Lesson 11-5

Example 1

Find factors for $6a^3 + 3a^2b^2 + 2ab + b^3$.

Solution

1. Check for a monomial factor for the whole expression. There is none.

2. Within the polynomial, make pairs of terms that share monomial factors.

   $(6a^3 + 3a^2b^2) + (2ab + b^3)$ or $(6a^3 + 2ab) + (3a^2b^2 + b^3)$

3. Extract the monomial factors in each pair.

   $3a^2(2a + b^2) + b(2a + b^2)$ or $2a(3a^2 + b) + b^2(3a^2 + b)$

4. The binomials left in each pair are identical, so they are a factor of the whole polynomial. The binomial can be extracted; the monomials create a second factor as follows.

   $(2a + b^2)(3a^2 + b)$ or $(3a^2 + b)(2a + b^2)$

Example 2

MANUFACTURING The volume of a box is $2wxz - 8vxz + 3wyz - 12vyz$. Find the dimensions of the box. (Hint: Volume is the product of three factors).

Solution

Check for a monomial factor for the whole expression. The constant z can be extracted: $z(2wx - 8vx + 3wy - 12vy)$.

$z[(2wx - 8vx) + (3wy - 12vy)]$
$= z[2x(w - 4v) + 3y(w - 4v)]$
$= z(w - 4v)(2x + 3y)$

or

$z[(2wx + 3wy) - (8vx + 12vy)]$
$= z[w(2x + 3y) - 4v(2x + 3y)]$
$= z(2x + 3y)(w - 4v)$

Example 3

Find factors for $m^4 + m^3n - 4mn^2 - 4n^3 + mp^3 + np^3$.

Solution

There is no shared monomial factor. Pair terms in the remaining polynomial, and factor if possible.

$$(m^4 + m^3n) - (4mn^2 + 4n^3) + (mp^3 + np^3) = m^3(m + n) - 4n^2(m + n) + p^3(m + n)$$
$$= (m + n)(m^3 - 4n^2 + p^3)$$