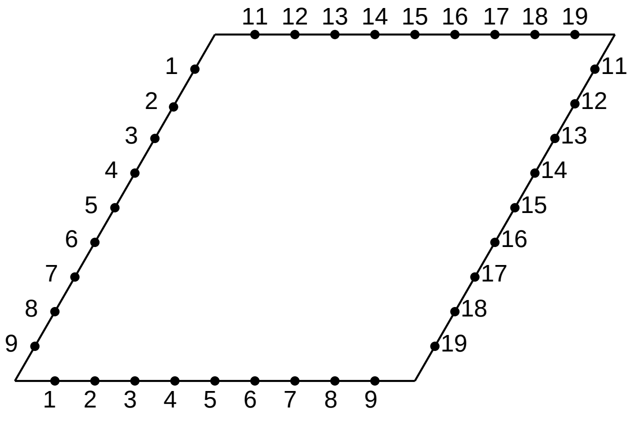
To begin the star design, draw a 60° angle. Mark eight equally-spaced points on each ray, and number the points as shown below. Then connect pairs of points that have the same number.



To make a complete star, make the same design in six 60° angles that have a common central vertex.

1. Complete the section of the star design above by connecting pairs of points that have the same number.

2. Complete the following design.



3. Create your own design. You may use several angles, and the angles may overlap.

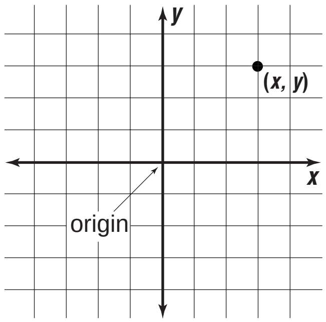
NAME ______________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 96–101

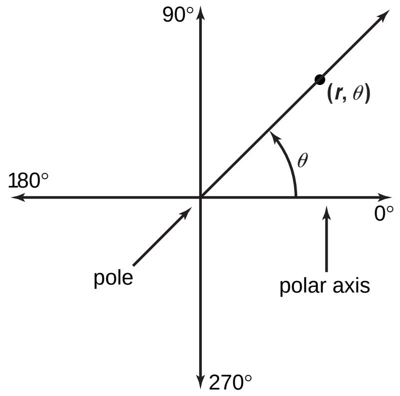
Polar Coordinates

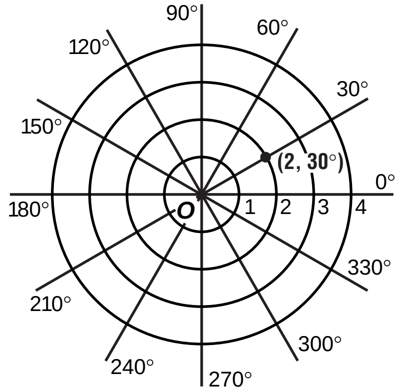
In a rectangular coordinate system, the ordered pair (x, y) describes the location of a point x units from the origin along the x-axis and y units from the origin along the y-axis.



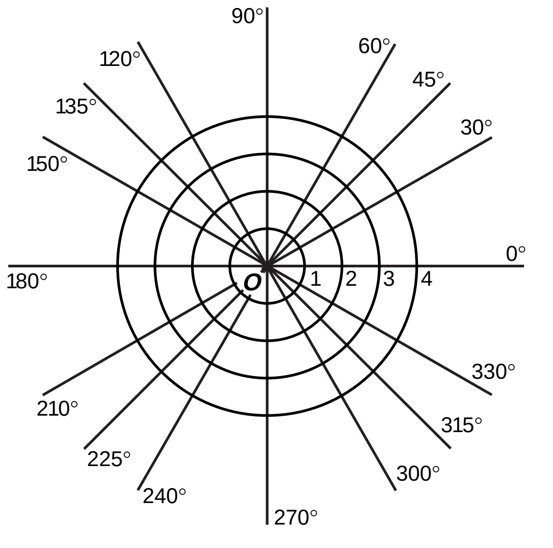
In a polar coordinate system, the ordered pair (r, θ) describes the location of a point r units from the pole on the ray (vector) whose endpoint is the pole and which forms an angle of θ with the polar axis.

The graph of (2, 30°) is shown on the polar coordinate system at the right below. Note that the concentric circles indicate the number of units from the pole.





Locate each point on the polar coordinate system below.



1. (3, 45°) **2.** (1, 135°) **3.** $(2\frac{1}{2}, 60°)$ **4.** (4, 120°)

5. (2, 225°) **6.** (3, –30°) **7.** (1, –90°) **8.** (–2, 30°)

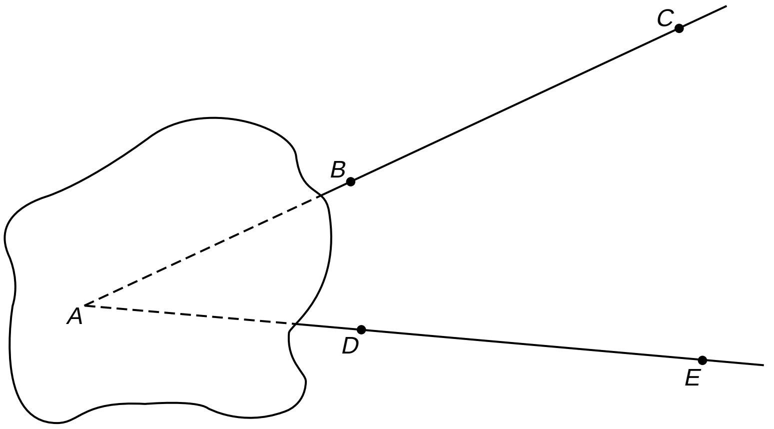
NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 104–109

Bisecting a Hidden Angle

The vertex of $\angle BAD$ at the right is hidden in a region. Within the region, you are not allowed to use a compass. Can you bisect the angle?



Follow these instructions to bisect $\angle BAD$.

1. Use a straightedge to draw lines *CE* and *BD*.

2. Construct the bisectors of $\angle DEC$ and $\angle BCE$.

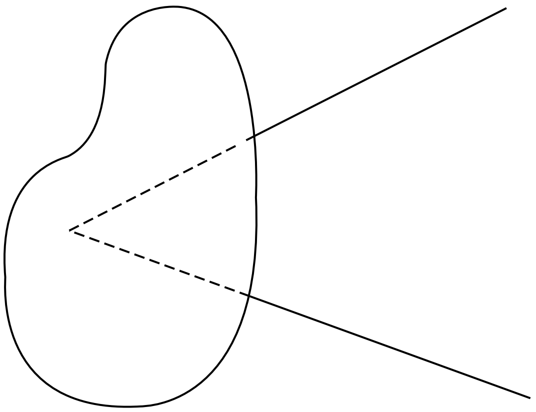
3. Label the intersection of the two bisectors as point *P*.

4. Construct the bisectors of $\angle BDE$ and $\angle DBC$.

5. Label the intersection of the two previous bisectors as point *Q*.

6. Use a straightedge to draw line *PQ*, which bisects the hidden angle.

7. Another hidden angle is shown at right. Construct the bisector using the method above, or devise your own method.



NAME ________________________________ DATE __________ PERIOD ______

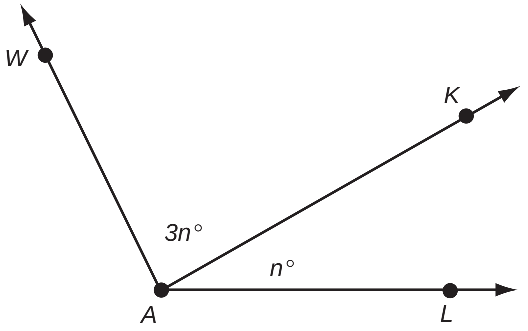
Enrichment

Using Ratios

Information about two numbers is often given by stating the ratio between them. Suppose x and y are in the ratio of 3 to 4 (or $\frac{3}{4}$). There are many possible values for x and y. For example, x may be 6 and y may be 8, since $\frac{6}{8} = \frac{3}{4}$. This is true if $x = 30$ and $y = 40$, or if $x = 1.5$ and $y = 2$, and so on. If x and y are in the ratio of 3 to 4, they may be represented as $x = 3n$ and $y = 4n$, where n is any real number. It is frequently necessary to represent unknown quantities to solve problems.

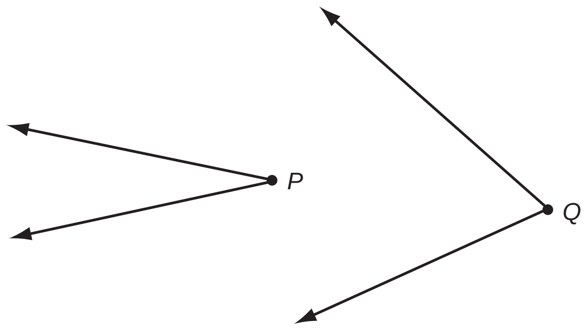
Solve.

1.



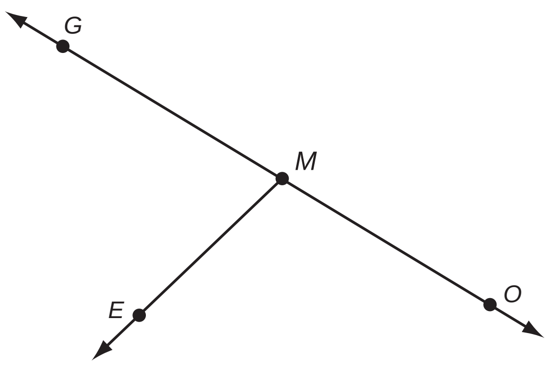
$\angle KAW$ and $\angle LAK$ are in the ratio of 3 to 1 and $m\angle WAL = 116$. Write an equation and find n. Then find $m\angle KAW$ and $m\angle LAK$.

2.



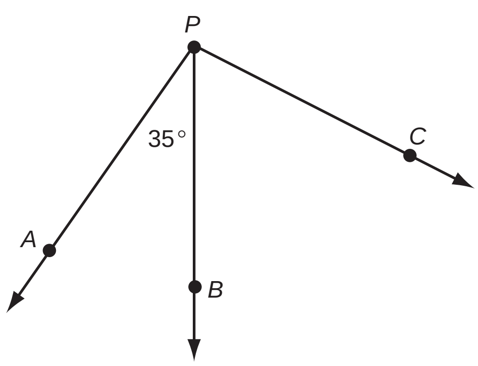
The measures of $\angle P$ and $\angle Q$ are in the ratio of 4 to 11. $\angle P$ and $\angle Q$ are complementary. Write an equation and find the measures of $\angle P$ and $\angle Q$.

3.



$\angle GME$ and $\angle EMO$ are supplementary and in the ratio of 5 to 7. Write an equation and find the measure of each angle.

4.



$\angle BPC$ and $\angle APC$ are in the ratio of 9 to 14. Write an equation and find the measure of each angle.

NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Angle Relationships

Angles are measured in degrees (°). Each degree of an angle is divided into 60 minutes (′), and each minute of an angle is divided into 60 seconds (″).

$$60' = 1^\circ$$

$$60'' = 1'$$

$$67\tfrac{1}{2}^\circ = 67^\circ 30'$$

$$70.4^\circ = 70^\circ 24'$$

$$90^\circ = 89^\circ 60'$$

Two angles are complementary if the sum of their measures is 90°. Find the complement of each of the following angles.

1. $35^\circ 15'$

2. $27^\circ 16'$

3. $15^\circ 54'$

4. $29^\circ 18' 22''$

5. $34^\circ 29' 45''$

6. $87^\circ 2' 3''$

Two angles are supplementary if the sum of their measures is 180°. Find the supplement of each of the following angles.

7. $120^\circ 18'$

8. $84^\circ 12'$

9. $110^\circ 2'$

10. $45^\circ 16' 24''$

11. $39^\circ 21' 54''$

12. $129^\circ 18' 36''$

13. $98^\circ 52' 59''$

14. $9^\circ 2' 32''$

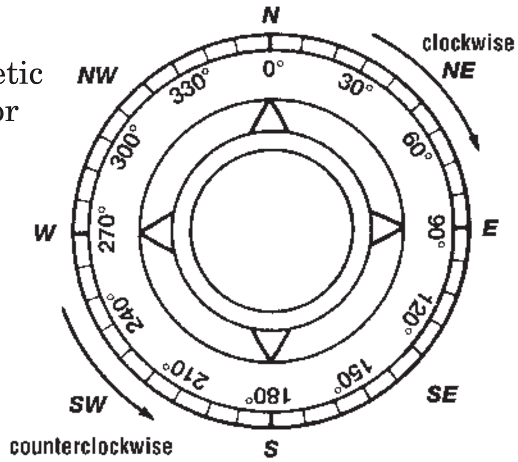
15. $1^\circ 2' 3''$

NAME ________________________________ DATE __________ PERIOD ______

Enrichment

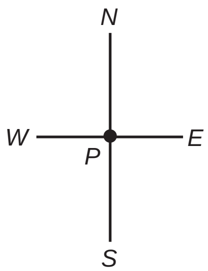
Using a Compass

Ships and airplanes navigate with the aid of a magnetic compass, which points due north. To find the course or bearing, the navigator measures the angle formed by due north and the line of travel of the plane or ship. This angle is measured clockwise (the direction in which the hands of a clock move). Bearings are between 0° and 360°, exclusively.

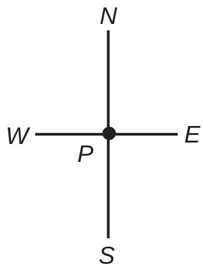


In Exercises 1–4, the perpendicular lines show north, south, east, and west. Draw a ray from point P to show the bearing.

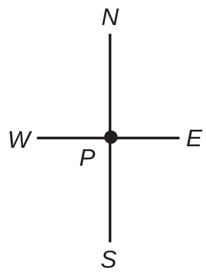
1. 23°



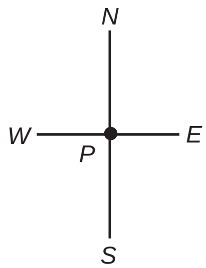
2. 167°



3. 210°

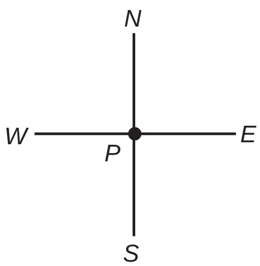


4. 325°

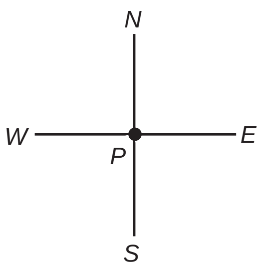


In Exercises 5–7, point P represents the position of a plane whose current bearing is given. Draw $\overrightarrow{PC}$ to show the direction the plane is currently traveling. The pilot wants to turn so that the plane will head at the new bearing. Draw $\overrightarrow{PN}$ to show the new bearing. Tell how many degrees clockwise or counterclockwise the pilot must turn the plane.

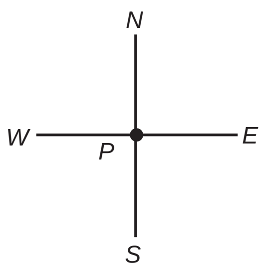
5. current heading: 135°
new heading: 62°



6. current heading: 160°
new heading: 255°



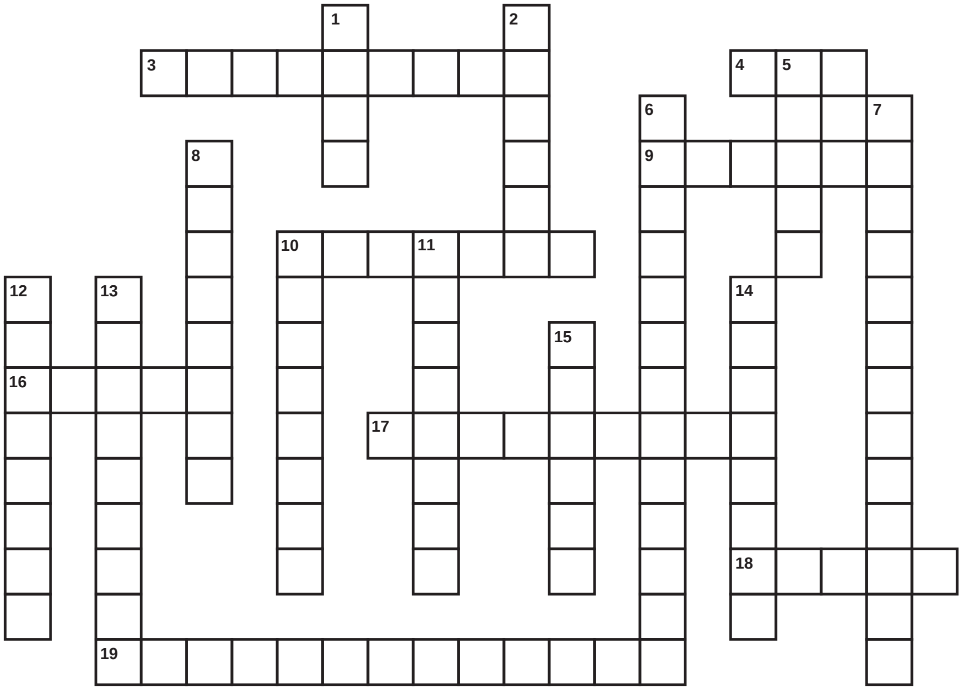
7. current heading: 215°
new heading: 140°



NAME ______________________ DATE __________ PERIOD ______

Enrichment

Geometry Crossword Puzzle



ACROSS

3. Points on the same line are ______.

4. A point on a line and all points of the line to one side of it.

9. An angle whose measure is greater than 90.

10. Two endpoints and all points between them.

16. A flat figure with no thickness that extends indefinitely in all directions.

17. Segments of equal length are ________ segments.

18. Two noncollinear rays with a common endpoint.

19. If $m\angle A + m\angle B = 180$, then $\angle A$ and $\angle B$ are ________ angles.

DOWN

1. The set of all points collinear to two points is a ________.

2. The point where the x- and y-axis meet.

5. An angle whose measure is less than 90.

6. If $m\angle A + m\angle D = 90$, then $\angle A$ and $\angle D$ are ________ angles.

7. Lines that meet at a 90° angle are ________.

8. Two angles with a common side but no common interior points are ________.

10. An "angle" formed by opposite rays is a ________ angle.

11. The middle point of a line segment.

12. Points that lie in the same plane are _______.

13. The four parts of a coordinate plane.

14. Two nonadjacent angles formed by two intersecting lines are ________ angles.

15. In angle ABC, point B is the ________.

4-1

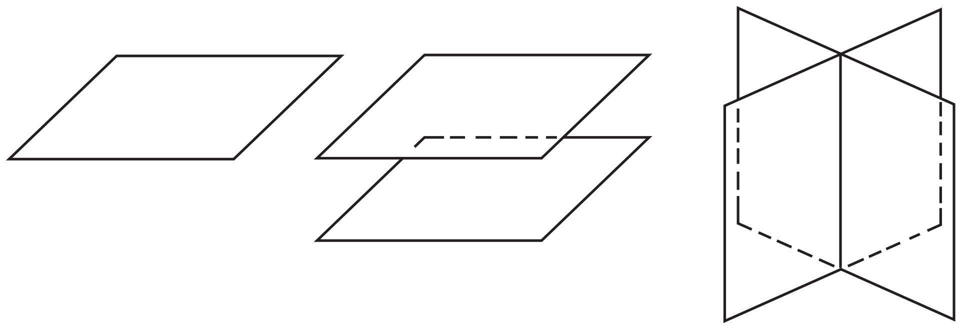
NAME ______________________________ DATE __________ PERIOD ______

Enrichment

Student Edition
Pages 142–147

Lines and Planes

Use the drawings below to help make a drawing that fits each description. If the situation cannot occur, write impossible.



1. a line parallel to a plane

2. a line intersecting a plane

3. a line intersecting each of two parallel planes

4. a line lying in one of two parallel planes and not intersecting the other

5. a line lying in one of two intersecting planes and intersecting the other in one point

6. a line intersecting each of two intersecting planes in one point

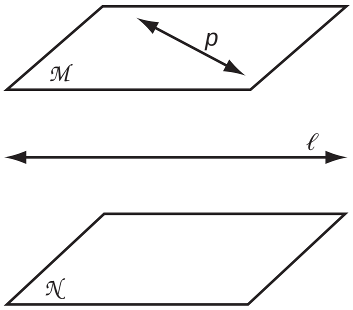
NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 148–153

Parallelism in Space

In space geometry, the concept of parallelism must be extended to include two planes and a line and a plane.



Definition: Two planes are parallel *if and only if* they do not intersect.

Definition: A line and a plane are parallel *if and only if* they do not intersect.

Thus, in space, two lines can be intersecting, parallel, or skew while two planes or a line and a plane can only be intersecting or parallel. In the figure at the right, $M \parallel N$, $\ell \parallel M$, $\ell \parallel N$, $p \parallel N$, and ℓ and p are skew.

The following six theorems are tests for and statements about parallel planes.

Theorem 1: If two planes are perpendicular to the same line, then the planes are parallel.

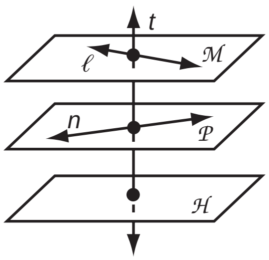
Theorem 2: If two planes are parallel to the same plane, then the two planes are parallel.

Theorem 3: If a line is perpendicular to one of two parallel planes, then it is perpendicular to the other.

Theorem 4: If a plane is perpendicular to one of two parallel planes, then it is perpendicular to the other.

Theorem 5: If two parallel planes each intersect a third plane, then the two lines of intersection are parallel.

***In the figure at the right, t ⊥ M, t ⊥ P, and P ∥ H. For each of the following, state* yes *or* no.**



1. $M \parallel P$

2. $\ell \parallel n$

3. $M \parallel H$

4. $\ell \parallel P$

5. $t \perp H$

6. $n \parallel H$

7. $\ell \perp P$

8. $t \parallel H$

***State whether each of the following is* true *or* false.**

9. If two lines are parallel to the same plane, then the lines are parallel.

10. If two planes are parallel to the same line, then the planes are parallel.

NAME ______________________________ DATE __________ PERIOD _____

4-3 Enrichment

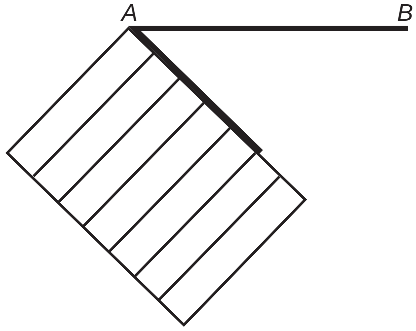
Student Edition
Pages 156–161

Parallel Lines and Congruent Parts

There is a theorem stating that if three parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on any transversal. This can be shown for any number of parallel lines. The following drafting technique uses this fact to divide a segment into congruent parts.

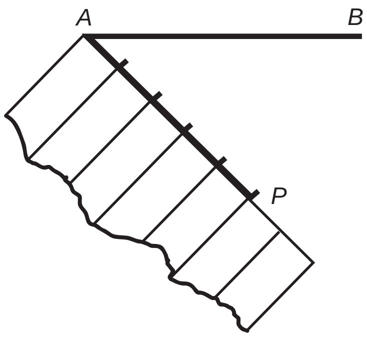
$\overline{AB}$ is to be separated into five congruent parts. This can be done very accurately without using a ruler. All that is needed is a compass and a piece of notebook paper.

Step 1 Hold the corner of a piece of notebook paper at point *A*.



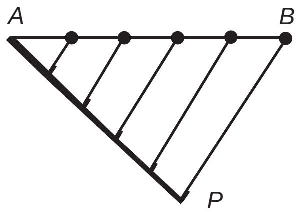
Step 2

From point *A*, draw a segment along the paper that is five spaces long. Mark where the lines of the notebook paper meet the segment. Label the fifth point, *P*.



Step 3

Draw $\overline{PB}$. Through each of the other marks on $\overline{AP}$, construct a line parallel to $\overline{BP}$. The points where these lines intersect $\overline{AB}$ will divide $\overline{AB}$ into five congruent segments.



Use a compass and a piece of notebook paper to divide each segment into the given number of congruent parts.

1. six congruent parts

2. seven congruent parts

NAME ______________________ DATE __________ PERIOD _____

Enrichment

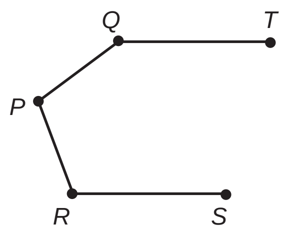
Auxiliary Lines

In solving a problem, it may be helpful to draw an **auxiliary line.** An auxiliary line is a line added to a diagram. Several postulates and theorems allow us to draw auxiliary lines. Two of these are shown below.

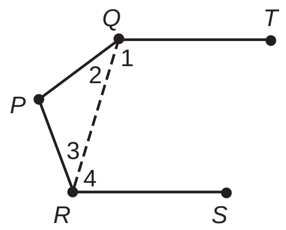
Postulate 1–1: Two points determine a unique line.

Theorem 3–9: If a line ℓ is in a plane and point T is a point on ℓ, then there exists exactly one line in that plane that is perpendicular to ℓ at T.

Example: In the figure at the right, $\overline{QT} \parallel \overline{RS}$. *Find* $m\angle P + m\angle Q + m\angle R$.

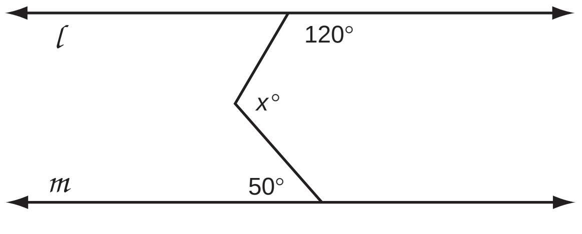


One way to solve the problem is to draw $\overline{QR}$ (Postulate 1–1) as an auxiliary line. By the Angle Sum Theorem, $m\angle P + m\angle 2 + m\angle 3 = 180$. Since $\angle 1$ and $\angle 4$ are consecutive interior angles, $m\angle 1 + m\angle 4 = 180$ by Theorem 7–2. Therefore, $m\angle P + m\angle 2 + m\angle 3 + m\angle 1 + m\angle 4 = 360$. By the Angle Addition Postulate, $m\angle 1 + m\angle 2 = m\angle Q$ and $m\angle 3 + m\angle 4 = m\angle R$. Thus, by substitution, $m\angle P + m\angle Q + m\angle R = 360$.

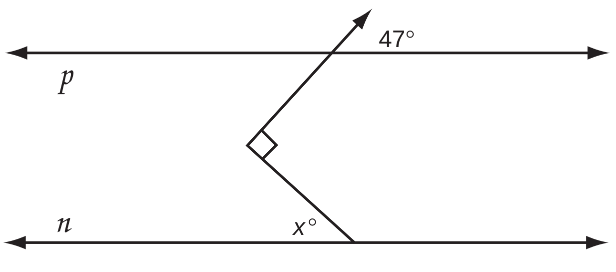


Solve each of the following using auxiliary lines.

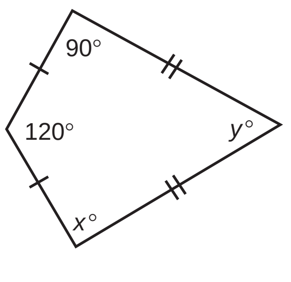
1. If $\ell \parallel m$, find x.



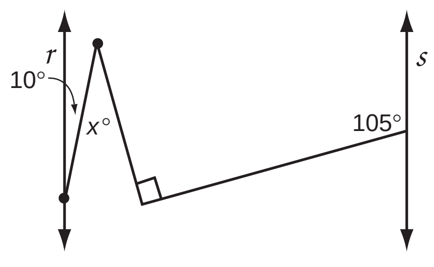
2. If $n \parallel p$, find x.



3. Find x and y.



4. If $r \parallel s$, find x.



NAME ______________________________ DATE __________ PERIOD ______

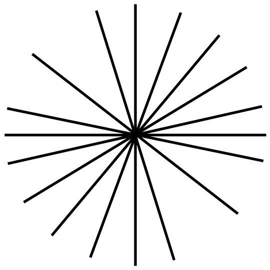
Enrichment

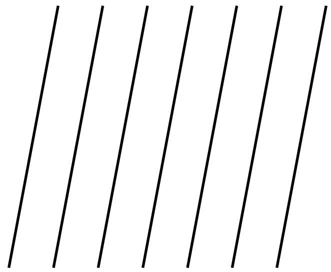
Student Edition
Pages 168–173

Pencils of Lines

All of the lines that pass through a single point in the same plane are called a **pencil of lines.**

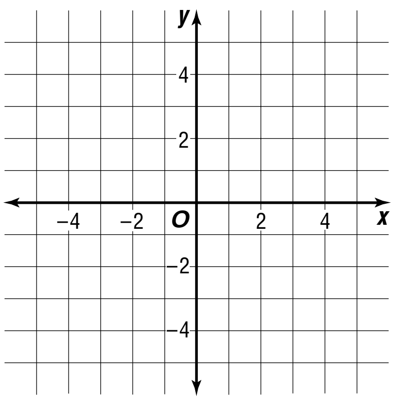
All lines with the same slope, but different intercepts, are also called a “pencil,” a **pencil of parallel lines.**



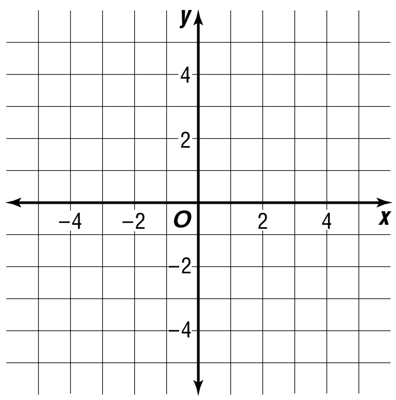


Graph some of the lines in each pencil.

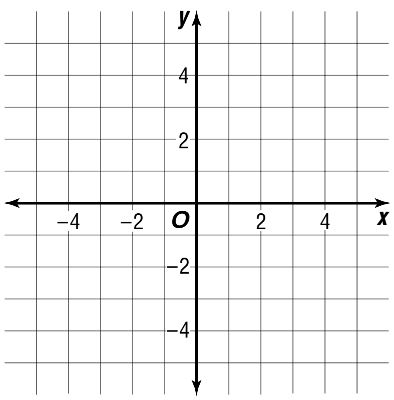
1. a pencil of lines through the point (1, 3)



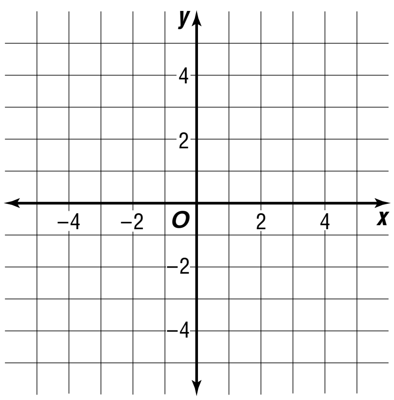
2. a pencil of lines described by $y - 4 = m(x - 2)$, where m is any real number



3. a pencil of lines parallel to the line $x - 2y = 7$



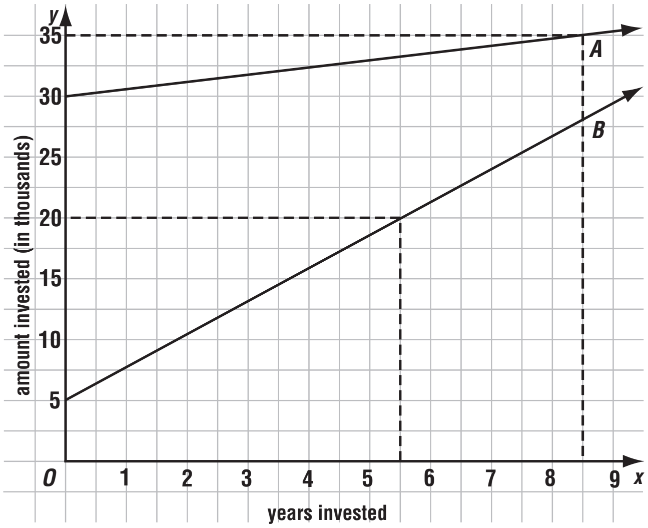
4. a pencil of lines described by $y = mx + 3m - 2$



Enrichment

Investments

The following graph represents two different investments. Line A represents an initial investment of \$30,000 with a bank paying passbook savings interest. Line B represents an initial investment of \$5,000 with a profitable mutual fund with dividends reinvested and capital gains accepted in shares. By deriving the linear equation, $y = mx + b$, for A and B, a projection of the future can be made.



Solve.

1. The y-intercept, b, is the initial investment. Find b for each of the following.

a. line A **b.** line B

2. The slope of the line, m, is the rate of return. Find m for each of the following.

a. line A **b.** line B

3. What are the equations of each of the following lines?

a. line A **b.** line B

Assume that the growth of each investment continues in the same pattern.

4. What will be the value of the mutual fund in the 11th year of investment?

5. What will be the value of the bank account in the 11th year of investment?

6. When will the mutual fund and the bank account have equal value?

7. In the long term, which investment has the greatest payoff?

NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 188–192

Reading Mathematics

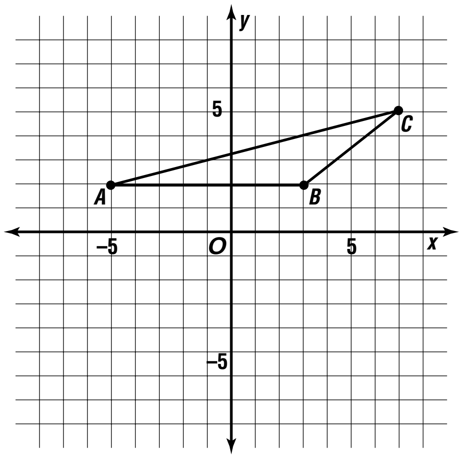
When you read geometry, you may need to draw a diagram to make the text easier to understand.

Example: Consider three points, A, B, and C on a coordinate grid. The y-coordinates of A and B are the same. The x-coordinate of B is greater than the x-coordinate of A. Both coordinates of C are greater than the corresponding coordinates of B. Is triangle ABC acute, right, or obtuse?

To answer this question, first draw a sample triangle that fits the description.

Side AB must be a horizontal segment because the y-coordinates are the same. Point C must be located to the right and up from point B.

From the diagram you can see that triangle ABC must be obtuse.



Answer each question. Draw a simple triangle on the grid above to help you.

1. Consider three points, R, S, and T on a coordinate grid. The x-coordinates of R and S are the same. The y-coordinate of T is between the y-coordinates of R and S. The x-coordinate of T is less than the x-coordinate of R. Is angle R of triangle RST acute, right, or obtuse?

2. Consider three noncollinear points, J, K, and L on a coordinate grid. The y-coordinates of J and K are the same. The x-coordinates of K and L are the same. Is triangle JKL acute, right, or obtuse?

3. Consider three noncollinear points, D, E, and F on a coordinate grid. The x-coordinates of D and E are opposites. The y-coordinates of D and E are the same. The x-coordinate of F is 0. What kind of triangle must $\triangle DEF$ be: scalene, isosceles, or equilateral?

4. Consider three points, G, H, and I on a coordinate grid. Points G and H are on the positive y-axis, and the y-coordinate of G is twice the y-coordinate of H. Point I is on the positive x-axis , and the x-coordinate of I is greater than the y-coordinate of G. Is triangle GHI scalene, isosceles, or equilateral?

NAME ______________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 193–197

Finding Angle Measures in Triangles

You can use algebra to solve problems involving triangles.

Example: In triangle ABC, $m\angle A$, is twice $m\angle B$, and $m\angle C$ is 8 more than $m\angle B$. What is the measure of each angle?

Write and solve an equation. Let $x = m\angle B$.

$$\begin{aligned} m\angle A + m\angle B + m\angle C &= 180 \\ 2x + x + (x + 8) &= 180 \\ 4x + 8 &= 180 \\ 4x &= 172 \\ x &= 43 \end{aligned}$$

So, $m\angle A = 2(43)$ or 86, $m\angle B = 43$, and $m\angle C = 43 + 8$ or 51.

Solve each problem.

1. In triangle DEF, $m\angle E$ is three times $m\angle D$, and $m\angle F$ is 9 less than $m\angle E$. What is the measure of each angle?

2. In triangle RST, $m\angle T$ is 5 more than $m\angle R$, and $m\angle S$ is 10 less than $m\angle T$. What is the measure of each angle?

3. In triangle JKL, $m\angle K$ is four times $m\angle J$, and $m\angle L$ is five times $m\angle J$. What is the measure of each angle?

4. In triangle XYZ, $m\angle Z$ is 2 more than twice $m\angle X$, and $m\angle Y$ is 7 less than twice $m\angle X$. What is the measure of each angle?

5. In triangle GHI, $m\angle H$ is 20 more than $m\angle G$, and $m\angle G$ is 8 more than $m\angle I$. What is the measure of each angle?

6. In triangle MNO, $m\angle M$ is equal to $m\angle N$, and $m\angle O$ is 5 more than three times $m\angle N$. What is the measure of each angle?

7. In triangle STU, $m\angle U$ is half $m\angle T$, and $m\angle S$ is 30 more than $m\angle T$. What is the measure of each angle?

8. In triangle PQR, $m\angle P$ is equal to $m\angle Q$, and $m\angle R$ is 24 less than $m\angle P$. What is the measure of each angle?

9. Write your own problem about measures of triangles.

5-3

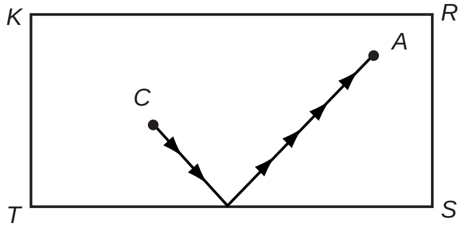
NAME ______________________________ DATE __________ PERIOD ______

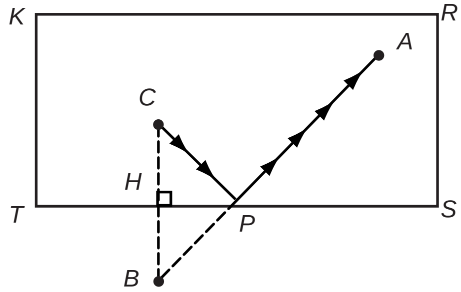
Enrichment

Student Edition
Pages 198–202

Billiards

Consider the billiard table depicted in the figure at the right. The object of the game of billiards is to use a cue stick to strike the cue ball at point C so that the ball will hit the sides (or cushions) of the table at least once before hitting another ball at point A. In playing the game, the question raised is how can point P on cushion $\overline{ST}$ be located such that cue ball C will strike the cushion and hit ball A. A skilled billiard player locates point P by picturing the following process.





Step 1 Find point B so that $\overline{BC} \perp \overline{ST}$ and $\overline{BH} \cong \overline{CH}$. B is called the reflected image of C in $\overline{ST}$.

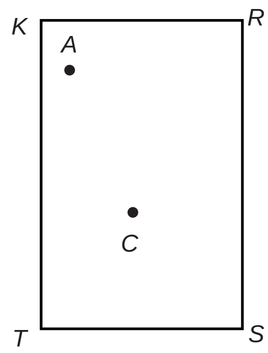
Step 2 Draw $\overline{AB}$.

Step 3 $\overline{AB}$ intersects $\overline{ST}$ at the desired point P.

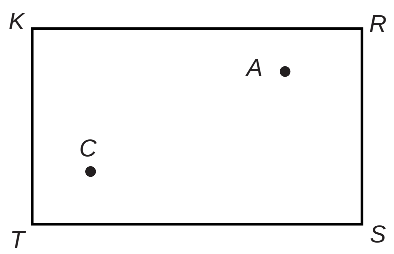
For each billiards problem, the cue ball at point C must strike the indicated cushion(s) and then strike the ball at point A. Draw and label the correct path for the cue ball using the process described above.

1. cushion $\overline{KR}$

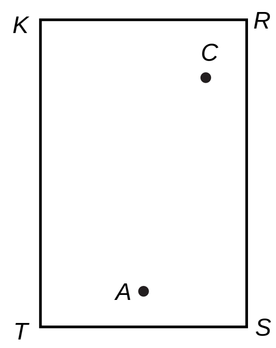
2. cushion $\overline{RS}$



3. cushion $\overline{TS}$, then cushion $\overline{RS}$



4. cushion $\overline{KT}$, then cushion $\overline{RS}$



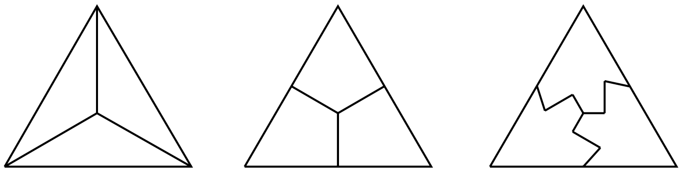
NAME ______________________ DATE __________ PERIOD ______

Enrichment

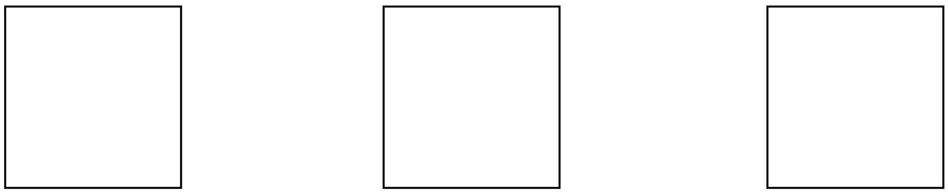
Student Edition
Pages 203–207

Congruent Parts of Regular Polygonal Regions

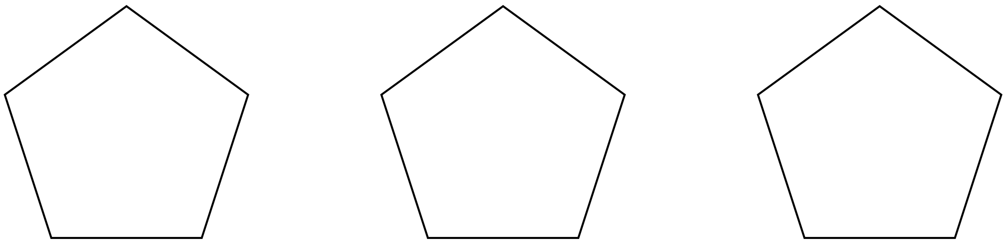
Congruent figures are figures that have exactly the same size and shape. There are many ways to divide regular polygonal regions into congruent parts. Three ways to divide an equilateral triangular region are shown. You can verify that the parts are congruent by tracing one part, then rotating, sliding, or reflecting that part on top of the other parts.



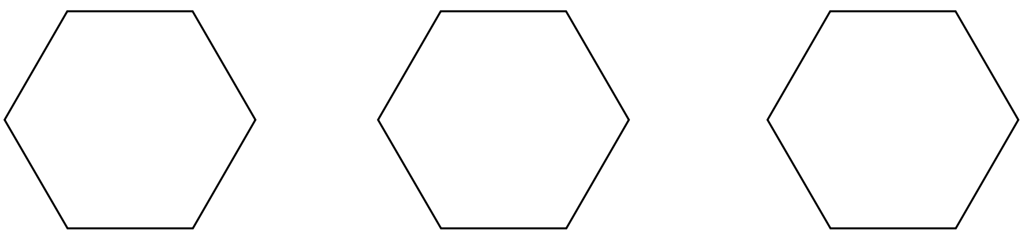
1. Divide each square into four congruent parts. Use three different ways.



2. Divide each pentagon into five congruent parts. Use three different ways.



3. Divide each hexagon into six congruent parts. Use three different ways.



4. What hints might you give another student who is trying to divide figures like those into congruent parts?

NAME ______________________________ DATE ____________ PERIOD ______

5-5 Enrichment

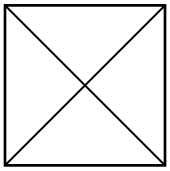
Student Edition
Pages 210–214

How Many Triangles?

Each puzzle below contains many triangles. Count them carefully. Some triangles overlap other triangles.

How many triangles in each figure?

1.



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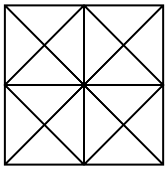

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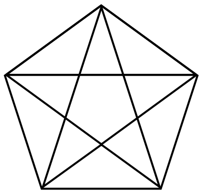

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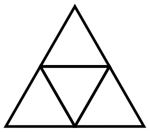

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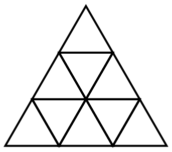

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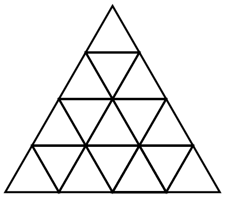
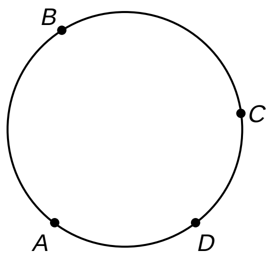


How many triangles can you form by joining points on each circle? List the vertices of each triangle.

7.



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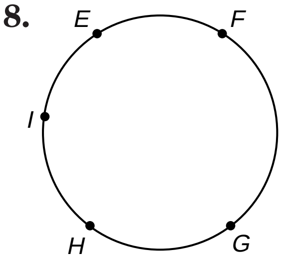
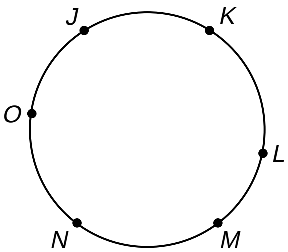
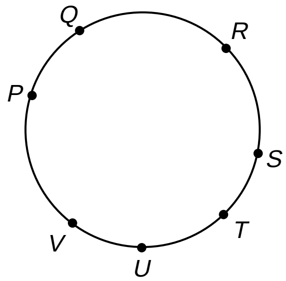


9.



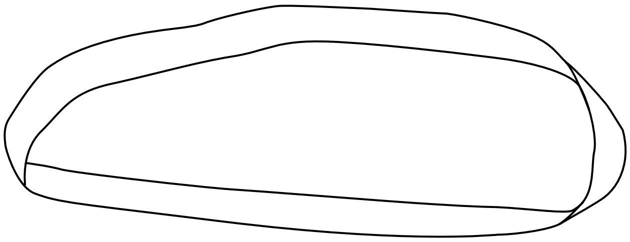
10.



Enrichment

The Möbius Strip

A Möbius strip is a special surface with only one side. It was discovered by August Ferdinand Möbius, a German astronomer and mathematician.



1. To make a Möbius strip, cut a strip of paper about 16 inches long and 1 inch wide. Mark the ends with the letters *A*, *B*, *C*, and *D* as shown below.

Twist the paper once, connecting *A* to *D* and *B* to *C*. Tape the ends together on both sides.

2. Use a crayon or pencil to shade one side of the paper. Shade around the strip until you get back to where you started. What happens?

3. What do you think will happen if you cut the Möbius strip down the middle? Try it.

4. Make another Möbius strip. Starting a third of the way in from one edge, cut around the strip, staying always the same distance in from the edge. What happens?

5. Start with another long strip of paper. Twist the paper twice and connect the ends. What happens when you cut down the center of this strip?

6. Start with another long strip of paper. Twist the paper three times and connect the ends. What happens when you cut down the center of this strip?

NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 228–233

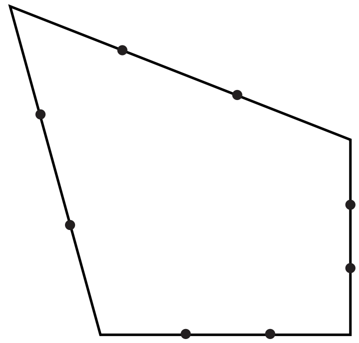
Finding the Centroid

The following method can be used to find the centroid or balance point of a four-sided figure.

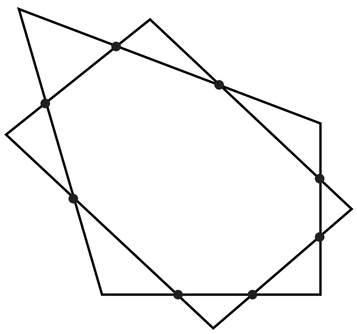
Step 1 Use a ruler to divide each side into three equal parts.

Step 2 Connect the points from Step 1 to make a parallelogram, a four-sided figure with two pairs of parallel sides.

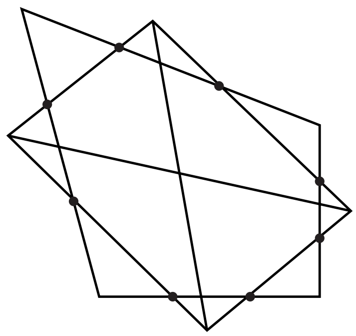
Step 3 Connect opposite vertices of the parallelogram, making the diagonals. The point where the diagonals intersect is the centroid.



Step 1



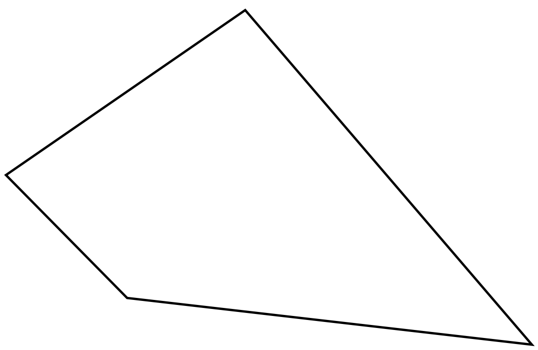
Step 2



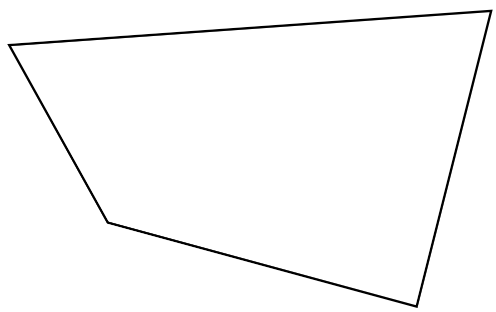
Step 3

Trace each figure on a piece of construction paper or cardboard. Use the method above to locate the centroid. Cut out the figure and test to see if it does balance on a pencil point.

1.



2.



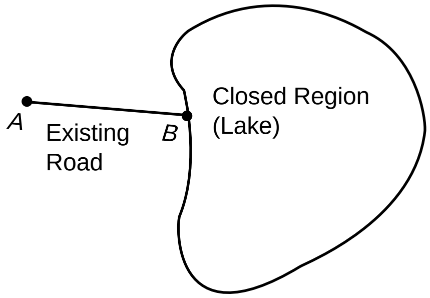
NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 234–239

Construction Problem

The diagram below shows segment AB adjacent to a closed region. The problem requires that you construct another segment XY to the right of the closed region such that points A, B, X, and Y are collinear. You are not allowed to touch or cross the closed region with your compass or straightedge.



Follow these instructions to construct a segment XY so that it is collinear with segment AB.

1. Construct the perpendicular bisector of $\overline{AB}$. Label the midpoint as point C, and the line as m.

2. Mark two points P and Q on line m that lie well above the closed region. Construct the perpendicular bisector n of $\overline{PQ}$. Label the intersection of lines m and n as point D.

3. Mark points R and S on line n that lie well to the right of the closed region. Construct the perpendicular bisector k of $\overline{RS}$. Label the intersection of lines n and k as point E.

4. Mark point X on line k so that X is below line n and so that $\overline{EX}$ is congruent to $\overline{DC}$.

5. Mark points T and V on line k and on opposite sides of X, so that $\overline{XT}$ and $\overline{XV}$ are congruent. Construct the perpendicular bisector ℓ of $\overline{TV}$. Call the point where the line ℓ hits the boundary of the closed region point Y. $\overline{XY}$ corresponds to the new road.

NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 240–243

Inscribed and Circumscribed Circles

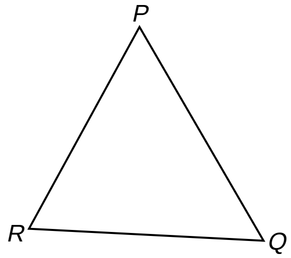
The three angle bisectors of a triangle intersect in a single point called the **incenter**. This point is the center of a circle that just touches the three sides of the triangle. Except for the three points where the circle touches the sides, the circle is inside the triangle. The circle is said to be inscribed in the triangle.

With a compass and a straightedge, construct the inscribed circle for △PQR by following the steps below.

1. Step 1 Construct the bisectors of $\angle P$ and $\angle Q$. Label the point where the bisectors meet A.

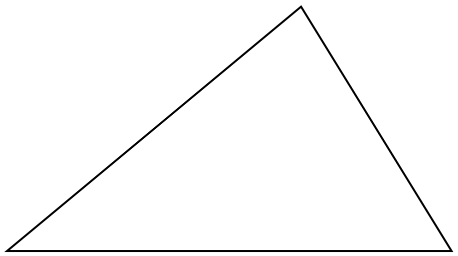
Step 2 Construct a perpendicular segment from A to $\overline{RQ}$. Use the letter B to label the point where the perpendicular segment intersects $\overline{RQ}$.

Step 3 Use a compass to draw the circle with center at A and radius $\overline{AB}$.

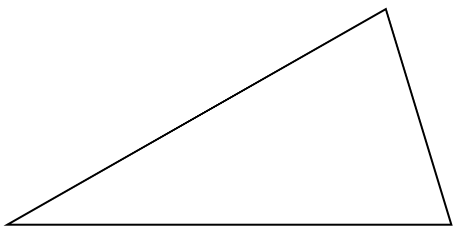


Construct the inscribed circle in each triangle.

2.



3.

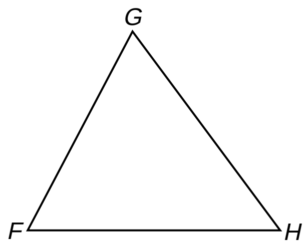


The three perpendicular bisectors of the sides of a triangle also meet in a single point. This point is the center of the circumscribed circle, which passes through each vertex of the triangle. Except for the three points where the circle touches the triangle, the circle is outside the triangle.

Follow the steps below to construct the circumscribed circle for △FGH.

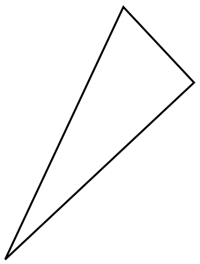
4. Step 1 Construct the perpendicular bisectors of $\overline{FG}$ and $\overline{FH}$. Use the letter A to label the point where the perpendicular bisectors meet.

Step 2 Draw the circle that has center A and radius $\overline{AF}$.



Construct the circumscribed circle for each triangle.

5.



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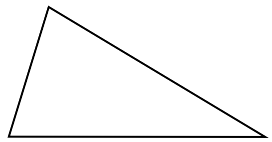
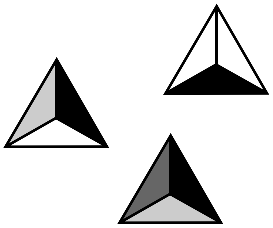


NAME ______________________ DATE __________ PERIOD _____

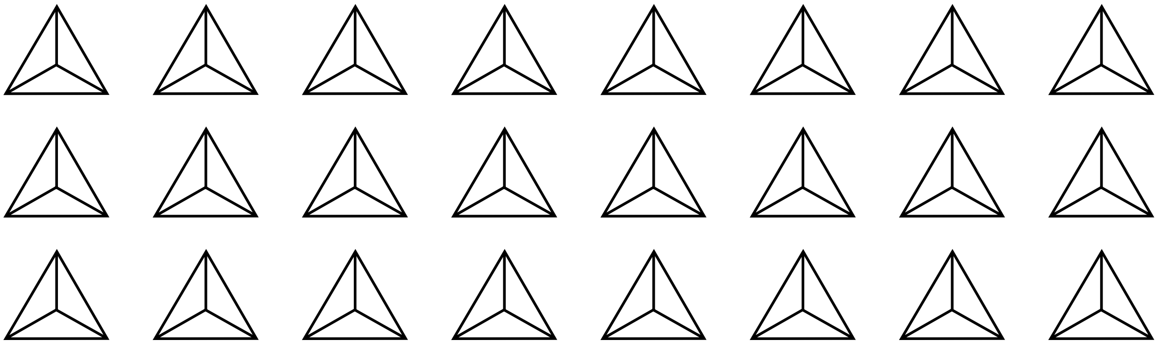
Enrichment

Three-Part Triangles

If an equilateral triangle is divided into three equal parts, the parts can be colored to create puzzle pieces. If each of the three parts is colored with one of four different colors, a set of 24 different triangles results. Reflections are considered different, but rotations are not.

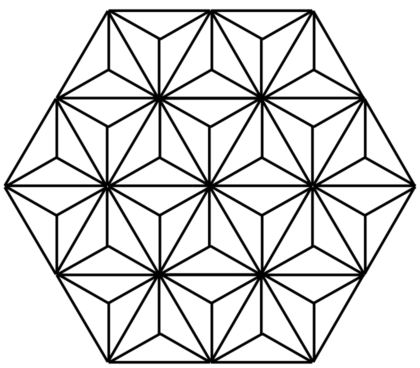


1. Three of the 24 possible triangles are shown above. Color these 24 triangles to show the complete set.

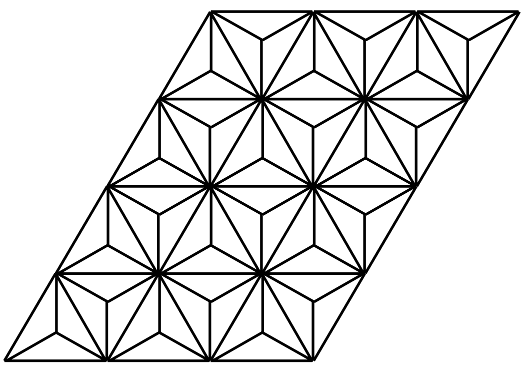


The 24 triangles can be used to make many shapes. Here are two for you to try. In both problems, the border of the shape must be all the same color.

2.



3.

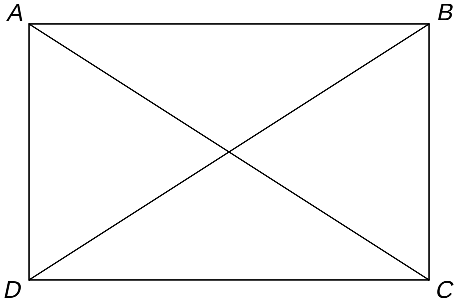


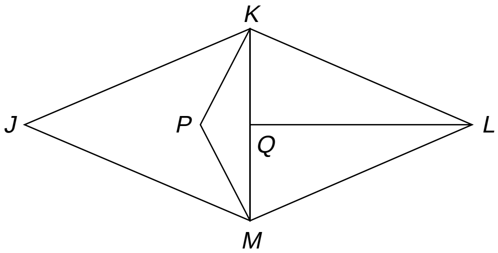
NAME ______________________________ DATE __________ PERIOD ______

Enrichment

Traceable Figures

Try to trace over each of the figures below without tracing the same segment twice.

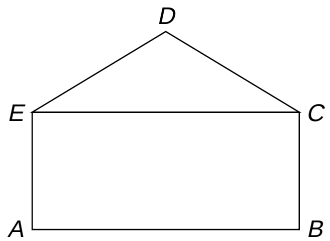




The figure at the left cannot be traced, but the one at the right can. The rule is that a figure is traceable if it has no points, or exactly two points where an odd number of segments meet. The figure at the left has three segments meeting at each of the four corners. However, the figure at the right has exactly two points, L and Q, where an odd number of segments meet.

Determine whether each figure can be traced. If it can, then name the starting point and number the sides in the order in which they should be traced.

1.



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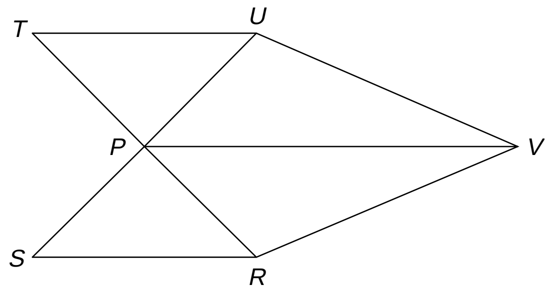

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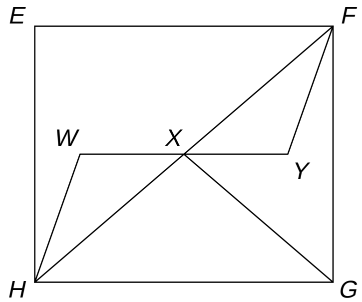

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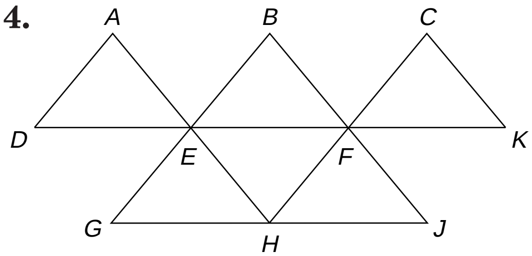
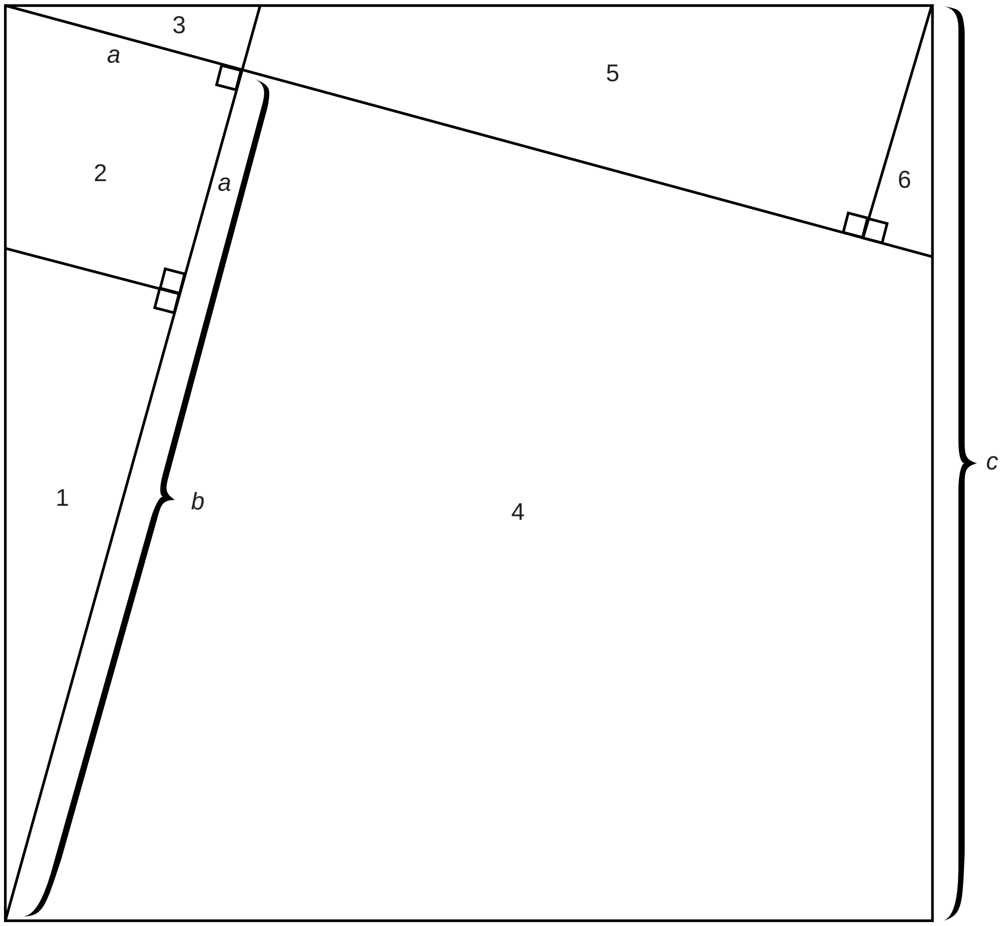


NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 256–261

Pythagorean Theorem



1. What is the area of the large square?

Cut out the figures in the square above. Then answer each of the following.

2. Form two separate squares from the six figures.

3. What is the area of each of the smaller squares?

4. What is the relationship between the areas of the three squares?

5. Does $a^2 + b^2 = c^2$?

NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Congruent Triangles in the Coordinate Plane

If you know the coordinates of the vertices of two triangles in the coordinate plane, you can often decide whether the two triangles are congruent. There may be more than one way to do this.

1. Consider $\triangle ABD$ and $\triangle CDB$ whose vertices have coordinates $A(0, 0)$, $B(2, 5)$, $C(9, 5)$, and $D(7, 0)$. Briefly describe how you can use what you know about congruent triangles and the coordinate plane to show that $\triangle ABD \cong \triangle CDB$. You may wish to make a sketch to help get you started.

2. Consider $\triangle PQR$ and $\triangle KLM$ whose vertices are the following points.

$P(1, 2)$	$Q(3, 6)$	$R(6, 5)$
$K(-2, 1)$	$L(-6, 3)$	$M(-5, 6)$

Briefly describe how you can show that $\triangle PQR \cong \triangle KLM$.

3. If you know the coordinates of all the vertices of two triangles, is it *always* possible to tell whether the triangles are congruent? Explain.

NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 276–281

Consecutive Integers and Inequalities

Consecutive integers follow one after another. For example, 4, 5, 6, and 7 are consecutive integers, as are $^-8$, $^-7$, and $^-6$. Each number to the right in the series is one greater than the one that comes before it. If x = the first consecutive integer, then $x + 1$ = the second consecutive integer, $x + 2$ = the third consecutive integer, $x + 3$ = the fourth consecutive integer, and so on.

Example: Find three consecutive positive integers whose sum is less than 12.

$$\underbrace{x}_{\text{first integer}} + \underbrace{x + 1}_{\text{second integer}} + \underbrace{x + 2}_{\text{third integer}} < 12$$

Simplify the expression by combining like terms.

$$3x + 3 < 12$$

$$3x + 3 - 3 < 12 - 3$$ *Subtract 3 from each side.*

$$3x < 9$$

$$\frac{3x}{3} < \frac{9}{3}$$ *Divide each side by 3.*

$$x < 3$$ So x could equal 1 or 2.

If $x = 1$, then $x + 1 = 2$, $x + 2 = 3$, and {1, 2, 3} is one solution.

If $x = 2$, then $x + 1 = 3$, $x + 2 = 4$, and {2, 3, 4} is another solution.

Each of the two solutions must be considered in the answer. The solution set is {1, 2, 3; 2, 3, 4}.

Solve. Show all possible solutions.

1. Find three consecutive positive integers whose sum is less than 15.

2. Find two consecutive positive even integers whose sum is less than 10.

3. Find three consecutive positive integers such that the second plus four times the first is less than 21.

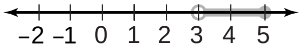
4. Find three consecutive positive even integers such that the third plus twice the second is less than 26.

NAME ________________________________ DATE __________ PERIOD ______

Enrichment

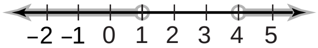
Compound Sentences

Example: $x > 3$ and $x \leq 5$



Think: To be a solution, values must be both greater than 3 <u>and</u> less than or equal to 5.

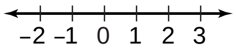
Example: $x > 4$ or $x < 1$



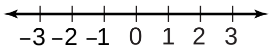
Think: To be a solution, values need only be greater than 4 <u>or</u> less than 1.

Graph the solution set of each compound sentence.

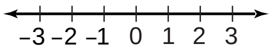
1. $r \geq 0$ or $r \leq 3$



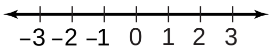
2. $d > 3$ or $d < {}^{-}3$



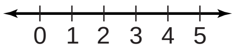
3. $m > 0$ and $m \leq 3$



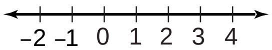
4. $x > 1$ and $x \leq {}^{-}1$



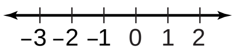
5. $b \geq 4$ and $b \leq 4$



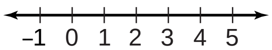
6. $q \geq 1$ or $q < {}^{-}2$



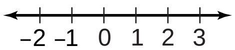
7. $0 < p$ and ${}^{-}3 < p$



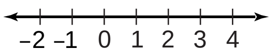
8. $2 \leq w$ or ${}^{-}1 \geq w$



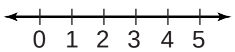
9. $x + 1 < 4$ and $x - ({}^{-}1) > {}^{-}1$



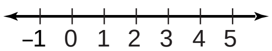
10. ${}^{-}3 \leq 2m + 1$ and $2m + 1 \leq 7$



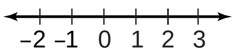
11. $r + 1 < 2r - 3$ and $2r - 3 < r + 2$



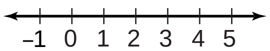
12. $2t + 1 > 7$ and $4t - 5 < 7$



13. ${}^{-}3q - 2 < 1$ and $7 - 4q > {}^{-}1$



14. $p + 2 \geq 4$ or $p + 5 \leq 7$



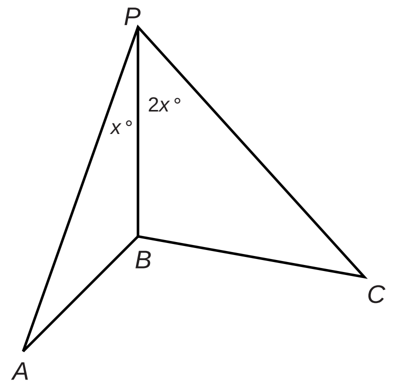
NAME ______________________________ DATE __________ PERIOD ______

Enrichment

Student Edition
Pages 290–295

Table of Triangles

In the table below, twelve values of x are given. For each value, find m∠APB and m∠BPC. Then use a ruler and protractor to draw a figure on the back of this page similar to the figure at the right. Use the angle measurements calculated and let AP = 5 cm, BP = 3 cm, and CP = 5 cm. Measure $\overline{AB}$ and $\overline{BC}$ to the nearest tenth of a centimeter. Fill in the table below.



	x	*m∠APB*	*m∠BPC*	*AB* (cm)	*BC* (cm)
1.	5				
2.	10				
3.	15				
4.	20				
5.	25				
6.	30				
7.	35				
8.	40				
9.	45				
10.	50				
11.	55				
12.	60				

NAME ______________________________ DATE __________ PERIOD _____

Enrichment

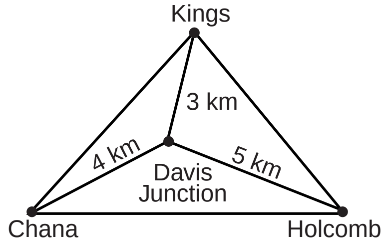
Student Edition
Pages 296–301

Drawing a Diagram

It is useful and often necessary to draw a diagram of the situation being described in a problem. The visualization of the problem is helpful in the process of problem solving.

Example: The roads connecting the towns of Kings, Chana, and Holcomb form a triangle. Davis Junction is located in the interior of this triangle. The distances from Davis Junction to Kings, Chana, and Holcomb are 3 km, 4 km, and 5 km, respectively. Jane begins at Holcomb and drives directly to Chana, then to Kings, and then back to Holcomb. At the end of her trip, she figures she has traveled 25 km altogether. Has she figured the distance correctly?

To solve this problem, a diagram can be drawn. Based on this diagram and the Triangle Inequality Theorem, the distance from Holcomb to Chana is less than 9 km. Similarly, the distance from Chana to Kings is less than 7 km, and the distance from Kings to Holcomb is less than 8 km.



Therefore, Jane must have traveled less than (9 + 7 + 8) km or 24 km versus her calculated distance of 25 km.

Explain why each of the following statements is true. Draw and label a diagram to be used in the explanation.

1. If an altitude is drawn to one side of a triangle, then the length of the altitude is less than one-half the sum of the lengths of the other two sides.

2. If point Q is in the interior of $\triangle ABC$ and on the angle bisector of $\angle B$, then Q is equidistant from $\overline{AB}$ and $\overline{CB}$. (*Hint:* Draw $\overline{QD}$ and $\overline{QE}$ such that $\overline{QD} \perp \overline{AB}$ and $\overline{QE} \perp \overline{CB}$.)

NAME ______________________________ DATE __________ PERIOD ______

Enrichment

Student Edition
Pages 310–315

Word Search

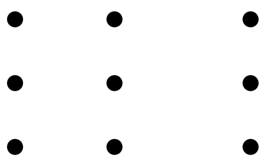
R	N	B	A	S	C	V	E	S	R	E	Q	P	O
G	E	O	M	E	T	R	Y	U	D	Z	A	H	B
A	T	E	J	L	S	A	I	P	T	R	U	G	T
T	R	I	A	N	G	L	E	P	A	M	E	H	U
B	A	D	V	M	X	Y	F	L	C	A	G	B	S
E	P	C	P	W	I	B	L	E	F	I	L	N	E
L	E	T	E	S	R	E	S	M	R	D	C	O	C
A	Z	U	R	U	L	Q	V	E	H	G	I	G	W
C	O	M	P	L	E	M	E	N	T	A	R	Y	H
I	I	L	E	A	F	O	W	T	J	N	C	L	G
T	D	X	N	K	R	F	E	A	M	G	L	O	E
R	N	T	D	M	A	X	N	R	O	L	E	P	L
E	E	N	I	L	Y	Z	L	Y	N	E	R	Q	G
V	A	U	C	S	B	P	O	L	L	K	P	I	N
K	S	Q	U	A	R	E	J	H	E	M	L	J	A
T	A	P	L	A	N	E	N	T	O	L	E	Y	T
N	S	V	A	T	C	W	U	X	Q	R	E	S	C
I	K	O	R	P	U	C	T	P	V	M	R	T	E
O	E	Z	Y	Y	A	X	S	U	B	M	O	H	R
P	W	Z	A	D	J	A	C	E	N	T	A	F	S
S	E	G	M	E	N	T	X	Q	W	D	G	Z	I

1. geometry
2. line
3. segment
4. plane
5. point
6. ray
7. angle
8. acute
9. right
10. obtuse
11. adjacent
12. complementary
13. supplementary
14. vertical
15. parallel
16. skew
17. perpendicular
18. triangle
19. polygon
20. rectangle
21. rhombus
22. square
23. trapezoid
24. circle

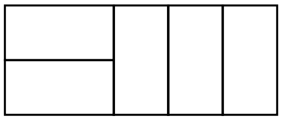
Enrichment

Polygon Puzzles

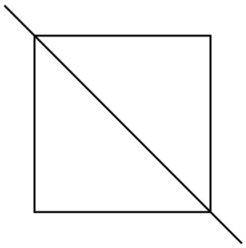
1. How many nonrectangular parallelograms can you draw that have vertices on the dots below? (Hint: Label the dots and make a list.)



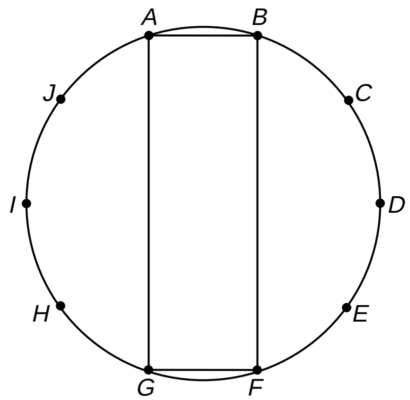
2. How many ways can the large rectangle be divided into five congruent rectangles? One way is shown.



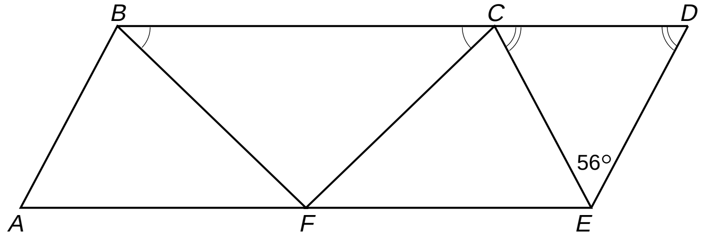
3. What polygons can be formed by drawing a single straight line across a square? For example, the line shown forms two isosceles right triangles.



4. How many rectangles can be formed by joining points on the circle? The points are equally spaced around the circle. For example, one rectangle is *ABFG*.



5. Suppose *ABDE* is a parallelogram and that in the figure, $m\angle BFC$ is 4 more than twice $m\angle EFC$. Find the measure of each angle of each triangle in the figure.

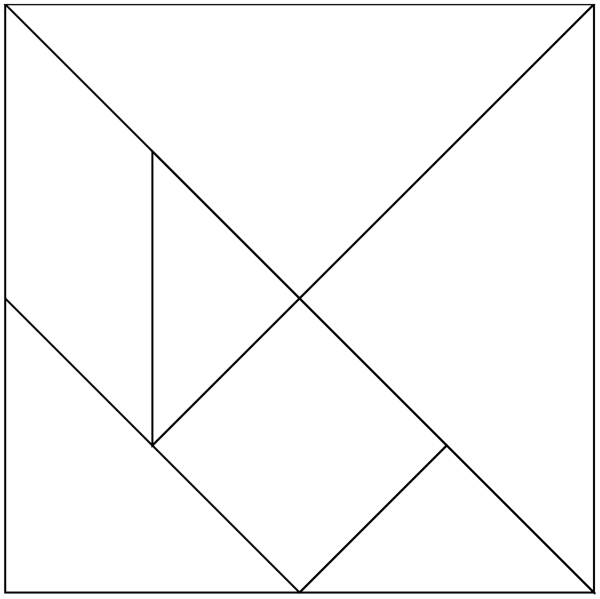


NAME ______________________ DATE ________ PERIOD _____

Enrichment

Tangrams

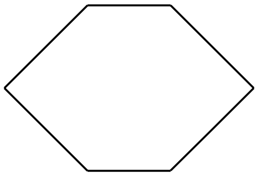
The tangram puzzle is composed of seven pieces that form a square, as shown at the right. This puzzle has been a popular amusement for Chinese students for hundreds and perhaps thousands of years.



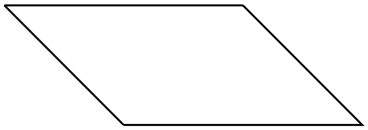
Make a careful tracing of the figure above. Cut out the pieces and rearrange them to form each figure below. Record each answer by drawing lines within each figure.

1.

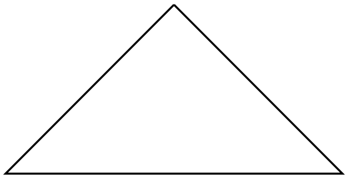
2.



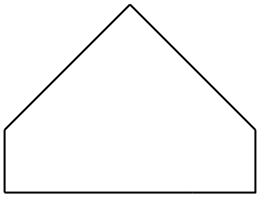
3.



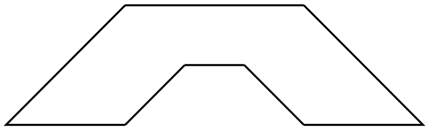
4.



5.



6.



7. Create a different figure using the seven tangram pieces. Trace the outline. Then challenge another student to solve the puzzle.

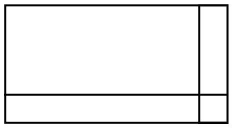
NAME ______________________________ DATE __________ PERIOD ______

Enrichment

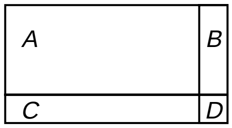
Counting Squares and Rectangles

Each puzzle below contains many squares and/or rectangles. Count them carefully. You may want to label each region so you can list all possibilities.

Example: How many rectangles are in the figure at the right?



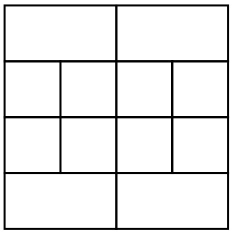
Label each small region with a letter. Then list each rectangle by writing the letters of regions it contains.
A, B, C, D, AB, CD, AC, BD, ABCD



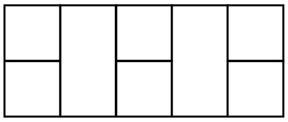
There are 9 rectangles.

How many squares are in each figure?

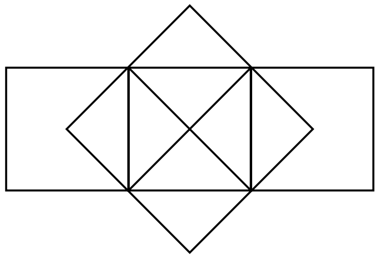
1.



2.

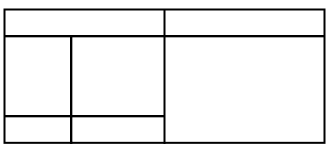


3.

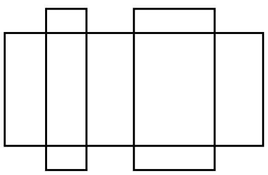


How many rectangles are in each figure?

4.



5.



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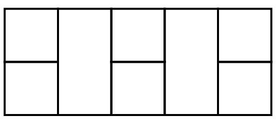
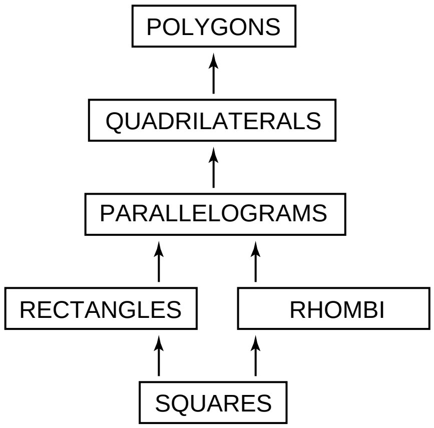


NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Reading Mathematics

A **hierarchy** is a ranking of classes or sets of things. Some classes of polygons are rectangles, rhombi, parallelograms, squares, and quadrilaterals. These classes are arranged in the hierarchy at the right.



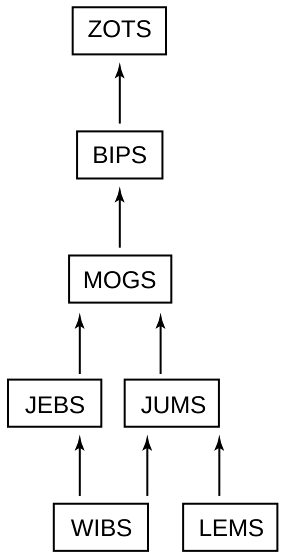
Use the following information to help read a hierarchy diagram.

(1) The class that is the broadest is listed at first, followed by other classes in order. For example, *polygons* is the broadest class in the hierarchy diagram at the right, and *squares* is the most specific.

(2) Each class is contained within any class above it in the hierarchy. For example, *all* squares are also rhombi, rectangles, parallelograms, quadrilaterals, and polygons.

(3) Some elements of each class are contained within lower classes in the hierarchy. For example, *some* parallelograms are rhombi, and *some* rectangles are squares.

Write* true, false, *or* not enough information *for each statement. Refer to the hierarchy diagram at the right. (All terms are fictitious.)



1. All bips are zots.

2. All mogs are jums.

3. Some jebs are jums.

4. All jums are lems.

5. All mogs are bips.

6. Some jums are wibs.

7. Some wibs are lems.

8. Draw a hierarchy diagram to show these classes: equilateral triangles, polygons, isosceles triangles, triangles.

NAME ________________________________ DATE __________ PERIOD ______

Enrichment

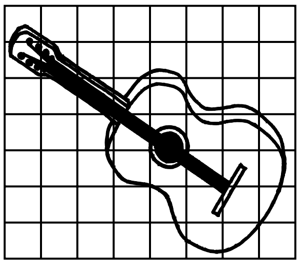
Using a Grid to Enlarge a Drawing

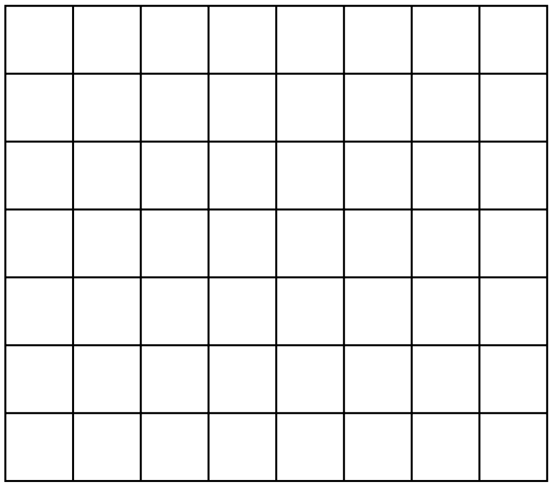
Here is method of enlarging a drawing or picture.

1. Lay a grid pattern of small squares over the picture.
2. On separate paper, draw a larger grid with the same number of squares. (If you want to double the dimensions of the picture, the sides of the squares of the larger grid should be twice as long as the sides of the original.)
3. Draw the contents of each square of the original picture on the corresponding square of the larger grid.

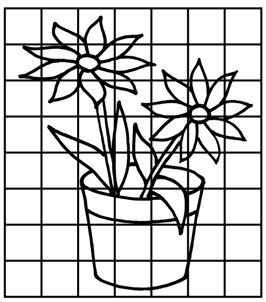
Enlarge each drawing.

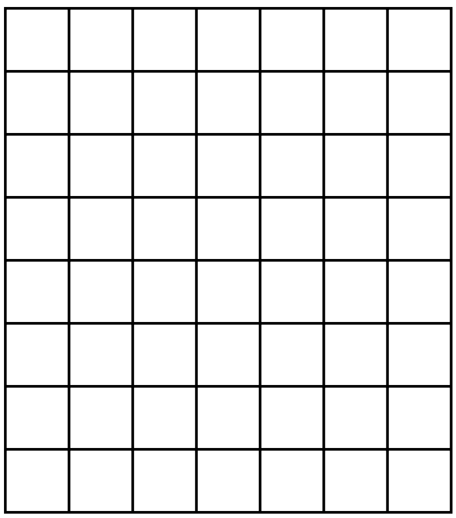
1.





2.



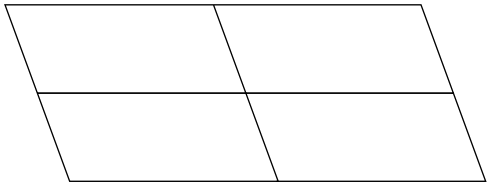


3. Draw a picture of your own on a small grid. Then enlarge the picture by reproducing each square on a larger grid.

NAME ________________________ DATE __________ PERIOD ______

Enrichment

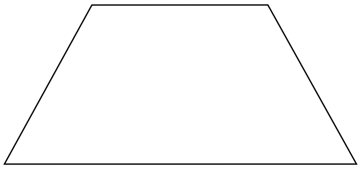
Rep-Tiles



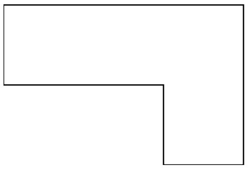
A rep-tile is a figure that can be subdivided into smaller copies of itself. The large figure is similar to the small ones and the small figures are all congruent.

Show that each figure is a rep-tile by subdividing it into four smaller and similar figures.

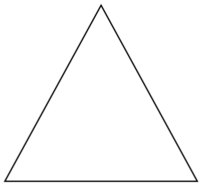
1.



2.



3.



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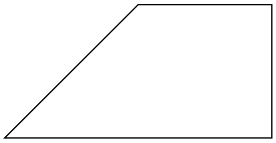

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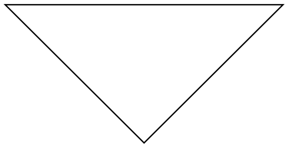

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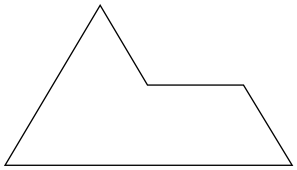
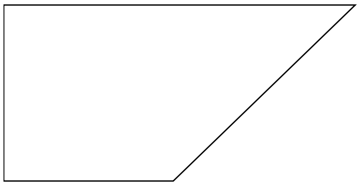


Subdivide each rep-tile into nine smaller and similar figures.

7.



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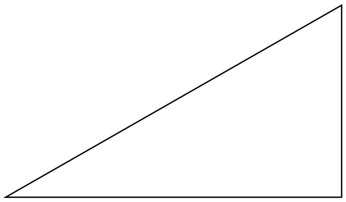

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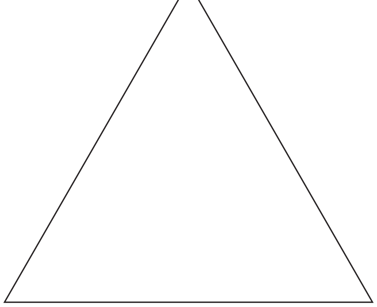

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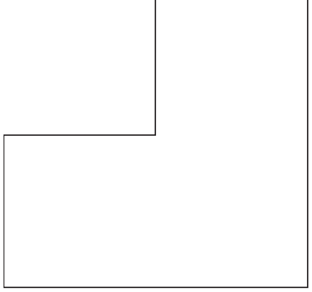


NAME ______________________________ DATE ____________ PERIOD _____

Enrichment

Constructing Similar Polygons

Here are four steps for constructing a polygon that is similar to and with sides twice as long as those of an existing polygon.

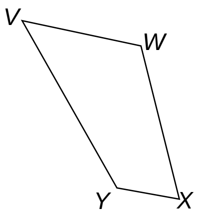
Step 1 Choose any point either inside or outside the polygon and label it O.

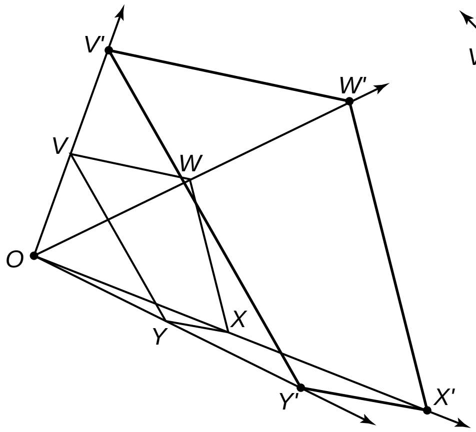
Step 2 Draw rays from O through each vertex of the polygon.

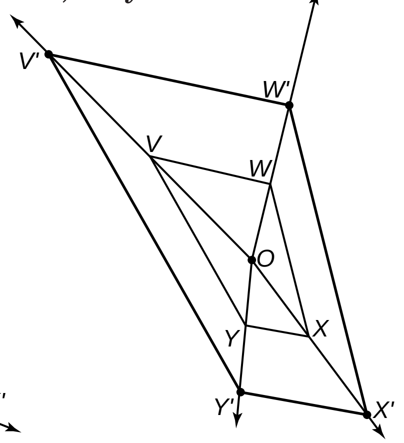
Step 3 For vertex V, set the compass to length OV. Then locate a new point V' on ray OV such that $VV' = OV$. Thus, $OV' = 2(OV)$.

Step 4 Repeat Step 3 for each vertex. Connect points V', W', X' and Y' to form the new polygon.

Two constructions of polygons similar to and with sides twice those of $VWXY$ are shown below. Notice that the placement of point O does not affect the size or shape of $V'W'X'Y'$, only its location.

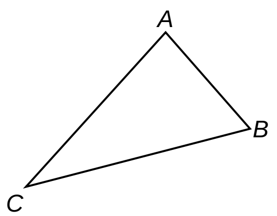






Trace each polygon. Then construct a similar polygon with sides twice as long as those of the given polygon.

1.



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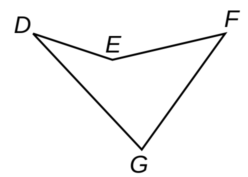
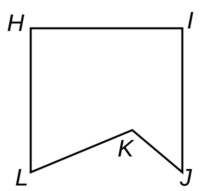
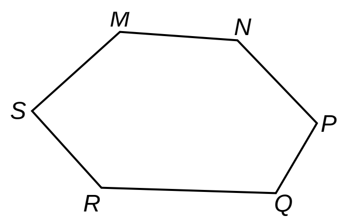


3. Explain how to construct a similar polygon with sides three times the length of those of polygon $HIJKL$. Then do the construction.



4. Explain how to construct a similar polygon with sides $1\frac{1}{2}$ times the length of those of polygon $MNPQRS$. Then do the construction.



NAME ______________________ DATE __________ PERIOD _____

Enrichment

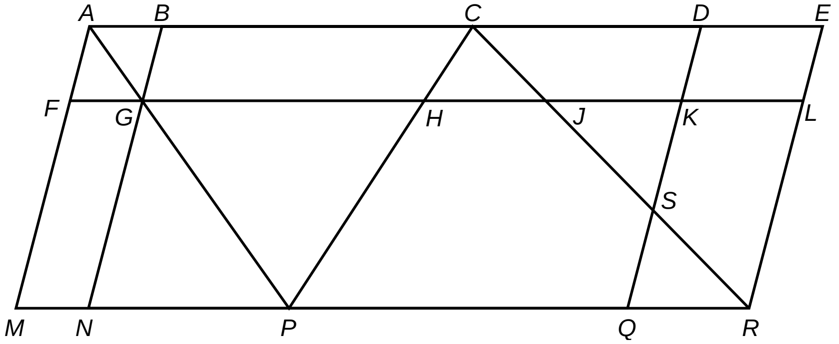
Student Edition
Pages 368–373

Proportions for Similar Triangles

Recall that if a line crosses two sides of a triangle and is parallel to the third side, then the line separates the two sides that it crosses into segments of proportional lengths.

You can write many proportions by identifying similar triangles in the following diagram. In the diagram,

$\overleftrightarrow{AM} \parallel \overleftrightarrow{BN}$, $\overleftrightarrow{AE} \parallel \overleftrightarrow{FL} \parallel \overleftrightarrow{MR}$, and $\overleftrightarrow{DQ} \parallel \overleftrightarrow{ER}$.



Answer each question. Use the diagram above.

1. Name a triangle similar to $\triangle GNP$.

2. Name a triangle similar to $\triangle CJH$.

3. Name two triangles similar to $\triangle JKS$.

4. Name a triangle similar to $\triangle ACP$.

Complete each proportion.

5. $\frac{AG}{AP} = \frac{AF}{?}$

6. $\frac{CP}{CH} = \frac{CR}{?}$

7. $\frac{JS}{JR} = \frac{?}{JL}$

8. $\frac{PH}{PC} = \frac{PG}{?}$

9. $\frac{ER}{LR} = \frac{?}{JR}$

10. $\frac{MN}{MP} = \frac{?}{AP}$

Solve.

11. If $CJ = 16$, $JR = 48$, and $LR = 30$, find EL.

12. If $DK = 5$, $KS = 7$, and $CJ = 8$, find JS.

13. If $MN = 12$, $NP = 32$, and $AP = 48$, find AG. Round to the nearest tenth.

14. If $CH = 18$, $HP = 82$, and $CR = 130$, find CJ.

15. Write three more problems that can be solved using the diagram above.

NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 374–379

Proportions Involving Quadratic Equations

To solve proportions such as $\frac{x}{5} = \frac{x+1}{3}$ and $\frac{y+5}{y+9} = \frac{4}{6}$, we use the Property of Proportions to generate an equivalent equation that can be solved more easily. In some cases, this equation is a quadratic equation.

Example: Solve the proportion $\frac{x+5}{x-7} = \frac{x+2}{5}$.

$$\frac{x+5}{x-7} = \frac{x+2}{5}$$

$(x + 5)5 = (x - 7)(x + 2)$ *Cross Products*

$5x + 25 = x^2 - 5x - 14$ *Multiplication*

$0 = x^2 - 10x - 39$ *Subtraction*

$0 = (x - 13)(x + 3)$ *Factoring quadratic equation*

$x - 13 = 0$ or $x + 3 = 0$

$x = 13 \qquad x = -3$

The solution set is (13, -3).

Solve each of the following proportions.

1. $\frac{8}{x+5} = \frac{x-3}{6}$

2. $\frac{2}{x+3} = \frac{x+20}{2x-5}$

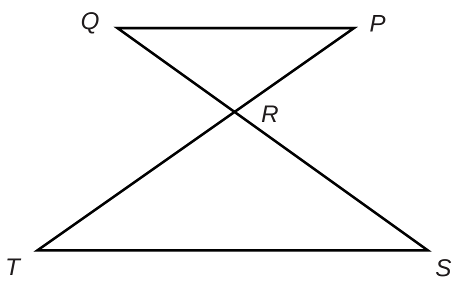
3. $\frac{x+9}{x+2} = \frac{-12}{x-8}$

4. $\frac{2x-3}{x-4} = \frac{x-12}{x-6}$

In the figure, $\overline{QP} \parallel \overline{ST}$. Use the given information to find the value(s) of x.

5. $QR = x$
$QP = 3$
$RS = 5$
$TS = 8 - x$

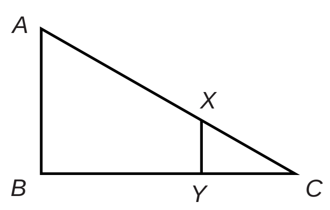
6. $PR = x + 5$
$TR = x + 8$
$QR = 2$
$SR = x + 6$



Determine the value(s) for x that would make $\overline{AB} \parallel \overline{XY}$ in the figure below.

7. $AX = 9$, $XC = x - 8$,
$BY = x$, $YC = 1$

8. $XC = x - 3$, $AX = x + 4$,
$XY = x - 6$, $AB = x + 18$



NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 382–387

Golden Rectangles

Use a straightedge, compass, and the instructions below to construct a golden rectangle.

1. Construct square *ABCD* with sides of 2 cm.

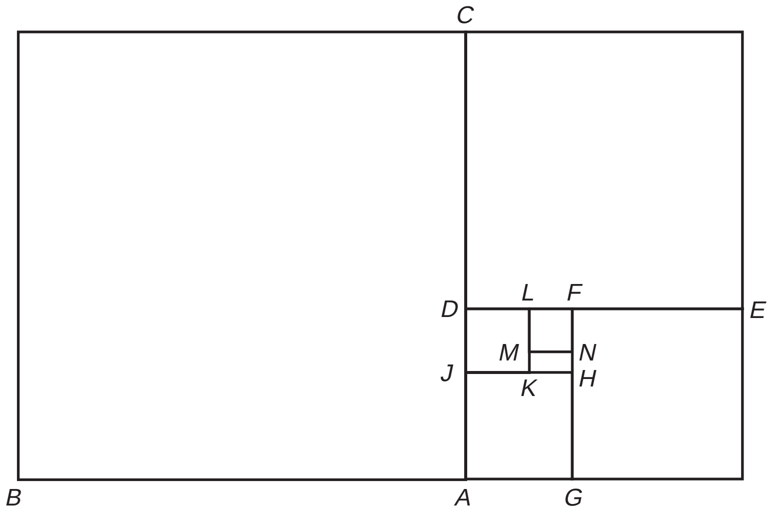
2. Construct the midpoint of $\overline{AB}$. Call the midpoint *M*.

3. Draw $\overleftrightarrow{AB}$. Set your compass at an opening equal to MC, Use *M* as the center to draw an arc that intersects $\overleftrightarrow{AB}$. Call the point of intersection *P*.

4. Construct a line through *P* that is perpendicular to $\overleftrightarrow{AB}$.

5. Draw $\overleftrightarrow{DC}$ so that it intersects the perpendicular line in step 4. Call the intersection point *Q*. *APQD* is a **golden rectangle** because the ratio of its length to its width is 1.618. Check this conclusion by finding the value of $\frac{QP}{AP}$. Rectangles whose sides have this ratio are, it is said, the most pleasing to the human eye.

A figure consisting of similar golden rectangles is shown below. Use a compass and the instructions below to draw quarter-circle arcs that form a spiral like that found in the shell of a chambered nautilus.



6. Using *A* as a center, draw an arc that passes through *B* and *C*.

7. Using *D* as a center, draw an arc that passes through *C* and *E*.

8. Using *F* as a center, draw an arc that passes through *E* and *G*.

9. Using *H* as a center, draw an arc that passes through *G* and *J*.

10. Using *K* as a center, draw an arc that passes through *J* and *L*.

11. Using *M* as a center, draw an arc that passes through *L* and *N*.

NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 388–393

Ratio Puzzles with Triangles

If you know the perimeter of a triangle and the ratios of the sides, you can find the lengths of the sides.

Example: The perimeter of a triangle is 84 units. The sides have lengths r, s, and t. The ratio of s to r is 5:3, and the ratio of t to r is 2:1. Find the length of each side.

Since both ratios contain r, rewrite one or both ratios to make r the same. You can write the ratio of t to r as 6:3. Now you can write a three-part ratio.

$$r:s:t = 3:5:6$$

There is a number x such that $r = 3x$, $s = 5x$, and $t = 6x$. Since you know the perimeter, 84, you can use algebra to find the lengths of the sides

$$r + s + t = 84$$
$$3x + 5x + 6x = 84$$
$$14x = 84$$
$$x = 6$$
$$13x = 18,\ 5x = 30,\ 6x = 36$$

So $r = 18$, $s = 30$, and $t = 36$.

Find the lengths of the sides of each triangle.

1. The perimeter of a triangle is 75 units. The sides have lengths a, b, and c. The ratio of b to a is 3:5, and the ratio of c to a is 7:5. Find the length of each side.

2. The perimeter of a triangle is 88 units. The sides have lengths d, e, and f. The ratio of e to d is 3:1, and the ratio of f to e is 10:9. Find the length of each side.

3. The perimeter of a triangle is 91 units. The sides have lengths p, q, and r. The ratio of p to r is 3:1, and the ratio of q to r is 5:2. Find the length of each side.

4. The perimeter of a triangle is 68 units. The sides have lengths g, h, and j. The ratio of j to g is 2:1, and the ratio of h to g is 5:4. Find the length of each side.

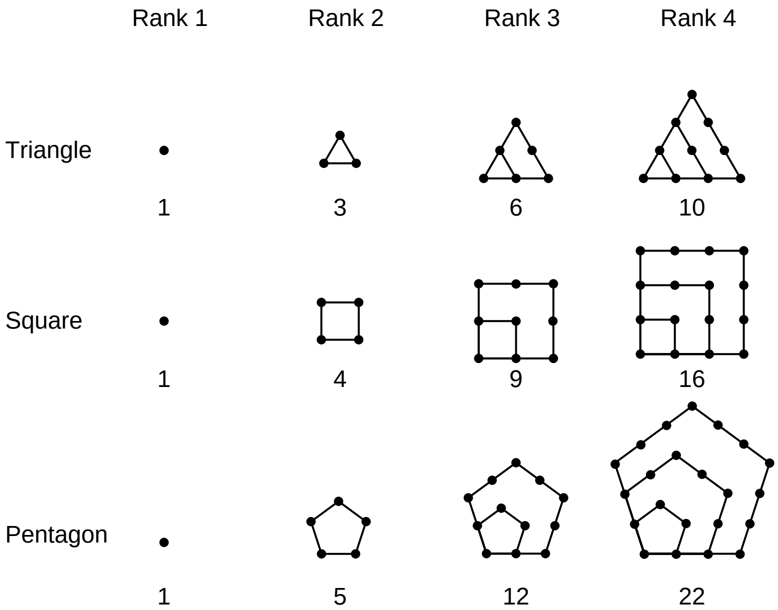
5. Write a problem similar to those above involving ratios in triangles.

NAME ______________________________ DATE __________ PERIOD ______

Enrichment

Polygonal Numbers

Certain numbers related to regular polygons are called **polygonal numbers**. The chart shows several triangular, square, and pentagonal numbers. The **rank** of a polygon number is the number of dots on each "side" of the outer polygon. For example, the pentagonal number 22 has a rank of 4.



Polygonal numbers can be described with formulas. For example, a triangular number T of rank r can be described by $T = \frac{r(r + 1)}{2}$.

Answer each question.

1. Draw a diagram to find the triangular number of rank 5.

2. Draw a diagram to find the pentagonal number of rank 5.

3. Write a formula for a square number S of rank r.

4. Write a formula for a pentagonal number P of rank r.

5. What is the rank of the pentagonal number 70?

6. List the hexagonal numbers for ranks 1 to 5. (Hint: Draw a diagram.)

10-2

NAME ______________________ DATE __________ PERIOD _____

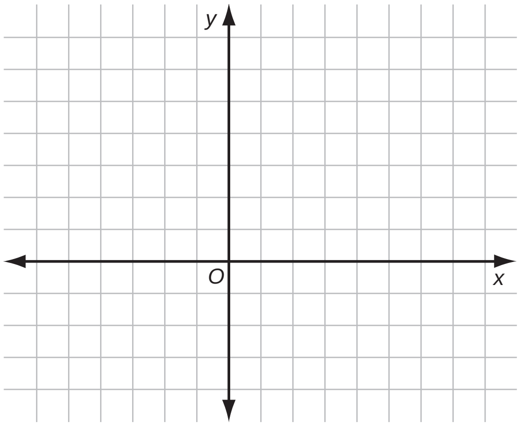
Enrichment

Student Edition
Pages 408–412

Slopes and Polygons

In coordinate geometry, the slopes of two lines determine if the lines are parallel or perpendicular. This knowledge can be useful when working with polygons.

1. The coordinates of the vertices of a triangle are $A(-6, 4)$, $B(8, 6)$, and $C(4, -4)$. Graph $\triangle ABC$.



2. J, K, and L are midpoints of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, respectively. Find the coordinates of J, K, and L. Draw $\triangle JKL$.

3. Which segments appear to be parallel?

4. Show that the segments named in Exercise 3 are parallel by finding the slopes of all six segments.

The coordinates of the vertices of right △PQR are given. Find the slope of each side of the triangle. Then name the hypotenuse.

5. $P(5, 1)$ $Q(1, -1)$ $R(-2, 5)$

slope of $\overline{PQ}$ = ______

slope of $\overline{QR}$ = ______

slope of $\overline{PR}$ = ______

hypotenuse: ______

6. $P(-2, -3)$ $Q(5, 1)$ $R(2, 3)$

slope of $\overline{PQ}$ = ______

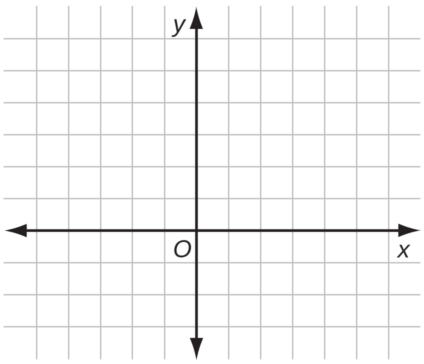
slope of $\overline{QR}$ = ______

slope of $\overline{PR}$ = ______

hypotenuse: ______

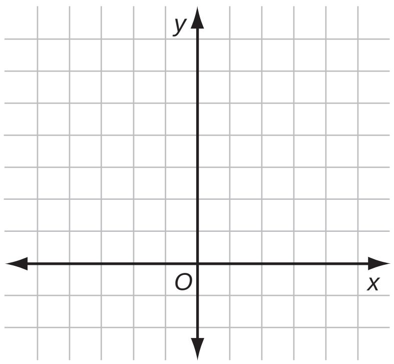
The coordinates of quadrilateral PQRS are given. Graph quadrilateral PQRS and find the slopes of the diagonals. State whether the diagonals are perpendicular.

7.



$P(-2, 6)$
$Q(4, 0)$
$R(1, -4)$
$S(-5, 2)$

8.



$P(0, 6)$
$Q(3, 0)$
$R(-4, -2)$
$S(-5, 4)$

10-3

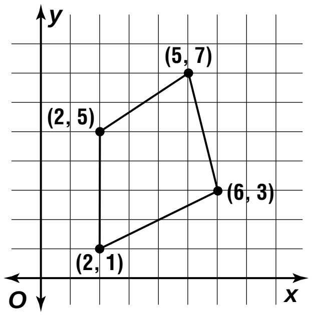
NAME ______________________ DATE __________ PERIOD ______

Enrichment

Student Edition
Pages 413–418

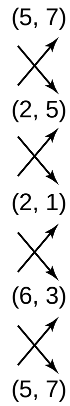
Aerial Surveyors and Area

Many land regions have irregular shapes. Aerial surveyors often use coordinates when finding areas of such regions. The coordinate method described in the steps below can be used to find the area of *any* polygonal region. Study how this method is used to find the area of the region at the right.



Step 1 List the ordered pairs for the vertices in counter-clockwise order, repeating the first ordered pair at the bottom of the list.

Step 2 Find D, the sum of the downward diagonal products (from left to right).
$D = (5 \cdot 5) + (2 \cdot 1) + (2 \cdot 3) + (6 \cdot 7)$
$= 25 + 2 + 6 + 42$ or 75



Step 3 Find U, the sum of the upward diagonal products (from left to right).
$U = (2 \cdot 7) + (2 \cdot 5) + (6 \cdot 1) + (5 \cdot 3)$
$= 14 + 10 + 6 + 15$ or 45

Step 4 Use the formula $A = \frac{1}{2}(D - U)$ to find the area.

$$A = \frac{1}{2}(D - U)$$

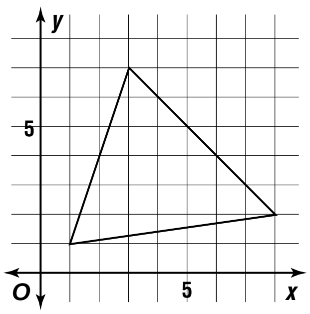
$$= \frac{1}{2}(75 - 45)$$

$$= \frac{1}{2}(30) \text{ or } 15$$

The area is 15 square units. Count the number of square units enclosed by the polygon. Does this result seem reasonable?

Use the coordinate method to find the area of each region in square units.

1.



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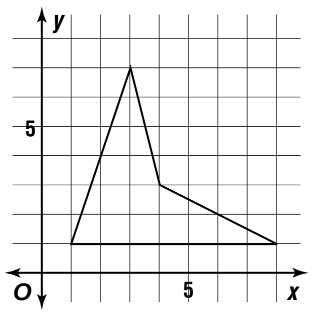

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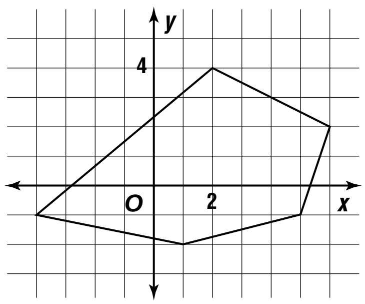


Enrichment

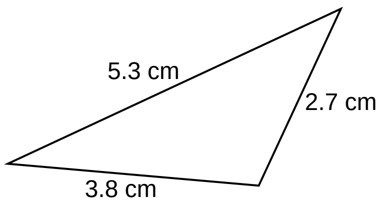
Student Edition
Pages 419–424

Heron's Formula

If you know the lengths of the sides of a triangle, you can use Heron's formula to find the area.

Heron's Formula:
The area, A, of a triangle with sides measuring a, b, and c is $A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{a+b+c}{2}$.

Example: Use Heron's formula to find the area of the triangle.



$$s = \frac{2.7 + 3.8 + 5.3}{2} = 5.9$$

$$A = \sqrt{5.9(5.9 - 2.7)(5.9 - 3.8)(5.9 - 5.3)}$$

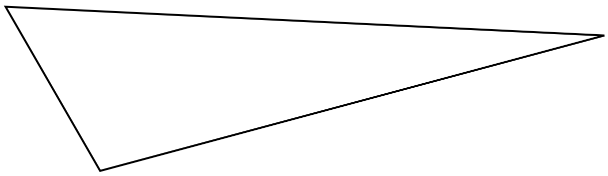
$$= \sqrt{5.9(3.2)(2.1)(0.6)}$$

$$= \sqrt{23.8} \approx 4.9$$

The area is about 4.9 cm^2.

Measure the sides of each triangle in centimeters. Then use Heron's formula to find the area. Round your answers to the nearest tenth.

1.



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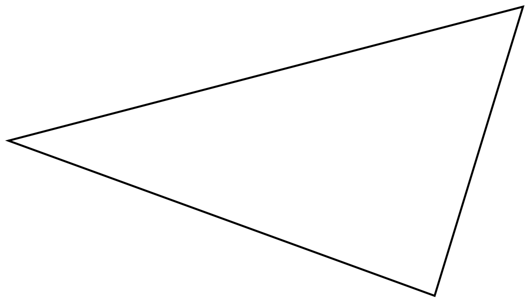

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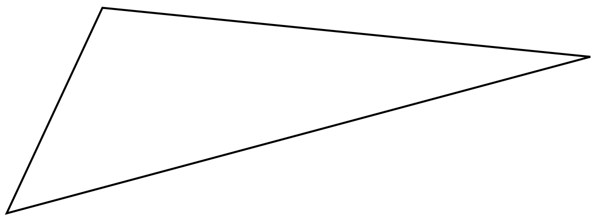


4. Construct an altitude for the triangle in problem 2. Then measure the altitude in centimeters and compute the area using the formula $A = \frac{1}{2}bh$. Did you get the same result as with Heron's formula?

NAME ______________________________ DATE __________ PERIOD _____

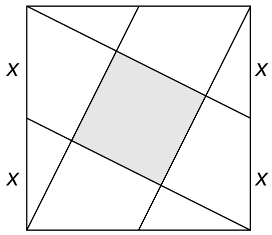
Enrichment

Polygon Probability

Each problem on this page involves one or more regular polygons. To find the probability of a point chosen at random being in the shaded region, you need to find the ratio of the shaded area to the total area. If you wish, you may substitute numbers for the variables.

Find the probability that a point chosen at random in each figure is in the shaded region. Assume polygons that appear to be regular are regular. Round your answer to the nearest hundredth.

1.



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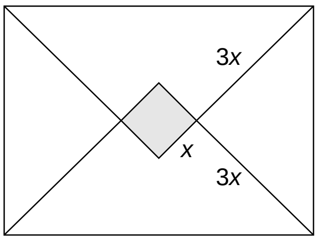

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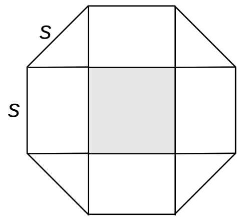

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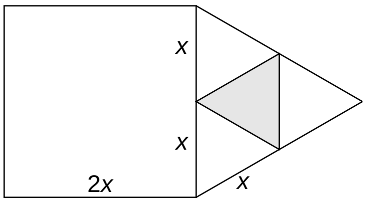

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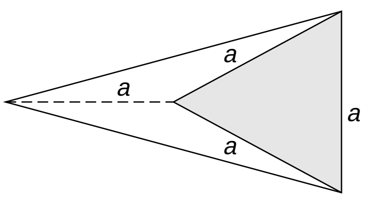

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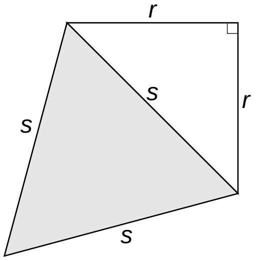


10-6

NAME ______________________________ DATE __________ PERIOD ______

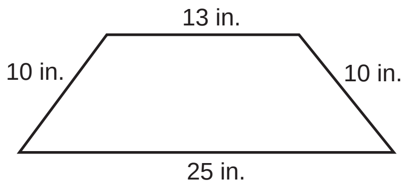
Enrichment

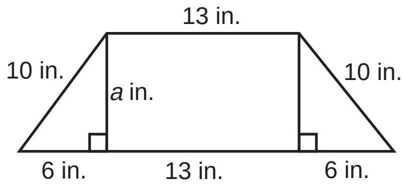
Student Edition
Pages 434–439

Trapezoids

Right triangles can often be useful when working with trapezoids.

Suppose you want to find the height of the isosceles trapezoid shown at the right. The first step is to draw perpendicular segments from the endpoints of one base to the other base. The perpendicular segments divide the trapezoid into two right triangles and a rectangle. Since the top of the rectangle has length 13 in., so does the bottom. Consequently, you can determine that the bases of the right triangles are each 6 in.



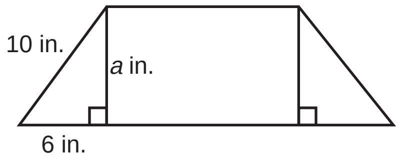


The Pythagorean Theorem can now be used to find a.

$$a^2 + 6^2 = 10^2$$

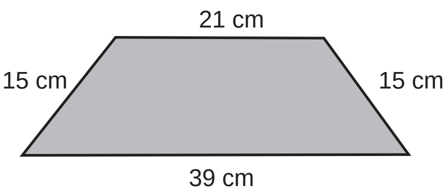
$$a^2 = 100 - 36 \text{ or } 64$$

$$a = \sqrt{64} \text{ or } 8$$

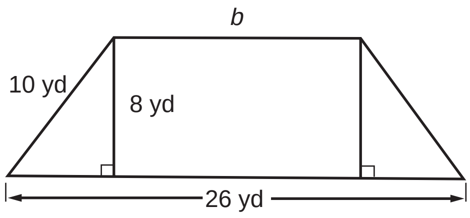


Therefore, the height of the isosceles trapezoid is 8 in.

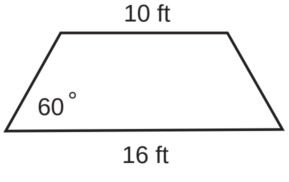
1. Find the altitude of the isosceles trapezoid. Then find the area.



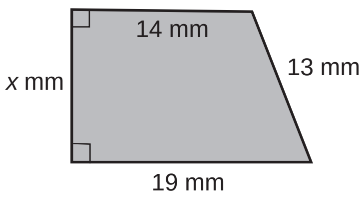
2. Find the length of the upper base of the isosceles trapezoid.



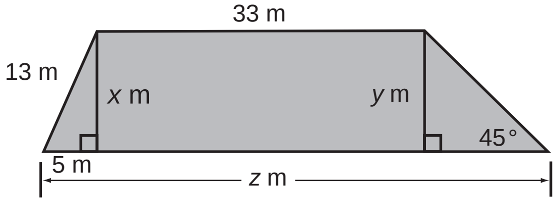
3. Find the perimeter of the isosceles trapezoid.



4. Find x and the area of the trapezoid.



5. Find x, y, z, and the area of the trapezoid shown at the right.



NAME ______________________________ DATE __________ PERIOD _____

Enrichment

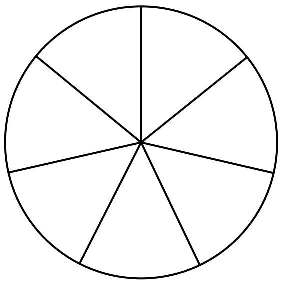
The Four-Color Problem

Mapmakers have long believed that only four colors are necessary to distinguish among any number of different countries on a plane map. Countries that meet only at a point may have the same color provided they do not have an actual border. The conjecture that four colors are sufficient for every conceivable plane map eventually attracted the attention of mathematicians and became known as the “four-color problem.” Despite extraordinary efforts over many years to solve the problem, no definite answer was obtained until the 1980s. Four colors are indeed sufficient, and the proof was accomplished by making ingenious use of computers.

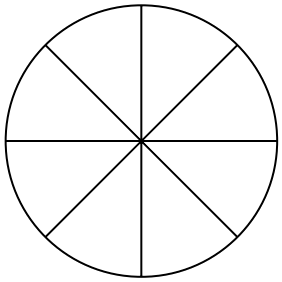
The following problems will help you appreciate some of the complexities of the four-color problem. For these “maps,” assume that each closed region is a different country.

1. What is the minimum number of colors necessary for each map?

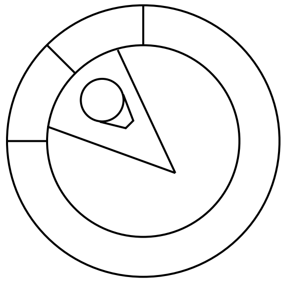
a.



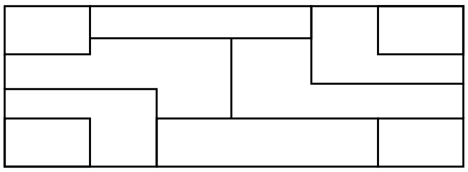
b.



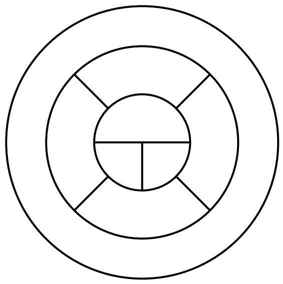
c.



d.



e.



2. Draw some plane maps on separate sheets. Show how each can be colored using four colors. Then determine whether fewer colors would be enough.

11-1

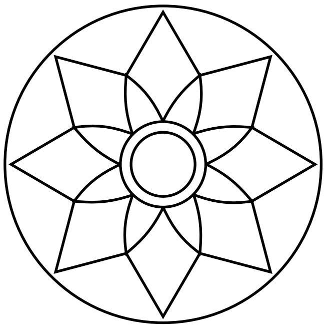
NAME ______________________________ DATE __________ PERIOD ______

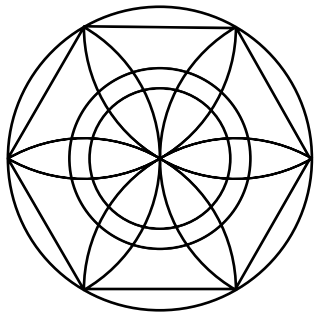
Enrichment

Student Edition
Pages 454–459

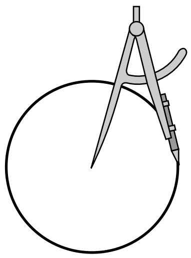
Constructing Designs

Many designs can be made using geometric constructions. Two examples are stained glass rose windows found in churches and Pennsylvania Dutch hex designs found on barns.

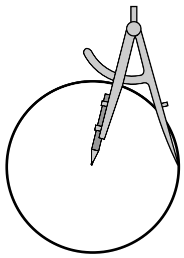




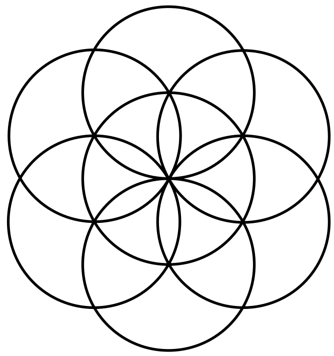
Use your compass to draw a circle.



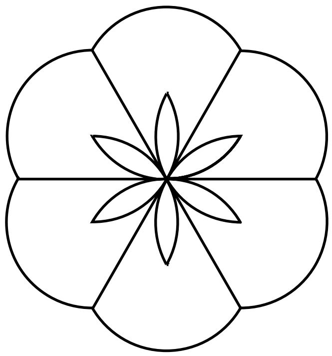
Then, without changing the compass setting, move the point to a point on the circle. Draw a second circle.



Place the point on one of the points where the two circles intersect. Draw a circle. Repeat five more times.



Use your compass and a straightedge to make a design like the one below. (HINT: See the design above.)



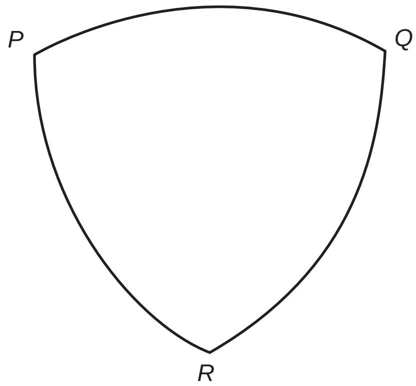
NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Curves of Constant Width

A circle is called a curve of constant width because no matter how you turn it, the greatest distance across it is always the same. However, the circle is not the only figure with this property.

The figure at the right is called a Reuleaux triangle.



1. Use a metric ruler to find the distance from P to any point on the opposite side.

2. Find the distance from Q to the opposite side.

3. What is the distance from R to the opposite side?

The Reuleaux triangle is made of three arcs. In the example shown, $\overset{\frown}{PQ}$ has center R, $\overset{\frown}{QR}$ has center P, and $\overset{\frown}{PR}$ has center Q.

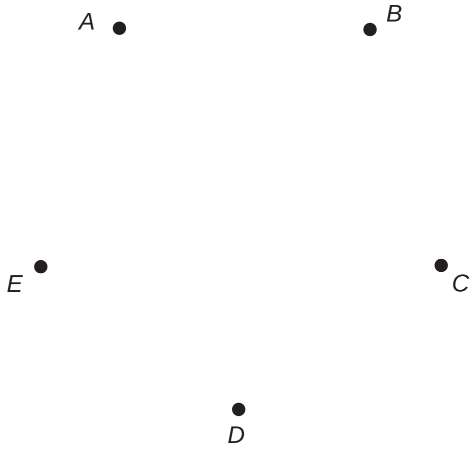
4. Trace the Reuleaux triangle above on a piece of paper and cut it out. Make a square with sides the length you found in Exercise 1. Show that you can turn the triangle inside the square while keeping its sides in contact with the sides of the square.

5. Make a different curve of constant width by starting with the five points below and following the steps given.

Step 1: Place the point of your compass on D with opening DA. Make an arc with endpoints A and B.

Step 2: Make another arc from B to C that has center E.

Step 3: Continue this process until you have five arcs drawn.



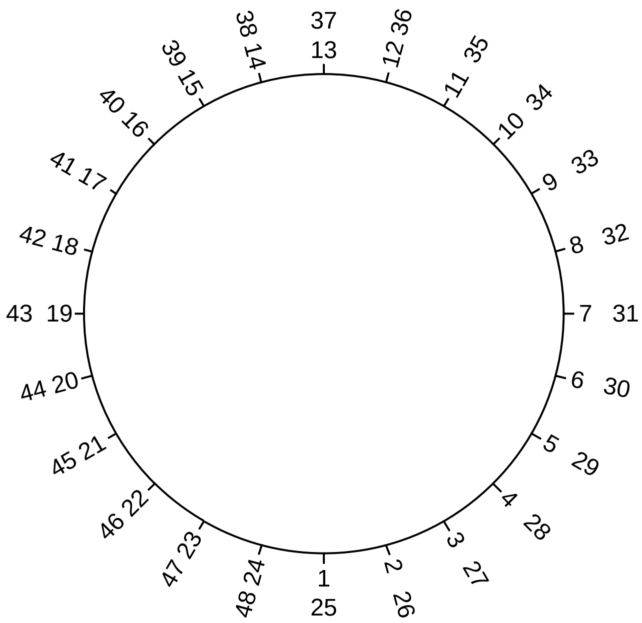
Some countries use shapes like this for coins. They are useful because they can be distinguished by touch, yet they will work in vending machines because of their constant width.

6. Measure the width of the figure you made in Exercise 5. Draw two parallel lines with the distance between them equal to the width you found. On a piece of paper, trace the five-sided figure and cut it out. Show that it will roll between the lines drawn.

Enrichment

Patterns from Chords

Some beautiful and interesting patterns result if you draw chords to connect evenly spaced points on a circle. On the circle shown below, 24 points have been marked to divide the circle into 24 equal parts. Numbers from 1 to 48 have been placed beside the points. Study the diagram to see exactly how this was done.



1. Use your ruler and pencil to draw chords to connect numbered points as follows: 1 to 2, 2 to 4, 3 to 6, 4 to 8, and so on. Keep doubling until you have gone all the way around the circle. What kind of pattern do you get?

2. Copy the original circle, points, and numbers. Try other patterns for connecting points. For example, you might try tripling the first number to get the number for the second endpoint of each chord. Keep special patterns for a possible class display.

NAME ______________________________ DATE __________ PERIOD ______

Enrichment

Student Edition
Pages 474–477

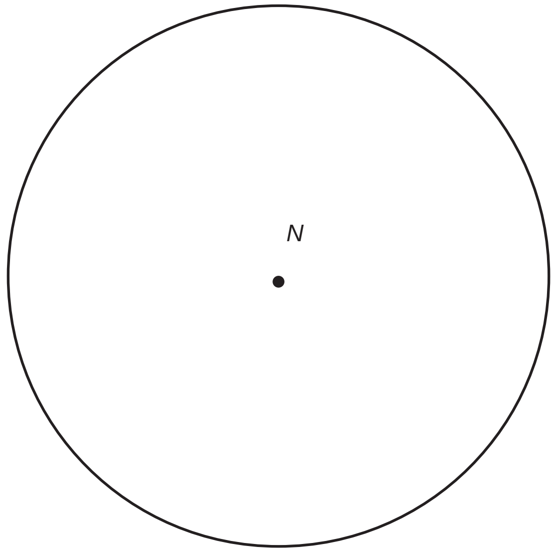
Area of Inscribed Polygons

A protractor can be used to inscribe a regular polygon in a circle. Follow the steps below to inscribe a regular nonagon in ⊙N.

Step 1: Find the degree measure of each of the nine congruent arcs.

Step 2: Draw 9 radii to form 9 angles with the measure you found in Step 1. The radii will intersect the circle in 9 points.

Step 3: Connect the nine points to form the nonagon.



1. Find the length of one side of the nonagon to the nearest tenth of a centimeter. What is the perimeter of the nonagon?

2. Measure the distance from the center perpendicular to one of the sides of the nonagon.

3. What is the area of one of the nine triangles formed?

4. What is the area of the nonagon?

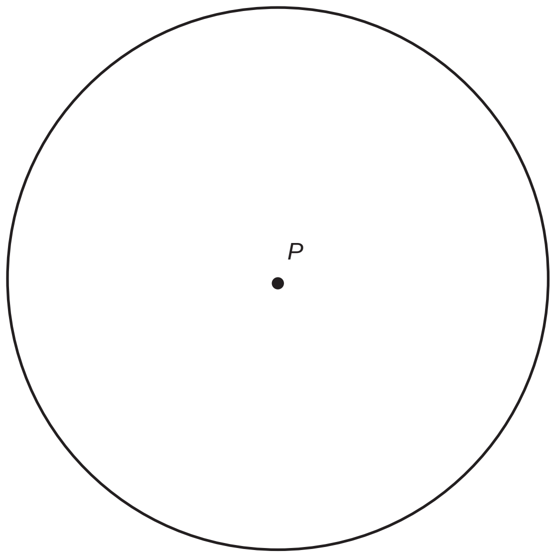
Make the appropriate changes in Steps 1–3 above to inscribe a regular pentagon in ⊙P. Answer each of the following.

5. Use a protractor to inscribe a regular pentagon in ⊙*P*.

6. What is the measure of each of the five congruent arcs?

7. What is the perimeter of the pentagon to the nearest tenth of a centimeter?

8. What is the area of the pentagon to the nearest tenth of a centimeter?



NAME ______________________________ DATE __________ PERIOD ______

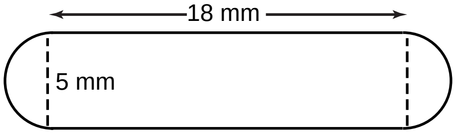
Enrichment

Student Edition
Pages 478–482

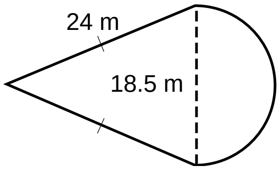
Finding Perimeter

Use a calculator to find the perimeter (the solid lines and curves) of each figure. Use $\pi \approx 3.14$.

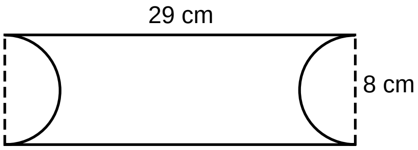
1.



2.



3.



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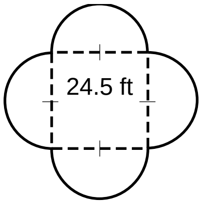

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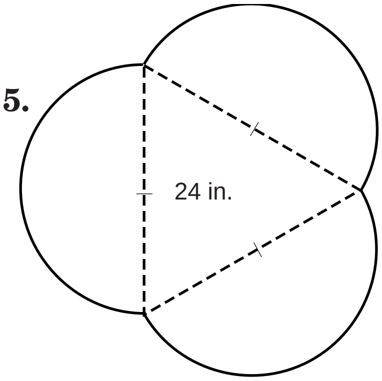

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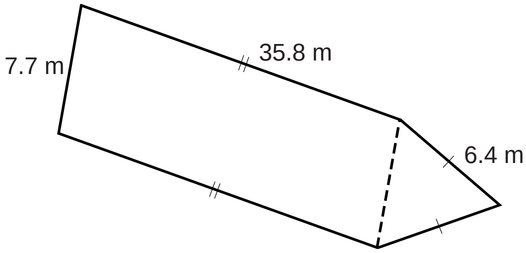

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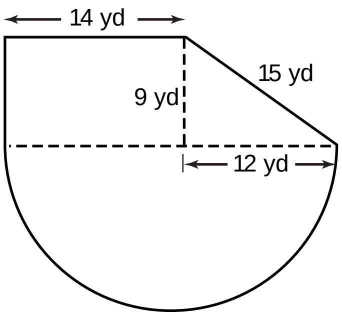

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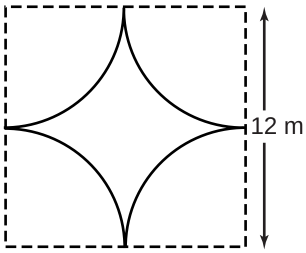

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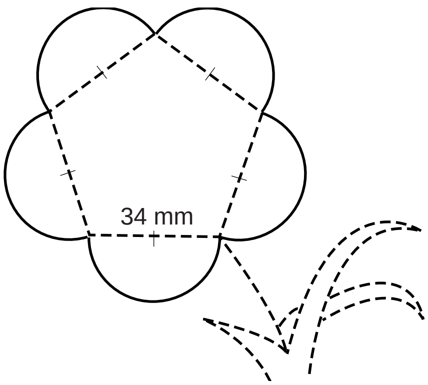

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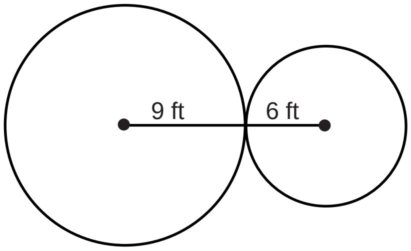


NAME ______________________________ DATE __________ PERIOD _____

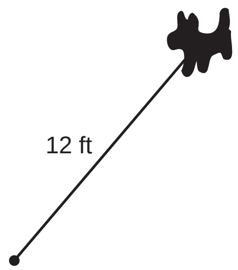
Enrichment

Student Edition
Pages 483–487

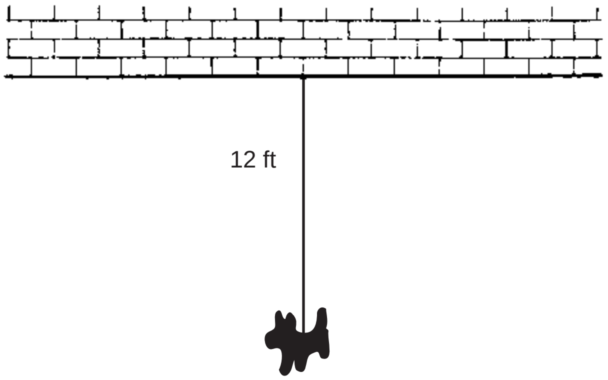
Area of Circular Regions

Robin is going to fix a chain to tie up his dog Rover. There are several places in the yard that Robin can attach the end of the chain. For each of the following, use a compass to draw the space that Rover can reach while on the end of a 12-foot chain. Then find the area.

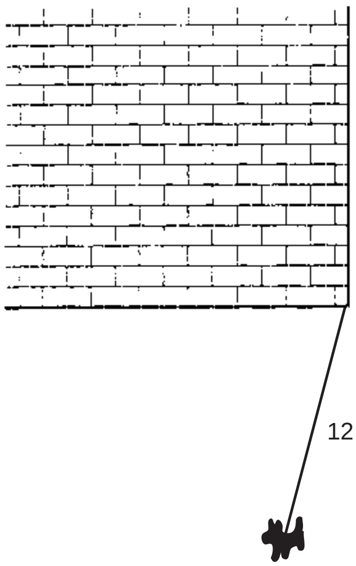
1. Rover's chain is attached to a stake in the middle of the yard.
area =



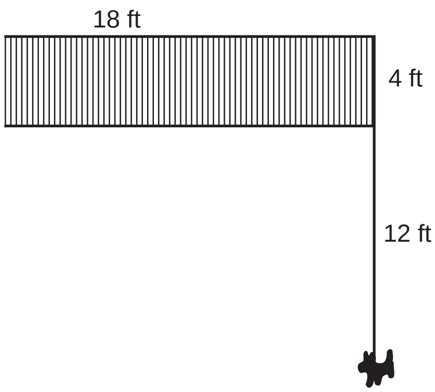
2. Rover's chain is attached to a long wall.
area =



3. Rover's chain is attached to the corner of the house.
area =



4. Rover's chain is attached to a 4-foot by 18-foot rectangular shed.
area =

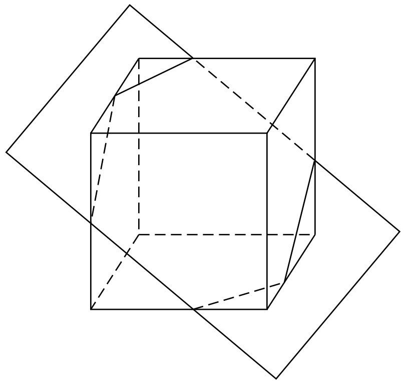


Enrichment

Student Edition
Pages 496–501

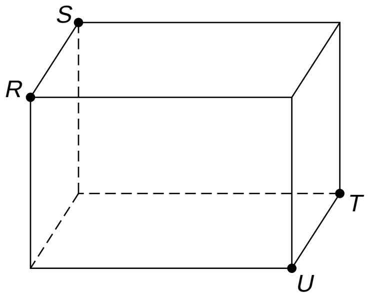
Cross Sections of Prisms

When a plane intersects a solid figure to form a two-dimensional figure, the results is called a **cross section**. The figure at the right shows a plane intersecting a cube. The cross section is a hexagon.

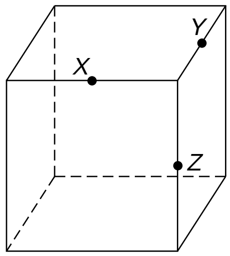


For each right prism, connect the labeled points in alphabetical order to show a cross section. Then identify the polygon.

1.



2.



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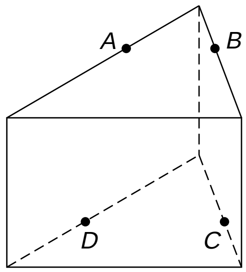


Refer to the right prisms shown below. In the rectangular prism, A and C are midpoints. Identify the cross-section polygon formed by a plane containing the given points.

4. A, C, H

5. C, E, G

6. H, C, E, F

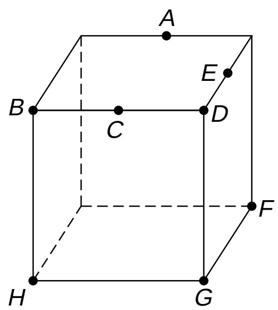
7. H, A, E

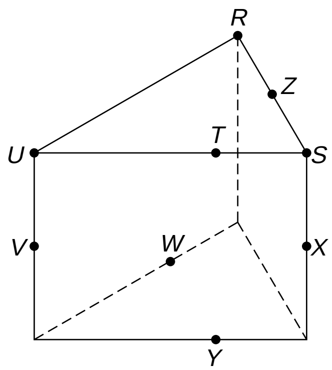
8. B, D, F

9. V, X, R

10. R, T, Y

11. R, S, W





NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 504–509

Can-didly Speaking

Beverage companies have mathematically determined the ideal shape for a 12-ounce soft-drink can. Why won't any shape do? Some shapes are more economical than others. In order to hold 12 ounces of soft drink, an extremely skinny can would have to be very tall. The total amount of aluminum used for such a shape would be greater than the amount used for the conventional shape, thus costing the company more to make the skinny can. The radius r chosen determines the height h needed to hold 12 ounces of liquid. This also determines the amount of aluminum needed to make the can. Companies also have to keep in mind that the top of the can is three times thicker than the bottom and sides. Why? So you won't tear off the entire top when you open the can! The following formulas can be used to find the height and amount of aluminum a needed for a 12-ounce soft-drink can.

$$h = \frac{17.89}{\pi r^2}$$

$$a = 0.02\pi\left[\frac{4r^2 + 35.78}{r}\right]$$

The values of *r* and *h* are measured in inches.

Find the height needed for a 12-ounce can for each radius. Round to the nearest tenth.

1. $2\frac{2}{3}$ in. **2.** 1 in. **3.** 3 in. **4.** $1\frac{3}{4}$ in.

Sketch the shape of each can in Exercises 1–4.

5. Exercise 1 **6.** Exercise 2 **7.** Exercise 3 **8.** Exercise 4

9a. Measure the radius of a soft-drink can

b. Use the formula to find the height of the can.

c. Measure the height of a soft-drink can.

d. How does this measure compare to your findings in part a?

10. Find the amount of aluminum used in making a soft-drink can.

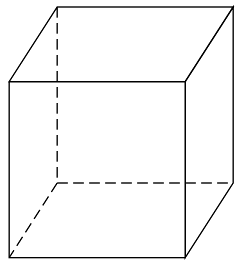
Enrichment

Prism Puzzles

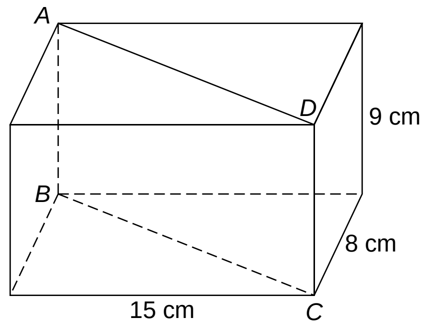
1. The surface area of a cube is 384 cm^2. Another cube is one unit longer on each edge. What is the surface area of the second cube?

2. The area of each end of a rectangular prism is 42 in^2. The area of the top is 77 in^2, and the area of the front is 66 in^2. What is the volume of the prism?

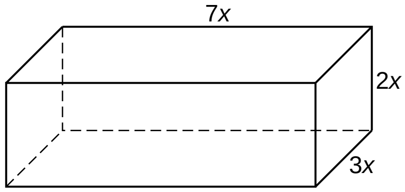
3. Place the digits 1 through 8 at the corners of this cube so the sum of the four numbers for each face is 18.



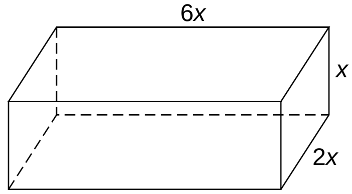
4. Find the area of the cross section of this right prism that contains points *A*, *B*, *C*, and *D*.



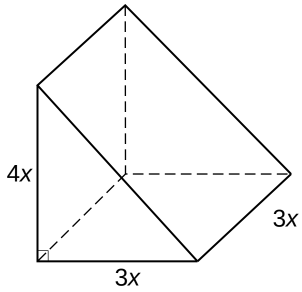
5. Find the value of x for the prism below so the surface area of the prism is 2050 square units.



6. Find the value of x for the prism below so the volume of the prism is 4116 cubic units.



7. Find the value of x for the prism below so the surface area of the prism is 768 square units.

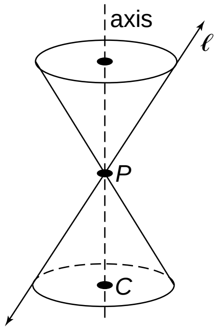


8. Find the value of x for the prism in Exercise 7 so the volume of the prism is 486 cubic units.

NAME ______________________________ DATE __________ PERIOD _____

Enrichment

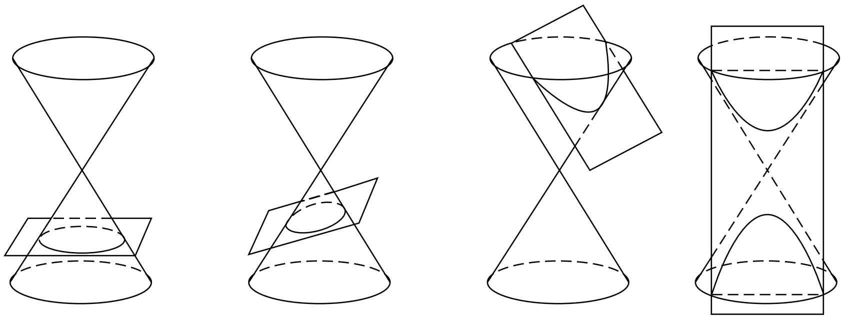
Conic Sections



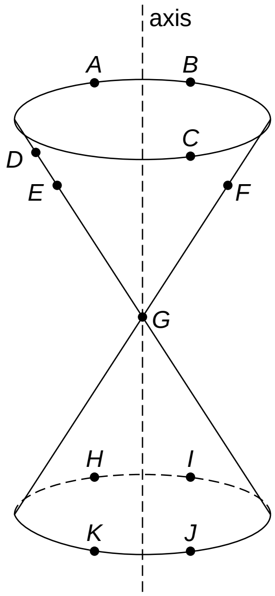
A double conical surface is formed by all the lines ℓ that pass through point P (not in the plane of $\odot C$) and that intersect $\odot C$. Each such line is an **element** of the surface. Cross sections formed by planes that cut the surface are called **conic sections**.

A circle or ellipse is formed by a plane that intersects one cone and is not parallel to an element. A **parabola** is formed by a plane parallel to an element. A **hyperbola** is formed by a plane that intersects both cones. Study the figures below.

Circle | Ellipse | Parabola | Hyperbola



Answer each question.



1. Points E and F are the same distance from the vertex, G. Which conic section would be formed by a plane through points E and F that is perpendicular to the axis?

2. Which conic section would be formed by a plane through points H and K that is parallel to edge FG.

3. Which conic section would be formed by a plane through points D and F that intersects only the top cone?

4. Which conic section would be formed by a plane through points B, C, I, and J?

5. What type of figure would be formed by a plane that contains the axis?

NAME ______________________ DATE __________ PERIOD ______

Enrichment

Student Edition
Pages 522–527

Frustums

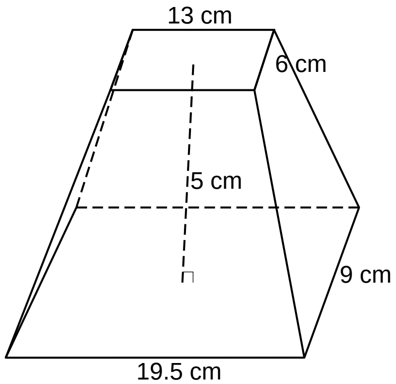
A **frustum** is a figure formed when a plane intersects a pyramid or cone so that the plane is parallel to the solid's base. The frustum is the part of the solid between the plane and the base. To find the volume of a frustum, the areas of both bases must be calculated and used in the formula

$$V = \frac{1}{3}h(B_1 + B_2 + \sqrt{B_1B_2}),$$

where h = height (perpendicular distance between the bases), B_1 = area of top base, and B_2 = area of bottom base.

Describe the shape of the bases of each frustum. Then find the volume. Round to the nearest tenth.

1.



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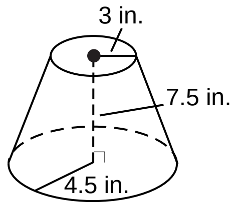

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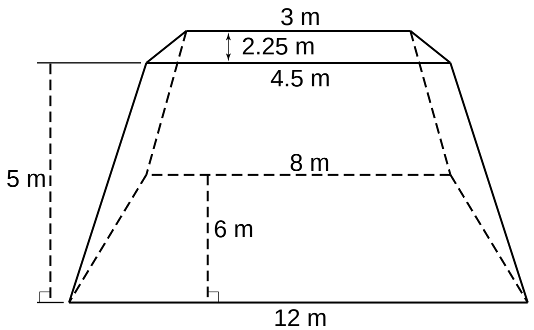

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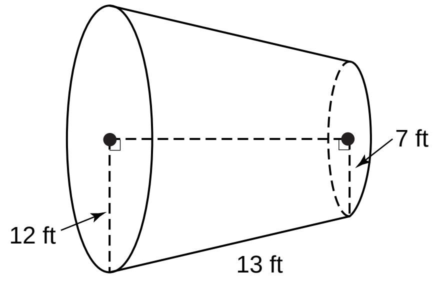


12-6

NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 528–533

Spheres and Density

The **density** of a metal is a ratio of its mass to its volume. For example, the mass of aluminum is 2.7 grams per cubic centimeter. Here is a list of several metals and their densities.

Aluminum	2.7 g/cm^3	Copper	8.96 g/cm^3
Gold	19.32 g/cm^3	Iron	7.874 g/cm^3
Lead	11.35 g/cm^3	Platinum	21.45 g/cm^3
Silver	10.50 g/cm^3		

To calculate the mass of a piece of metal, multiply volume by density.

Example: Find the mass of a silver ball that is 0.8 cm in diameter.

$$M = D \cdot V$$
$$= 10.5 \cdot \frac{4}{3}\pi(0.4)^3$$
$$\approx 10.5\ (0.27)$$
$$\approx 2.83$$

The mass is about 2.83 grams.

Find the mass of each metal ball described. Assume the balls are spherical. Round your answers to the nearest tenth.

1. a copper ball 1.2 cm in diameter

2. a gold ball 0.6 cm in diameter

3. an aluminum ball with radius 3 cm

4. a platinum ball with radius 0.7 cm

Solve. Assume the balls are spherical. Round your answers to the nearest tenth.

5. A lead ball weighs 326 g. Find the radius of the ball to the nearest tenth of a centimeter.

6. An iron ball weighs 804 g. Find the diameter of the ball to the nearest tenth of a centimeter.

7. A silver ball and a copper ball each have a diameter of 3.5 cm. Which weighs more? How much more?

8. An aluminum ball and a lead ball each have a radius of 1.2 cm. Which weighs more? How much more?

NAME ______________________________ DATE __________ PERIOD ______

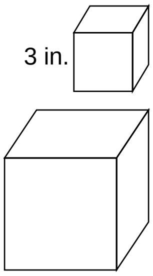
Enrichment

Student Edition
Pages 534-539

Doubling Sizes

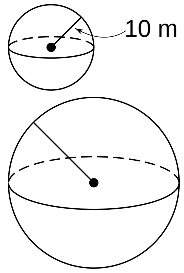
Consider what happens to surface area when the sides of a figure are doubled.

The sides of the large cube are twice the size of the sides of the small cube.



1. How long are the edges of the large cube?
2. What is the surface area of the small cube?
3. What is the surface area of the large cube?
4. The surface area of the large cube is how many times greater than that of the small cube?

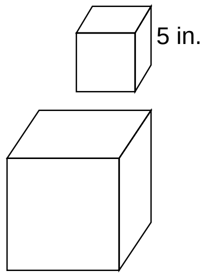
The radius of the large sphere at the right is twice the radius of the small sphere.



5. What is the surface area of the small sphere?
6. What is the surface area of the large sphere?
7. The surface area of the large sphere is how many times greater than the surface area of the small sphere?
8. It appears that if the dimensions of a solid are doubled, the surface area is multiplied by ______.

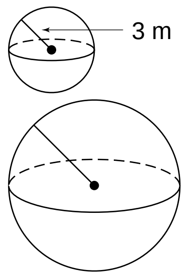
Now consider how doubling the dimensions affects the volume of a cube.

The sides of the large cube are twice the size of the small cube.



9. How long are the edges of the large cube?
10. What is the volume of the small cube?
11. What is the volume of the large cube?
12. The volume of the large cube is how many times greater than that of the small cube?

The large sphere at the right has twice the radius of the small sphere.



13. What is the volume of the small sphere?
14. What is the volume of the large sphere?
15. The volume of the large sphere is how many times greater than the volume of the small sphere?
16. It appears that if the dimensions of a solid are doubled, the volume is multiplied by ______.

NAME ______________________________ DATE __________ PERIOD ______

13-1 Enrichment

Student Edition
Pages 548–553

Properties of the Geometric Mean

The square root of the product of two numbers is called their **geometric mean.**

Numbers	**Geometric Mean**
a and c	$b = \sqrt{ac}$
12 and 48	$\sqrt{12 \cdot 48} = \sqrt{576} = 24$

The geometric mean has many interesting properties. For example, two numbers and their geometric mean satisfy the proportion at the right. $\frac{a}{b} = \frac{b}{c}$

Find the geometric mean, b, for each pair of numbers.

1. $a = 2$ and $c = 8$
$b =$

2. $a = 4$ and $c = 9$
$b =$

3. a = 9 and $c = 16$
$b =$

4. $a = 16$ and $c = 4$
$b =$

5. $a = 16$ and $c = 36$
$b =$

6. $a = 12$ and $c = 3$
$b =$

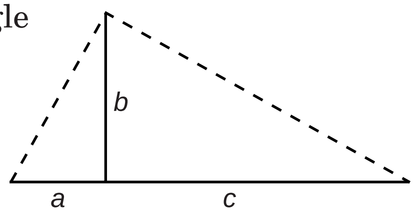
7. $a = 18$ and $c = 8$
$b =$

8. $a = 2$ and $c = 18$
$b =$

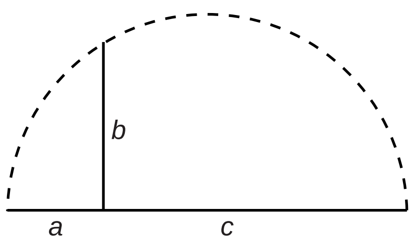
9. $a = 27$ and $c = 12$
$b =$

Solve.

10. For each triple of numbers in Exercises 1–9, draw a triangle like the one shown at the right. What property is shown?



11. Now make this drawing for each triple of numbers. The semicircle has a diameter equal to the sum of a and c. What property do you find?



NAME ______________________________ DATE __________ PERIOD _____

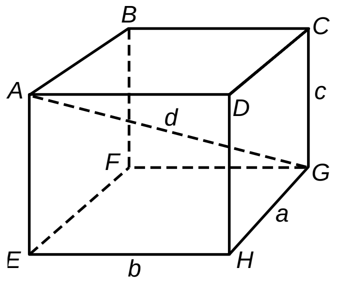
Enrichment

Diagonals

To find the length of diagonals in cubes and rectangular solids, a formula can be applied. In the example below, the length of diagonal $\overline{AG}$ or d can be found using the formula

$$d^2 = a^2 + b^2 + c^2 \text{ or } d = \sqrt{a^2 + b^2 + c^2}.$$

Example 1:



The diagonal, d, is equal to the square root of the sum of the squares of the length, a, the width, b, and the height, c.

Example 2: Find the length of the diagonal of a rectangular prism with length of 8 meters, width of 6 meters, and height of 10 meters.

$d = \sqrt{8^2 + 6^2 + 10^2}$	*Substitute the dimensions into the equation.*
$= \sqrt{64 + 36 + 100}$	*Square each value. Add.*
$= \sqrt{200}$	*Find the square root of the sum.*
$= 14.1$ m	*Round the answer to the nearest tenth.*

Solve. Use $d = \sqrt{a^2 + b^2 + c^2}$. Round answers to the nearest tenth.

1. Find the diagonal of a cube with sides of 6 inches.

2. Find the diagonal of a cube with sides of 2.4 meters.

3. Find the diagonal of a rectangular solid with length of 18 meters, width of 16 meters, and height of 24 meters.

4. Find the diagonal of a rectangular solid with length of 15.1 meters, width of 8.4 meters, and height of 6.3 meters.

5. Find the diagonal of a cube with sides of 34 millimeters.

6. Find the diagonal of a rectangular solid with length of 8.9 millimeters, width of 6.7 millimeters, and height of 14 millimeters.

13-3

NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 559–563

Roots

The symbol $\sqrt{\ }$ indicates a square root. By placing a number in the upper left, the symbol can be changed to indicate higher roots.

$\sqrt[3]{8} = 2$ because $2^3 = 8$
$\sqrt[4]{81} = 3$ because $3^4 = 81$
$\sqrt[5]{100{,}000} = 10$ because $10^5 = 100{,}000$

Find each of the following.

1. $\sqrt[3]{125}$

2. $\sqrt[4]{16}$

3. $\sqrt[8]{1}$

4. $\sqrt[3]{27}$

5. $\sqrt[5]{32}$

6. $\sqrt[3]{64}$

7. $\sqrt[3]{1000}$

8. $\sqrt[3]{216}$

9. $\sqrt[6]{1{,}000{,}000}$

10. $\sqrt[3]{1{,}000{,}000}$

11. $\sqrt[4]{256}$

12. $\sqrt[3]{729}$

13. $\sqrt[6]{64}$

14. $\sqrt[4]{625}$

15. $\sqrt[5]{243}$

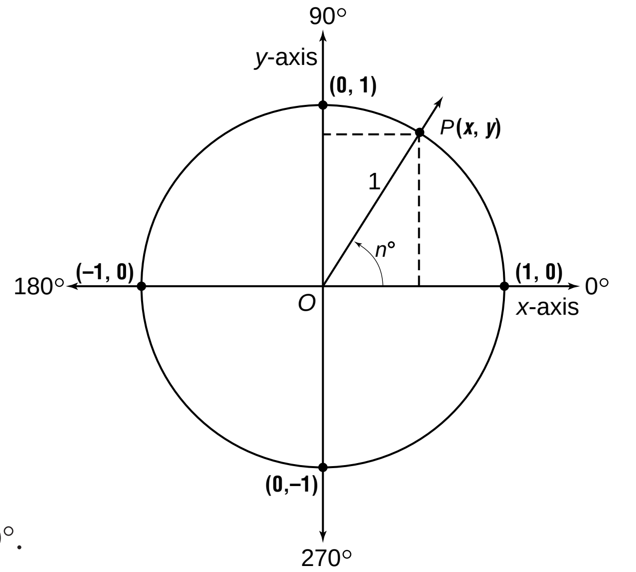
NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 564–569

The Unit Circle

A **unit circle** has a radius of 1. By using a unit circle with its center at the origin of a coordinate plane, the sine and cosine ratios can be defined for measures from 0° to 360°.



Consider a ray rotated through $n°$ from the positive x-axis and intersecting the circle at P, as shown at the right. Point $P(x, y)$ where the ray intersects the unit circle has coordinates ($\cos n°$, $\sin n°$).

$$x = \cos n° \qquad y = \sin n°$$

Example 1: Use the unit circle to find cos 180°.

The cosine of 180° is the x-coordinate of the point where the ray rotated 180° intersects the unit circle. So, $\cos 180° = -1$.

Example 2: For measures between 270° and 360°, is the sine positive or negative?

The sine is the y-coordinate of a point on the unit circle in the fourth quadrant. So, the sine is negative.

Use the unit circle to find each value.

1. sin 180° **2.** cos 270° **3.** sin 270° **4.** sin 90°

5. cos 90° **6.** sin 360° **7.** cos 360° **8.** sin 0°

Use the unit circle to tell if each value is positive or negative. Write + or –.

9. sin 200° **10.** cos 100° **11.** sin 130° **12.** cos 310°

13. What is the range of possible values for $\sin n°$? What is the range of possible values for $\cos n°$?

14. Consider a measure $n°$ between 0° and 180°. Compare $\sin n°$ and $\sin (360° - n°)$. Use a calculator to find several values. Then compare $\cos n°$ and $\cos (360 - n°)$. What do you notice?

NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 572–577

Law of Sines

Trigonometric ratios can also be used to solve problems involving triangles that are not right triangles. Since the measure of an angle in such a triangle may be greater than 90°, but less than 180°, the following theorem is needed.

Theorem: The sine of an obtuse angle is equal to the sine of its supplement.

Example 1: $\sin 150° = 0.5$, since $\sin (180° - 150°) = \sin 30°$ or 0.5.

If the measures of two angles and the measure of the included side are given, or if the measures of two angles and the measure of a nonincluded side are given, then the Law of Sines can be used to find the unknown measures.

Law of Sines: Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite angles with measures A, B, and C respectively. Then

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

Example 2: Using the figure at the right, find AB.

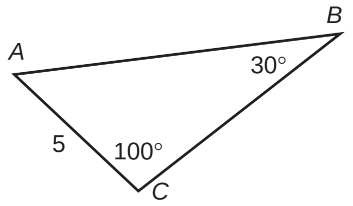
$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{c}{\sin 100°} = \frac{5}{\sin 30°}$$

$$c = \frac{5}{\sin 30°}(\sin 100°)$$

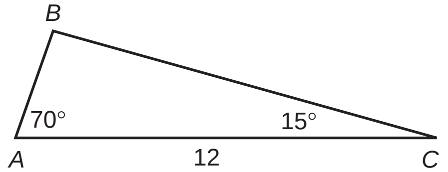
$$c \approx 9.8$$

Therefore, AB is approximately 9.8.

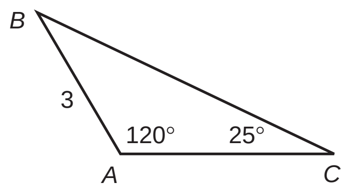


Find the indicated measures. Round answers to the nearest tenth.

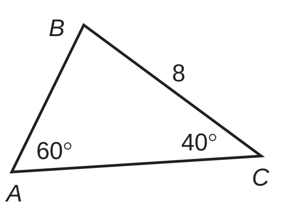
1. Find BC.



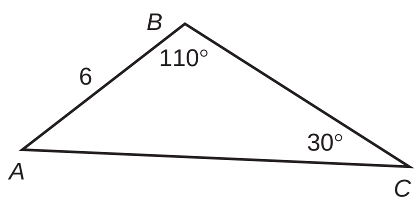
2. Find BC.



3. Find AB.



4. Find AC and BC.



NAME ______________________________ DATE __________ PERIOD ______

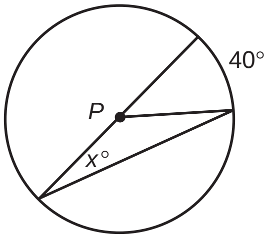
Enrichment

Student Edition
Pages 586–591

Angle Measures and Arc Measures

For each ⊙P, use isosceles triangles to find x. Note that x may be an angle or an arc measure.

1.



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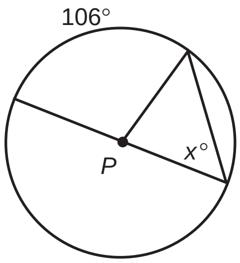

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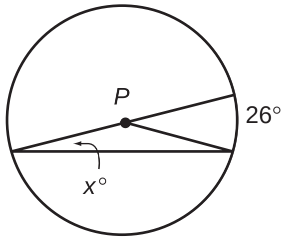

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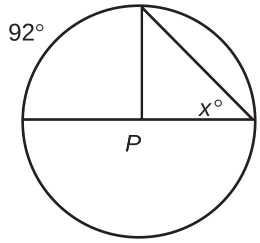

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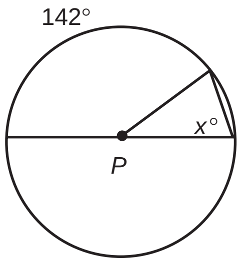

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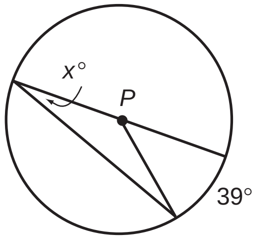

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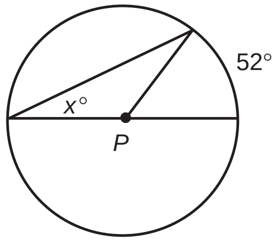

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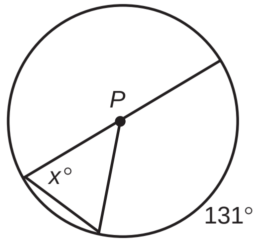

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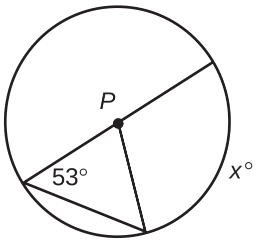

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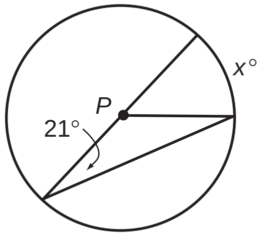

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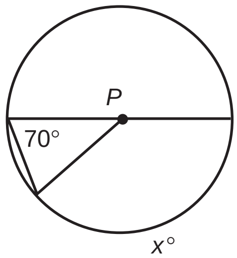

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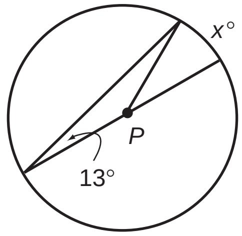

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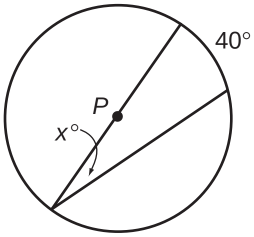

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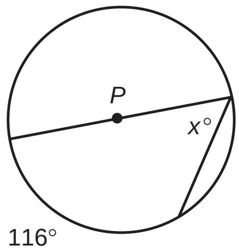

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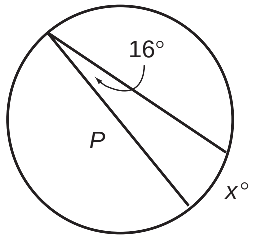

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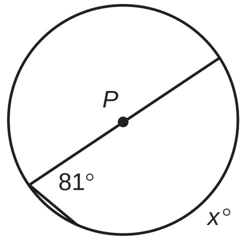


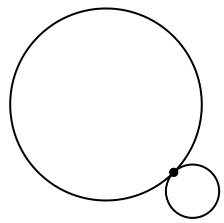
NAME ______________________________ DATE __________ PERIOD _____

14-2 Enrichment

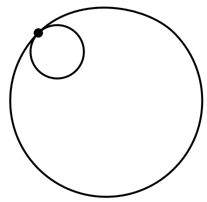
Student Edition
Pages 592–597

Tangent Circles

Two circles in the same plane are **tangent circles** if they have exactly one point in common. Tangent circles with no common interior points are **externally tangent**. If tangent circles have common interior points, then they are **internally tangent**. Three or more circles are **mutually tangent** if each pair of them are tangent.



Externally Tangent Circles



Internally Tangent Circles

1. Make sketches to show all possible positions of three mutually tangent circles.

2. Make sketches to show all possible positions of four mutually tangent circles.

3. Make sketches to show all possible positions of five mutually tangent circles.

4. Write a conjecture about the number of possible positions for n mutually tangent circles if n is a whole number greater than four.

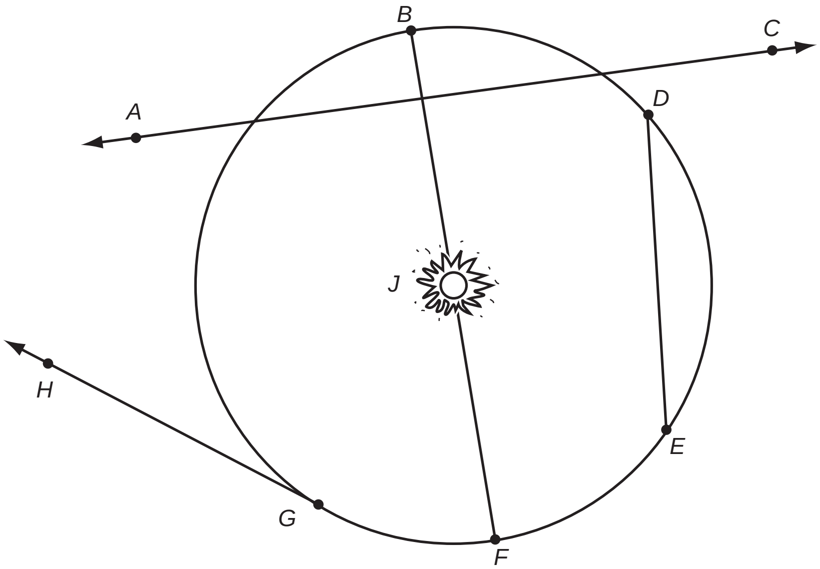
NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 600–605

Orbiting Bodies

The path of the Earth's orbit around the sun is elliptical. However, it is often viewed as circular.



Use the drawing above of the Earth orbiting the sun to name the line or segment described. Then identify it as a radius, diameter, chord, tangent, or secant of the orbit.

1. the path of an asteroid

2. the distance between the Earth's position in July and the Earth's position in October

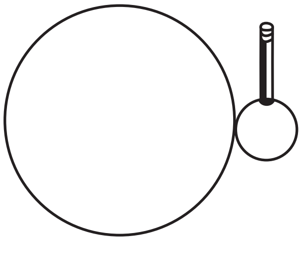
3. the distance between the Earth's position in December and the Earth's position in June

4. the path of a rocket shot toward Saturn

5. the path of a sunbeam

6. If a planet has a moon, the moon circles the planet as the planet circles the sun. To visualize the path of the moon, cut two circles from a piece of cardboard, one with a diameter of 4 inches and one with a diameter of 1 inch.

Tape the larger circle firmly to a piece of paper. Poke a pencil point through the smaller circle, close to the edge. Roll the small circle around the outside of the large one. The pencil will trace out the path of a moon circling its planet. This kind of curve is called an epicycloid. To see the path of the planet around the sun, poke the pencil through the center of the small circle (the planet), and roll the small circle around the large one (the sun).

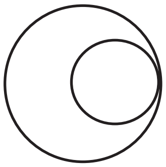


Enrichment

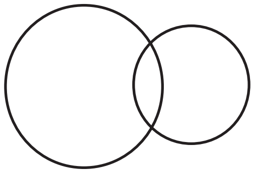
Student Edition
Pages 606–611

Tangents, Secants, and Intercepted Arcs

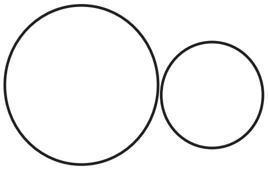
1. Draw a line that is tangent to both circles.



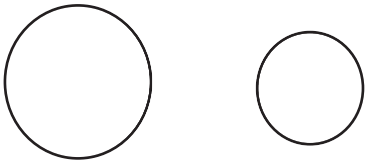
2. Draw the two lines that are tangent to both circles.



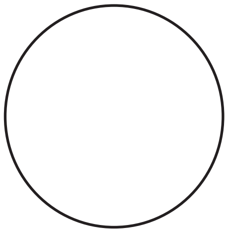
3. Draw all lines that are tangent to both circles.



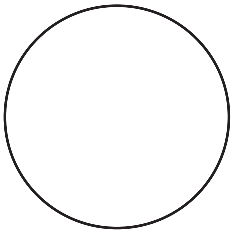
4. Draw all lines that are tangent to both circles.



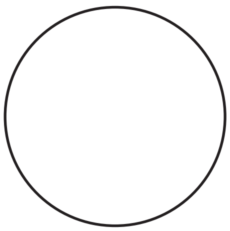
5. Draw $\angle ABC$ so that it is an inscribed angle and its measure is 25°. Find the measure of the intercepted arc.



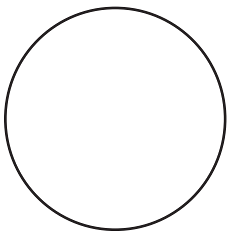
6. Draw $\angle JKL$ so that $m\angle JKL = 37$ and $\overleftrightarrow{KJ}$ and $\overleftrightarrow{KL}$ are secants. Find the measure of the two intercepted arcs.



7. Draw $\angle PQR$ so that it is formed by a tangent and a secant and $m\angle PQR = 70$. Find the measures of the two intercepted arcs.



8. Draw $\angle TXW$ so that it is a 110° angle formed by two tangents. Find the measures of the two intercepted arcs.



NAME ______________________________ DATE __________ PERIOD _____

Enrichment

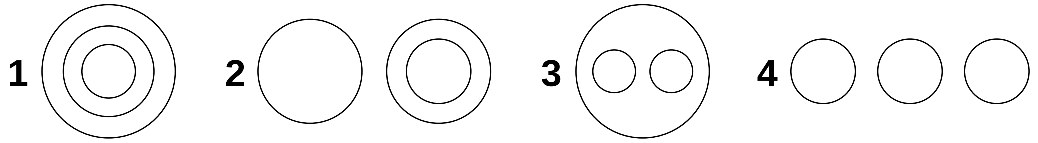
Student Edition
Pages 612–617

Curious Circles

Two circles can be arranged in four ways: one circle can be inside the other, they can be separate, they can overlap, or they can coincide.

In how many ways can a given number of circles be either separate or inside each other? (The situations in which the circles overlap or coincide are not counted here.)

Here is the answer for 3 circles. There are 4 different possibilities.



Solve each problem. Make drawings to show your answers.

1. Show the different ways in which 2 circles can be separate or inside each other. How many ways are there?

2. Show the different ways for 4 circles. How many ways are there?

3. Use your answer for Exercise 2 to show that the number of ways for 5 circles is at least 18.

4. Find the number of ways for 5 circles. Show your drawings on a separate sheet of paper.

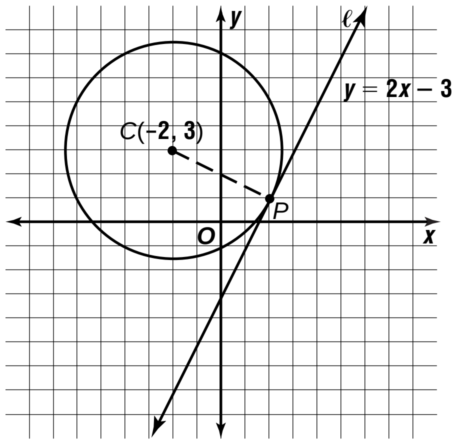
NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 618–623

Equations of Circles and Tangents

Recall that the circle whose radius is r and whose center has coordinates (h, k) is the graph of $(x - h)^2 + (y - k)^2 = r^2$. You can use this idea and what you know about circles and tangents to find an equation of the circle that has a given center and is tangent to a given line.



Use the following steps to find an equation for the circle that has center C(−2, 3) and is tangent to the graph y = 2x − 3. Refer to the figure.

1. State the slope of the line ℓ that has equation $y = 2x - 3$.

2. Suppose $\odot C$ with center $C(^-2, 3)$ is tangent to line ℓ at point P. What is the slope of radius $\overline{CP}$?

3. Find an equation for the line that contains $\overline{CP}$.

4. Use your equation from Exercise 3 and the equation $y = 2x - 3$. At what point do the lines for these equations intersect? What are its coordinates?

5. Find the measure of radius $\overline{CP}$.

6. Use the coordinate pair $C(^-2, 3)$ and your answer for Exercise 5 to write an equation for $\odot C$.

NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 632–637

Counterexamples

When you make a conclusion after examining several specific cases, you have used **inductive reasoning**. However, you must be cautious when using this form of reasoning. By finding only one **counterexample**, you disprove the conclusion.

Example: Is the statement $\frac{1}{x} \leq 1$ true when you replace x with 1, 2, and 3? Is the statement true for all reals? If possible, find a counterexample.
$\frac{1}{1} = 1$, $\frac{1}{2} < 1$, and $\frac{1}{3} < 1$. But when $x = \frac{1}{2}$, then $\frac{1}{x} = 2$. This counterexample shows that the statement is not always true.

Answer each question.

1. The coldest day of the year in Chicago occurred in January for five straight years. Is it safe to conclude that the coldest day in Chicago is always in January?

2. Suppose John misses the school bus four Tuesdays in a row. Can you safely conclude that John misses the school bus every Tuesday?

3. Is the equation $\sqrt{k^2} = k$ true when you replace k with 1, 2, and 3? Is the equation true for all integers? If possible, find a counterexample.

4. Is the statement $2x = x + x$ true when you replace x with $\frac{1}{2}$, 4, and 0.7? Is the statement true for all real numbers? If possible, find a counterexample.

5. Suppose you draw four points A, B, C, and D and then draw $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$. Does this procedure give a quadrilateral always or only sometimes? Explain your answers with figures.

6. Suppose you draw a circle, mark three points on it, and connect them. Will the angles of the triangle be acute? Explain your answers with figures

NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 638–643

Valid and Faulty Arguments

Consider the statements at the right. What conclusions can you make?

(1) Boots is a cat.
(2) Boots is purring.
(3) A cat purrs if it is happy.

From statements 1 and 3, it is correct to conclude that Boots purrs if it is happy. However, it is faulty to conclude from only statements 2 and 3 that Boots is happy. The if-then form of statement 3 is *If a cat is happy, then it purrs*.

Advertisers often use faulty logic in subtle ways to help sell their products. By studying the arguments, you can decide whether the argument is valid or faulty.

Decide if each argument is valid or faulty.

1. (1) If you buy Tuff Cote luggage, it will survive airline travel.
(2) Justin buys Tuff Cote luggage.
Conclusion: Justin's luggage will survive airline travel.

2. (1) If you buy Tuff Cote luggage, it will survive airline travel.
(2) Justin's luggage survived airline travel.
Conclusion: Justin has Tuff Cote luggage.

3. (1) If you use Clear Line long distance service, you will have clear reception.
(2) Anna has clear long distance reception.
Conclusion: Anna uses Clear Line long distance service.

4. (1) If you read the book *Beautiful Braids*, you will be able to make beautiful braids easily.
(2) Nancy read the book *Beautiful Braids*.
Conclusion: Nancy can make beautiful braids easily.

5. (1) If you buy a word processor, you will be able to write letters faster.
(2) Tania bought a word processor.
Conclusion: Tania will be able to write letters faster.

6. (1) Great swimmers wear AquaLine swimwear.
(2) Gina wears AquaLine swimwear.
Conclusion: Gina is a great swimmer.

7. Write an example of faulty logic that you have seen in an advertisement.

NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 644–648

Logic Problems

The following problems can be solved by eliminating possibilities. It may be helpful to use charts such as the one shown in the first problem. Mark an X in the chart to eliminate a possible answer.

Solve each problem.

1. Nancy, Olivia, Mario, and Kenji each have one piece of fruit in their school lunch. They have a peach, an orange, a banana, and an apple. Mario does not have a peach or a banana. Olivia and Mario just came from class with the student who has an apple. Kenji and Nancy are sitting next to the student who has a banana. Nancy does not have a peach. Which student has each piece of fruit?

	Nancy	Olivia	Mario	Kenji
Peach				
Orange				
Banana				
Apple				

2. Victor, Leon, Kasha, and Sheri each play one instrument. They play the viola, clarinet, trumpet, and flute. Sheri does not play the flute. Kasha lives near the student who plays flute and the one who plays trumpet. Leon does not play a brass or wind instrument. Which student plays each instrument?

3. Mr. Guthrie, Mrs. Hakoi, Mr. Mirza, and Mrs. Riva have jobs of doctor, accountant, teacher, and office manager. Mr. Mirza lives near the doctor and the teacher. Mrs. Riva is not the doctor or the office manager. Mrs. Hakoi is not the accountant or the office manager. Mr. Guthrie went to lunch with the doctor. Mrs. Riva's son is a high school student and is only seven years younger than his algebra teacher. Which person has each occupation?

4. Yvette, Lana, Boris, and Scott each have a dog. The breeds are collie, beagle, poodle, and terrier. Yvette and Boris walked to the library with the student who has a collie. Boris does not have a poodle or terrier. Scott does not have a collie. Yvette is in math class with the student who has a terrier. Which student has each breed of dog?

NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 649–653

More Counterexamples

Some statements in mathematics can be proven false by **counterexamples.** Consider the following statement.

For any numbers a and b, $a - b = b - a$.

You can prove that this statement is false in general if you can find one example for which the statement is false.

Let $a = 7$ and $b = 3$. Substitute these values in the equation above.

$$7 - 3 \stackrel{?}{=} 3 - 7$$
$$4 \neq -4$$

In general, for any numbers a and b, the statement $a - b = b - a$ is false. You can make the equivalent verbal statement: subtraction is *not* a commutative operation.

In each of the following exercises a, b, and c are any numbers. Prove that the statement is false by counterexample.

1. $a - (b - c) \stackrel{?}{=} (a - b) - c$

2. $a \div (b \div c) \stackrel{?}{=} (a \div b) \div c$

3. $a \div b \stackrel{?}{=} b \div a$

4. $a \div (b + c) \stackrel{?}{=} (a \div b) + (a \div c)$

5. $a + (bc) \stackrel{?}{=} (a + b)(a + c)$

6. $a^2 + a^2 \stackrel{?}{=} a^4$

7. Write the verbal equivalents for Exercises 1, 2, and 3.

8. For the Distributive Property $a(b + c) = ab + ac$ it is said that multiplication distributes over addition. Exercises 4 and 5 prove that some operations do not distribute. Write a statement for each exercise that indicates this.

NAME ______________________________ DATE __________ PERIOD _____

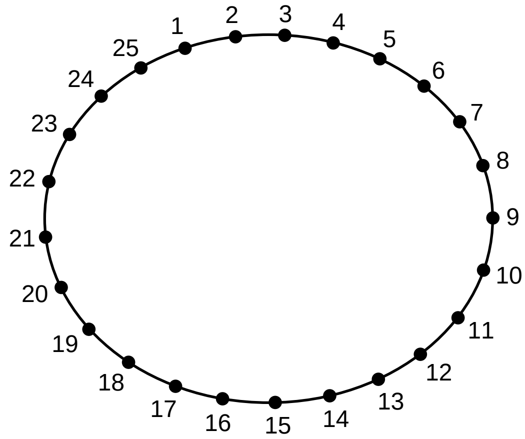
Enrichment

Student Edition
Pages 654–659

Counting-Off Puzzles

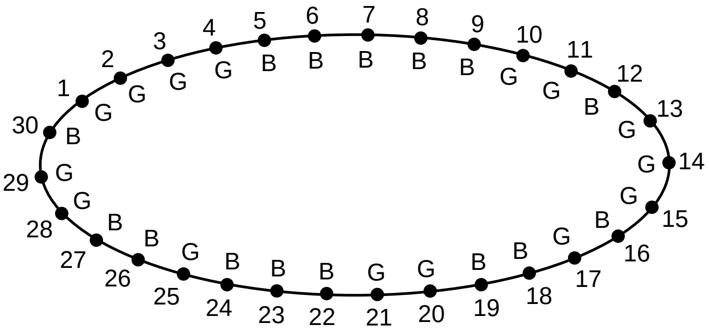
Solve each puzzle.

1. Twenty-five people are standing in a circle. Starting with person 1, they count off from 1 to 7 and then start over with 1. Each person who says "7" drops out of the circle. Who is the last person left?



2. Forty people stand in a circle. They count off so that every third person drops out. Which two people are the last ones left?

3. Only half of the 30 students in Sharon's class can go on a field trip. Sharon arranges the boys and girls as shown. They count off from 1 to 9 and every ninth person drops out until only 15 people are left. Who gets to go on the field trip.



A group of people stand in a circle and count off 1, 2, 1, 2, 1 and so on. Every second person drops out. Person number 1 is the last person left.

4. Draw a diagram to show why the number of people in the circle must be even. Then, explain your answer.

5. When the count returns to person number 1 for the first time, how many people have dropped out?

6. Find the number of people in the circle if the number is between 10 and 20. Do the same if the number is between 30 and 40. What can you conclude about the original number of people?

NAME ______________________ DATE __________ PERIOD ______

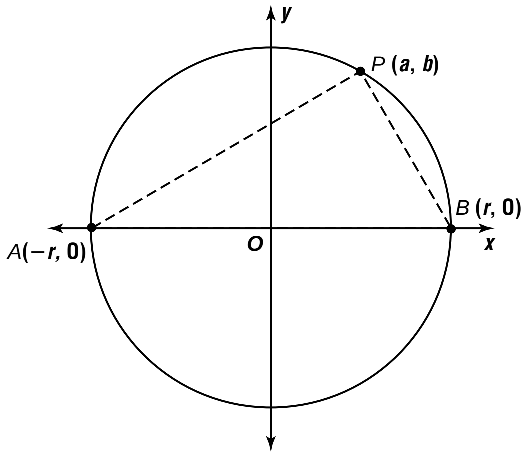
Enrichment

Student Edition
Pages 660–665

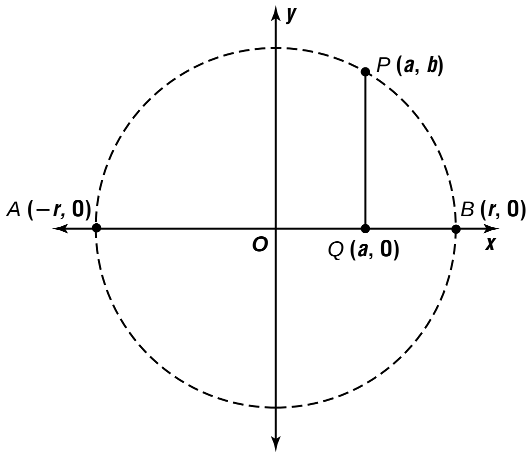
Coordinate Proofs with Circles

You can prove many theorems about circles by using coordinate geometry. Whenever possible locate the circle so that its center is at the origin.

1. Prove that an angle inscribed in a semicircle is a right angle. Use the figure at right. (Hint: Write an equation for the circle.
Use your equation to help show that (slope of $\overline{AP}$) · (slope of $\overline{PB}$) = −1).



2. Suppose $\overline{PQ} \perp \overline{AB}$, Q is between A and B, and $\overline{PQ}$ is the geometric mean between $\overline{AQ}$ and $\overline{QB}$. Prove that P is on the circle that has $\overline{AB}$ as a diameter. Use the figure at the right.

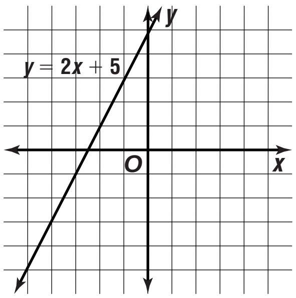


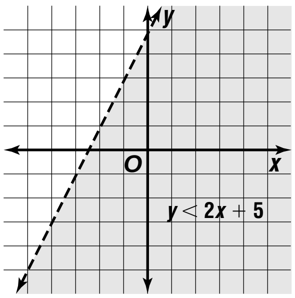
NAME ______________________________ DATE __________ PERIOD _____

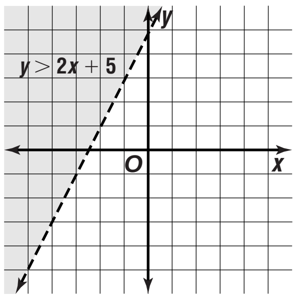
Enrichment

Graphing Linear Inequalities

The graph of $y = 2x + 5$ separates the coordinate plane into two **half-planes**. The half-plane below the line for $y = 2x + 5$ consists of all points (x, y) whose coordinates satisfy the inequality $y < 2x + 5$. The half-plane above the line consists of all points that satisfy $y > 2x + 5$. The graph of $y = 2x + 5$ is the **boundary** of the half-planes.



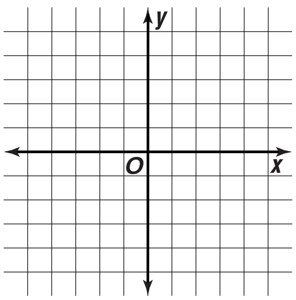




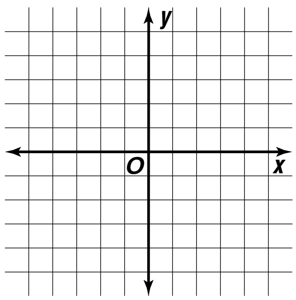
Notice the dashed line used for $y = 2x + 5$ in the graphs of the inequalities. This boundary line is not part of either of the half-planes.

Graph each inequality. Use shading to show half-planes.

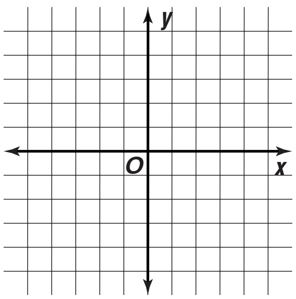
1. $y > \frac{1}{2}x + 3$



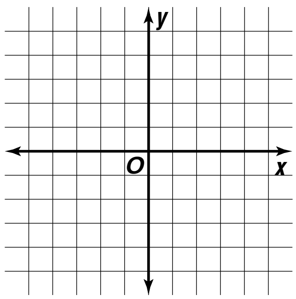
2. $y < {}^-x$



3. $y < {}^-2x + 4$



4. $y > x + 1$



NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 681–686

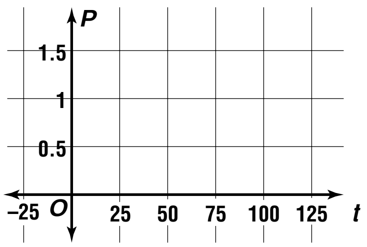
Absolute Zero

All matter is made up of atoms and molecules that are in constant motion. Temperature is one measure of this motion. Absolute zero is the theoretical temperature limit at which the motion of the molecules and atoms of a substance is the least possible.

Experiments with gaseous substances yield data that allow you to estimate just how cold absolute zero is. For any gas of a constant volume, the pressure, expressed in a unit called atmospheres, varies linearly as the temperature. That is, the pressure P and the temperature t are related by an equation of the form $P = mt + b$, where m and b are real numbers.

1. Sketch a graph for the data in the table. Use the axis shown below.

t (in °C)	P (in atmospheres)
−25	0.91
0	1.00
25	1.09
100	1.36



2. Use the data and your graph to find values for m and b in the equation $P = mt + b$ which relates temperature to pressure.

3. Estimate absolute zero in degrees Celsius by setting P equal to 0 in the equation above and using the values m and b that you obtained in Exercise 2.

NAME ______________________________ DATE __________ PERIOD ______

Enrichment

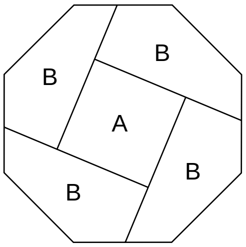
Student Edition
Pages 687–691

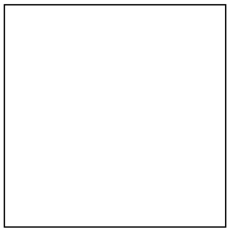
Dissection Puzzles

In a dissection puzzle you are to cut apart one figure and then rearrange the pieces to make a new figure. Only straight cuts are allowed. Usually the puzzle-solver must figure out where to make a given number of cuts. However, for these puzzles, the cut lines are shown. You must find out how to rearrange the pieces.

Cut apart the figure shown. Then rearrange the pieces to form a square. Record your solution in the square at the right.

1.





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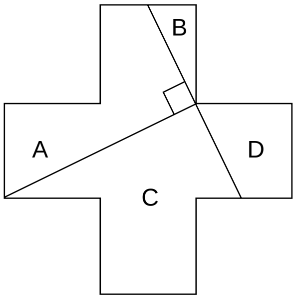
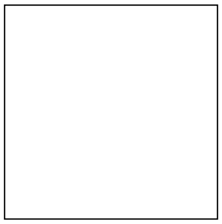
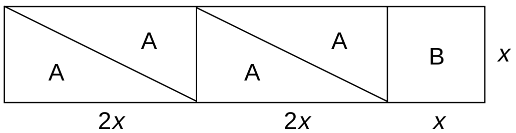
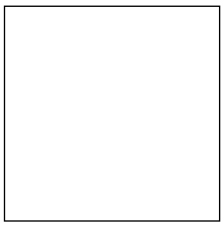


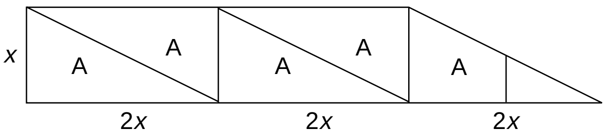


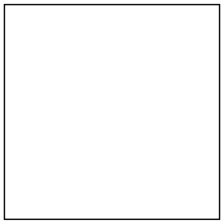
3.





4. For this dissection, you must cut one of the triangles into two pieces to make the square.

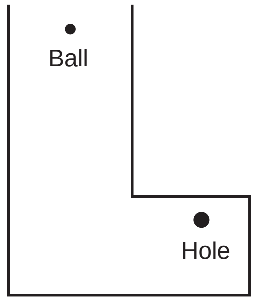




Enrichment

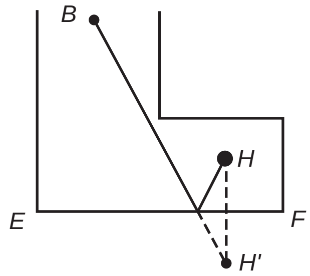
Miniature Golf

In miniature golf, the object of the game is to putt the golf ball into the hole in as few shots as possible. As in the diagram at the right, the hole is often placed so that a direct shot is impossible. If the ball does not have much spin, it will rebound off a wall in such a way that the two angles formed by the path of the ball and the wall will be congruent. Reflections can be used to help determine the direction that the ball should be struck in order to score a hole-in-one.



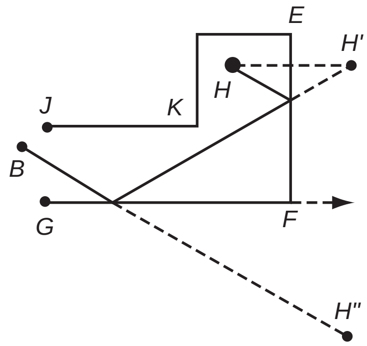
Example 1: Using wall $\overline{EF}$, find the path to use to score a hole-in-one.

Find the reflection image of the “hole” with respect to $\overline{EF}$ and label it H'. The intersection of $\overline{BH'}$ with wall $\overline{EF}$ is the point at which the shot should be directed.



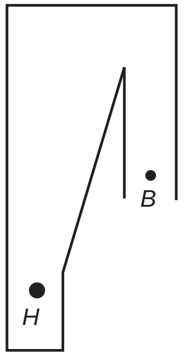
Example 2: For the hole at the right, find a path to score a hole-in-one.

Find the reflection image of H with respect to $\overline{EF}$ and label it H'. In this case, $\overline{BH'}$ intersects $\overline{JK}$ before intersecting $\overline{EF}$. Therefore, this path cannot be used. To find a usable path, find the reflection image of H' over $\overrightarrow{GF}$ and label it H''. Now, the intersection of $\overline{BH''}$ with wall $\overline{GF}$ is the point at which the shot should be directed. Notice how the path of the ball is generated using B, H'', H', and H.

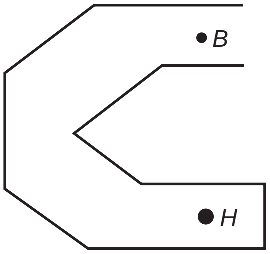


Use reflections to determine a possible path for a hole-in-one.

1.



2.



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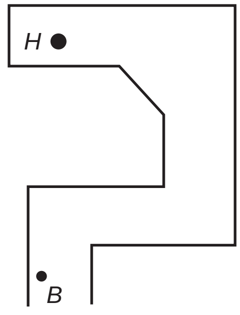


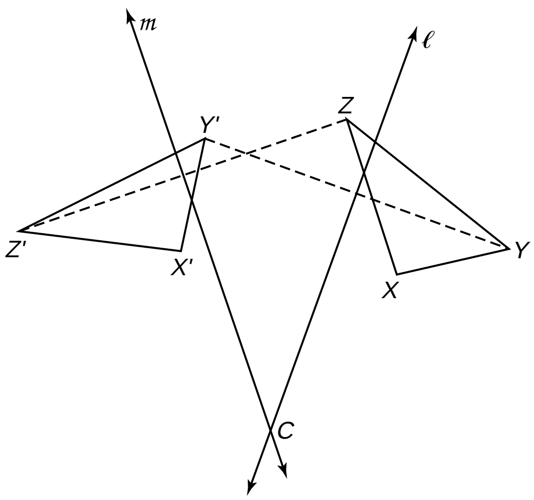
NAME ______________________________ DATE __________ PERIOD _____

Enrichment

Student Edition
Pages 697–702

Finding the Center of Rotation

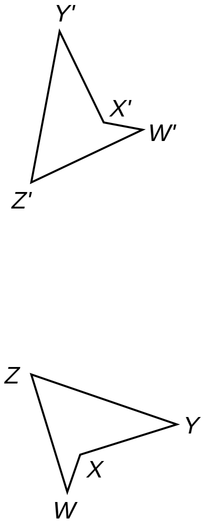
Suppose you are told that $\Delta X'Y'Z'$ is the rotation image of ΔXYZ, but you are not told where the center of rotation is nor the measure of the angle of rotation. Can you find them? Yes, you can. Connect two pairs of corresponding vertices with segments. In the figure, the segments YY' and ZZ' are used. Draw the perpendicular bisectors, ℓ and m, of these segments. The point C where ℓ and m intersect is the center of rotation.



1. How can you find the measure of the angle of rotation in the figure above?

Locate the center of rotation for the rotation that maps WXYZ onto W′X′Y′Z′. Then find the measure of the angle of rotation.

2.



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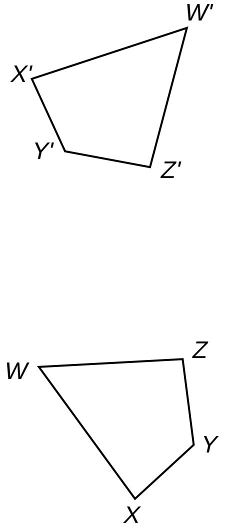


NAME ______________________________ DATE __________ PERIOD _____

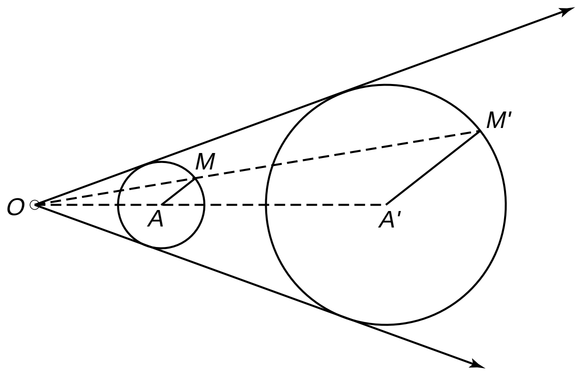
Enrichment

Student Edition
Pages 703–707

Similar Circles

You may be surprised to learn that two noncongruent circles that lie in the same plane and have no common interior points can be mapped one onto the other by more than one dilation.

1. Here is diagram that suggests one way to map a smaller circle onto a larger one using a dilation. The circles are given. The lines suggest how to find the center for the dilation. Describe how the center is found. Use segments in the diagram to name the scale factor.



2. Here is another pair of noncongruent circles with no common interior point. From Exercise 1, you know you can locate a point off to the left of the smaller circle that is the center for a dilation mapping $\odot C$ onto $\odot C'$. Find another center for another dilation that maps $\odot C$ onto $\odot C'$. Mark and label segments to name the scale factor.