Lesson 3-5

Example 1

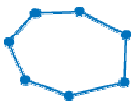
a. Draw the next figure in this pattern.

b. Describe the fifteenth figure in the pattern.

Solution

Each figure in the pattern consists of a number of segments that intersect only at their endpoints. It is important to note that, in each figure, no three of the endpoints are collinear.
The 1st figure consists of 3 segments.
The 2nd figure consists of 4 segments.
The 3rd figure consists of 5 segments.
The 4th figure consists of 6 segments.

a. The 5th figure consists of 7 segments, as shown.



b. From the discussion above, you see that the *n*th figure consists of $(n + 2)$ segments. So, the 15th figure will consist of $(15 + 2)$ segments or 17 segments. The 17 segments intersect only at their endpoints, and no three of the endpoints are collinear.

Example 2

CITY PLANNING A traffic engineer is using line segments determined by 12 collinear points to represent the stops on a bus route. How many different segments of the route are there?

Solution

Solving the problem directly would involve identifying and counting all the segments in a figure such as this.

The process would be time-consuming, and it would be very easy to make a mistake. As an alternative, count the number of segments formed in a sequence of simpler cases and try to find a pattern in the results. Then use inductive reasoning to make a conjecture about the given problem. Organize the date into a table like this.

Number of Collinear Points		Number of Segments Formed	
	2	$\overline{AB}$	1
	3	$\overline{AB}, \overline{AC}, \overline{BC}$	3
	4	$\overline{AB}, \overline{AC}, \overline{AD}, \overline{BC}, \overline{BD}, \overline{CD}$	6
	5	$\overline{AB}, \overline{AC}, \overline{AD}, \overline{AE}, \overline{BC}, \overline{BD}, \overline{BE}, \overline{CD}, \overline{CE}, \overline{DE}$	10
	6	$\overline{AB}, \overline{AC}, \overline{AD}, \overline{AE}, \overline{AF}, \overline{BC}, \overline{BD}, \overline{BE}, \overline{BF}, \overline{CD}, \overline{CE}, \overline{CF}, \overline{DE}, \overline{DF}, \overline{EF}$	15

The numbers in the right-hand column of this table form the pattern of add 2, add 3, add 4, add 5 and so on. So, extend the sequence until the number of points is ten.

Number of collinear points	2	3	4	5	6	7	8	9	10	11	12
Number of segments formed	1	3	6	10	15	21	28	36	45	55	66
		+ 2	+ 3	+ 4	+ 5	+ 6	+ 7	+ 8	+ 9	+ 10	+ 11

Using this table, you can now make the following conjecture: The number of segments determined by twelve collinear points is 66.