5

Measures of Variability

Humans seem to have a built-in need for competition. The legendary Green Bay Packers coach Vince Lombardi once said, "Winning isn't everything; it's the only thing." Many people agree wholeheartedly with his statement, and even the most noncompetitive find themselves acceding to it in at least some circumstances. We compete in many aspects of our lives: in business to earn a living, in research to make the first discovery, in school to get the highest grades, in shopping to find the shortest line, in sports to have fun.

Even though humans spend much of their lives competing, it is not clear that competition always brings out the best in people. Indeed, most of the psychological literature on games stresses the need for cooperation rather than competition to improve performance (Buskist & Morgan, 1988). Still, there is something about competition that incites some people to new heights of performance, particularly in sports. What makes competition so compelling is that even heavily favored people or teams often lose. They lose because people's behavior, whether academic, athletic, or cooperative team behavior, is not always the same; it varies. With this in mind, see if you can answer the following question.

Ami Chin, with a diving average of 9.5, and Jamie Fowler, who averages 9.3, are vying for the world diving championship. Who will win? An academic college bowl team that averages 150 points per game is competing against a team that averages 140 points. Which team do you think will win? The water polo teams from Cucamonga College and Buckaroo University both average 6 points a match. Do you think that the match will end in a 6-6 tie? To predict the outcomes of these matches, it is certainly helpful to know the competitors' averages. However, just because Ami averages 9.5 does not mean that she always gets exactly 9.5. Just because a team averages 6 points per match does not mean that they score six goals every time they take to the pool. People aren't perfect, and they certainly aren't perfectly consistent. Some, however, are more consistent than others. For example, let's look at the Cucamonga and Buckaroo water polo teams.

Knowing the average scores of the Cucamonga and Buckaroo water polo teams leads you to assume that the game will be a close one, but look at their scores in Table 5.1. Cucamonga College unfailingly scores just about six goals in every contest, whereas Buckaroo University scores either a lot of goals or none at all, and it has never scored exactly six. Knowing the point spread for each team, now what do you think about the possibility of a 6-6 tie?

There are statistical tools that can be used to measure the spread of scores in such distributions as the water polo teams' performances; they are called *measures of variability.* When used in conjunction with measures of central tendency, they disclose additional information about any distribution.

Measures of central tendency yield a single average score that represents the center of the distribution. **Measures of variability** give an idea of how much scores in the distribution vary from that one average score. For instance, by knowing that Ami Chin averages 9.5 on her dives, you know that she is an overall better diver than Jamie Fowler, who averages 9.3. Also, if each diver's scores almost never vary by more than 0.1 point, you can safely assume that Chin will have a better chance of winning the diving championship. Thus, measures of variability can reveal the consistency or similarity of the scores in a distribution. They can

Table 5.1 Goals Scored in the Past 10 Games by the Cucamonga College and Buckaroo University Water Polo Teams

Game	Cucamonga College (X_1)	Buckaroo University (X_2)
1	6	0
2	5	11
3	6	13
4	7	0
5	6	12
6	6	0
7	6	0
8	8	14
9	5	10
10	5	0
	$\Sigma X_1 = 60$	$\Sigma X_2 = 60$
	$\overline{X}_1 = 6$	$\overline{X}_2 = 6$

also indicate how much the average score truly represents all the scores in the distribution. If there is a large spread among the scores, as with Buckaroo's water polo scores, there is quite a different picture of the team's performance than there would be if the scores clustered around the average, as do Cucamonga's. In some cases, especially cases that involve planning for the future, it is more important to know the spread of the distribution than the center of the distribution. Examples are determining peak electrical loads for power companies, designing treatments for patients with mental disorders, and staffing for colleges and universities.

In this chapter, four basic measures of variability are discussed: the range, the average mean deviation, the variance, and the standard deviation. The range, though by far the easiest to compute, is quite unstable because one extreme score can have a radical effect on it. Therefore, it is seldomly used as a measure of variability. By comparison, the variance and the standard deviation, though more complicated than the range, are less affected by extreme scores and are much more widely used.

The Range

The **range** is a measure of the full extent of the scores in a distribution, from the highest to the lowest. It is computed by subtracting the low score from the high score.

$$\text{Range} = \text{high score} - \text{low score} \tag{5.1}$$

The range is extremely easy to compute. Quite often, in fact, you can compute it in your head. For example, look at the data in Table 5.1 and calculate the range for the Cucamonga and Buckaroo water polo teams. If you need help, use Formula 5.1:

$$\text{Range}_{\text{Cucamonga}} = 8 - 5 = 3$$

$$\text{Range}_{\text{Buckaroo}} = 14 - 0 = 14$$

Even if you didn't have access to the actual scores, knowing the range would tell you that Cucamonga's team must be fairly consistent because all its final scores vary within only 3 points of one another. Buckaroo's team is much more unpredictable because its scores are highly scattered and vary quite a bit. However, what if Cucamonga played the worst team in the league, and that team had an exceptionally bad day, resulting in Cucamonga scoring 19 points on that day? Then its distribution of points would vary from a low of 5 to a high of 19. All the other scores in the distribution are the same as before, but note that Cucamonga's range has increased from 3 to 14, which is the same as Buckaroo's. If you were now shown the means and the range for each of the teams, you would assume that their performances were quite similar. It is clear that even though the range is easy to compute, it is severely affected by extreme scores; even one score can alter the range to a large degree. Thus, although the range is sometimes used to gain a quick and easy picture of the data, it is not used as a reliable measure of variability.

Question: Is there some measure of variability that takes into account each of the scores so that one extreme score doesn't have so much influence?

Yes, the other measures of variability do just this: They incorporate all the scores in their calculations. Using the mean as a sort of reference point, they determine how much the other scores differ from the mean.

Mean Deviation

Average Mean Deviation: It All Adds Up to Nothing

The **average mean deviation (*AMD*)** is the average deviation of each score from the mean of the distribution. To compute the average mean deviation, first find the mean, then subtract the mean from each score. This will give you the **deviation scores,** or the mean deviations, which are represented by the symbol x (we call them "little xs"). Next, sum the deviation scores (Σx) and divide that by the total number of scores (n). Thus, you compute the average mean deviation as you would any average, but instead of computing an average of several scores, you are computing an average of the deviations from the mean. Here is the formula:

$$\text{Average mean deviation } (AMD) = \frac{\Sigma(X - \overline{X})}{n} = \frac{\Sigma x}{n} \qquad (5.2)$$

To illustrate the calculation of the average mean deviation, let's pretend you are conducting a preliminary study on conformity. In conformity experiments, it is customary to place participants in situations where a group of confederates (people hired by the experimenter) try to convince the participant to give a response that he or she knows is wrong or false. Suppose that there are five groups, each group consisting of one participant and three confederates. The task of each participant is to determine which of three lines displayed on a computer screen is the same length as a line previously displayed. Everyone in the group sees the same computer screen, and the experimenter asks each person in turn which of the lines on the screen matches the line previously displayed. The confederates are always asked their opinion before the participant, and the confederates always agree on the same line. During eight of the trials, the critical trials, the confederates name the wrong line as matching the previous line. The dependent variable in this research project is the number of conforming trials: the number of times the participant agrees with the confederates on the critical trials. Table 5.2 lists the number of conforming trials for each of the five participants in the study.

Table 5.2 Computation of the Deviation Scores and Average Mean Deviation for the Study on Conformity

X	$X - \bar{X}$	x
1	1 − 3	−2
2	2 − 3	−1
3	3 − 3	0
4	4 − 3	1
5	5 − 3	2
$\Sigma X = 15$		$\Sigma x = 0$
$\bar{X} = 3$		

$$\text{Average mean deviation} = \frac{\Sigma x}{n} = \frac{0}{5} = 0$$

As you can see, the average mean deviation is equal to zero. Compare this to the average mean deviation for the two water polo teams, which is computed in Table 5.3. Do you notice the similarity? These are not special cases. *The average mean deviation is always equal to zero.* Because you are computing the average

Table 5.3 Computation of the Deviation Scores and the Average Mean Deviation for the Cucamonga College and Buckaroo University Water Polo Teams

	Cucamonga College			Buckaroo University		
Game	X_1	$X_1 - \bar{X}_1$	x_1	X_2	$X_2 - \bar{X}_2$	x_2
1	6	6 − 6	0	0	0 − 6	−6
2	5	5 − 6	−1	11	11 − 6	5
3	6	6 − 6	0	13	13 − 6	7
4	7	7 − 6	1	0	0 − 6	−6
5	6	6 − 6	0	12	12 − 6	6
6	6	6 − 6	0	0	0 − 6	−6
7	6	6 − 6	0	0	0 − 6	−6
8	8	8 − 6	2	14	14 − 6	8
9	5	5 − 6	−1	10	10 − 6	4
10	5	5 − 6	−1	0	0 − 6	−6
	$\Sigma X_1 = 60$		$\Sigma x_1 = 0$	$\Sigma X_2 = 60$		$\Sigma x_2 = 0$
	$\bar{X}_1 = 6$			$\bar{X}_2 = 6$		
	$AMD = \frac{\Sigma x_1}{n_2} = \frac{0}{5} = 0$			$AMD = \frac{\Sigma x_2}{n_2} = \frac{0}{5} = 0$		

deviation, about half the deviations are positive. These are counterbalanced by the other half, which are negative. When added together, they equal zero.

The basic idea of finding the average amount each score deviates from the mean is quite sound. However, when the end result always equals zero, the procedure is worthless as a measure of variability because it does not allow for any meaningful comparisons between different distributions. If all distributions have the same average mean deviation, then another way must be found to compute the variability of scores within a distribution. The way out of this dilemma is to eliminate the minus signs of the deviation scores, but how can you do this? One way is to use absolute value: Sum the absolute values of the mean deviations, then divide by the total number of scores. Although this method does get rid of the minus signs, this absolute mean deviation method is not as useful as the method that is discussed after the Concept Quiz. After completing the Concept Quiz, try to think of another way to get rid of those minus signs in front of half the deviation scores.

Concept Quiz

Personal injury damage awards are made to help alleviate people's pain and suffering when they are harmed in some way. Among the findings of Marti and Wissler's (2000) study was the discovery that the amount of money people are awarded could be influenced by the presence and size of the minimum and maximum award people thought was reasonable. In this study, mock jurors' perception of the severity of the injury did not differ, but the size of the award varied depending on the boundaries of the request. Awards increased as the requested amounts increased and decreased when the requests were extreme.

1. The distribution of all monetary awards from the lowest award to the highest award is called the ________________.
2. The problem with using the range as a measure of variability is that it can be unduly influenced by ________________ scores.
3. If the lowest award was $5,000 and the highest award was $90,000, the range is ________________.
4. Subtracting the mean from each score creates ________________ scores.
5. The average mean deviation for any distribution is equal to ________________.

Answers

1. range
2. extreme
3. $85,000
4. deviation
5. zero

The Variance: The Mean of the Squared Deviations

Question: How about squaring the deviation scores? When you square negative numbers, the results are positive.

This is exactly what you do when you compute the variance and the standard deviation. Both measures take advantage of the fact that whenever any number, positive or negative, is squared, the resulting value is positive. The variance, in fact, is simply the mean of the squared deviations. The variance of the entire population is represented by σ^2 (lowercase Greek letter sigma squared), and the variance of a sample is represented by S^2.

The formula for the population variance is shown in Formula 5.3.

$$\text{Variance of a population} = \sigma^2 = \frac{\Sigma(X - \mu)^2}{N} = \frac{\Sigma x^2}{N} \qquad (5.3)$$

The **variance** of a population is the actual computed variance of the population, whereas the sample variance is an estimate of the population variance using only the values in the sample. Because samples rarely contain all the extreme scores of a population, their variances are generally smaller than those of the population. In other words, the scores in the sample are likely to be more closely packed around the mean. Therefore, the variance of a sample cannot be considered a good estimate of the variance of the population unless there is some kind of correction. The usual correction for this underestimation is to divide by $n - 1$ rather than n. The effect of this change from n to $n - 1$ is to increase the size of the variance. The amount of this increase varies depending on the size of n. If n is small, subtracting 1 may make a large difference in the computed value of the variance; if n is large, subtracting 1 may make no noticeable difference. For example, if $n = 5$, then dividing by 4 $(n - 1)$, rather than 5 (n) will create a 25% increase in the size of the variance. On the other hand, if $n = 100$, then dividing by 99 $(n - 1)$ rather than by 100 (n) will create only a 1% increase. Because it is rare to have access to scores for an entire population, the variance of the sample is more widely used than the variance of the population. Thus, when the term *variance* is used in the rest of this chapter, it means the variance of the sample, as in Formula 5.4. The formula and the procedure for computing the variance of a sample are shown here.

$$\text{Variance of a sample} = S^2 = \frac{\Sigma(X - \overline{X})^2}{n - 1} = \frac{\Sigma x^2}{n - 1} \qquad (5.4)$$

In Table 5.4, the variance is computed from the distribution of scores from the conformity study. In Table 5.5, the variance for the water polo data is calculated.

Table 5.4 Computation of the Variance in the Study on Conformity

Participant	X	$X - \overline{X}$	x	x^2
A	1	$1 - 3$	-2	4
B	2	$2 - 3$	-1	1
C	3	$3 - 3$	0	0
D	4	$4 - 3$	1	1
E	5	$5 - 3$	2	4
	$\Sigma X = 15$			$\Sigma x^2 = 10$
	$\overline{X} = 3$			

$$S^2 = \frac{\Sigma x^2}{n - 1} = \frac{10}{5 - 1} = \frac{10}{4} = 2.5$$

Although you may not have noticed, there is a problem with the variance: It is not in the same units as the original scores. Because all the deviations are squared, the variance is in squared units; this fact can be quite misleading when you try to relate the variance to the raw scores. The solution to this dilemma is quite simple: Compute the square root of the variance. The result of this is known as the standard deviation.

Standard Deviation: The Square Root of the Variance

The **standard deviation** is the square root of the variance, and it is represented by either σ or S. Thus, the standard deviation is in the same units as the original scores. The formula for the standard deviation of a population is

$$\sigma = \sqrt{\frac{\Sigma(X - \mu)^2}{N}} = \sqrt{\frac{\Sigma x^2}{N}} \quad \text{or} \quad \sigma = \sqrt{\sigma^2} \tag{5.5}$$

The formula for the standard deviation of a sample is

$$S = \sqrt{\frac{\Sigma(X - \overline{X})^2}{n - 1}} = \sqrt{\frac{\Sigma x^2}{n - 1}} \quad \text{or} \quad S = \sqrt{S^2} \tag{5.6}$$

To find the standard deviation of the conformity distribution from Table 5.4, you merely calculate the square root of the variance:

$$S = \sqrt{S^2} = \sqrt{2.5} = 1.581$$

Table 5.5 Computation of the Variance and the Standard Deviation for the Cucamonga College and Buckaroo University Water Polo Teams

	Cucamonga College			Buckaroo University		
Game	X_1	x_1	x_1^2	X_2	x_2	x_2^2
1	6	0	0	0	−6	36
2	5	−1	1	11	5	25
3	6	0	0	13	7	49
4	7	1	1	0	−6	36
5	6	0	0	12	6	36
6	6	0	0	0	−6	36
7	6	0	0	0	−6	36
8	8	2	4	14	8	64
9	5	−1	1	10	4	16
10	5	−1	1	0	−6	36

Cucamonga College:

$\Sigma X_1 = 60$ $\quad \Sigma x_1^2 = 8$

$\overline{X}_1 = 6$

$$S_1^2 = \frac{\Sigma x_1^2}{n_1 - 1} = \frac{8}{10 - 1} = \frac{8}{9} = .889$$

$$S_1 = \sqrt{S_1^2} = \sqrt{.889} = .943$$

Buckaroo University:

$\Sigma X_2 = 60$ $\quad \Sigma x_2^2 = 370$

$\overline{X}_2 = 6$

$$S_2^2 = \frac{\Sigma x_2^2}{n_2 - 1} = \frac{370}{10 - 1} = \frac{370}{9} = 41.111$$

$$S_2 = \sqrt{S_2^2} = \sqrt{41.111} = 6.412$$

Table 5.5 shows the calculation of the standard deviation for the water polo scores.

> **Question:** I still don't really understand the variance and the standard deviation. Without using formulas, can you tell me what they are?

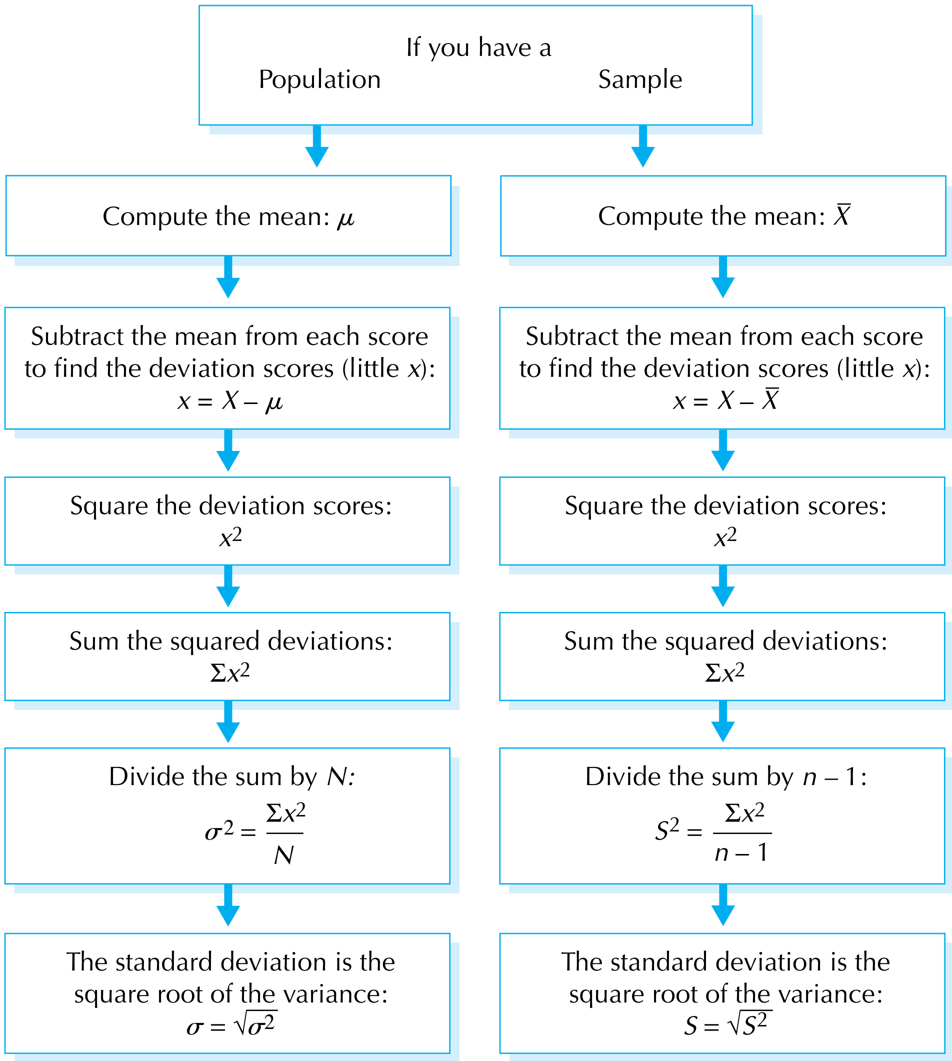
Both the standard deviation and the variance are essentially averages. The variance is the average of the squared deviations from the mean. Because the standard deviation is the square root of the variance, it represents an average measure of the amount each score deviates from the mean.

> **Question:** Computing the variance and standard deviation is so complicated and time consuming! Isn't there an easier way to calculate them?

There are several alternative methods for computing the variance and standard deviation. One of these is to use computational formulas rather than the deviation score formulas we gave earlier. These are discussed in the next section.

VISUAL SUMMARY

Computing the Variance and Standard Deviation



Computational Formulas

Computational formulas are designed to take advantage of the features of pocket calculators to make calculations faster. However, computational formulas can be dreadfully tedious when done with paper and pencil because of the large numbers involved in the computations. Therefore, the answer to the preceding question is yes, there are formulas that can make your life easier, *if* you have an electronic calculator. The following are the computational formulas for the variance and standard deviation.

$$\text{Variance} = S^2 = \frac{\Sigma X^2 - \frac{(\Sigma X)^2}{n}}{n - 1} \tag{5.7}$$

$$\text{Standard deviation} = S = \sqrt{\frac{\Sigma X^2 - \frac{(\Sigma X)^2}{n}}{n - 1}} \tag{5.8}$$

These formulas introduce a new term: ΣX^2. This term is read as "the sum of the X squares," and it equals $X_1^2 + X_2^2 + X_3^2 + \cdots + X_n^2$. It means that you should square each original score and then compute the sum of these squared scores. (Remember that this is the *sum of the X squares,* not the sum of the "little *x*" squares. "Little *x*" squares are deviation scores; X squares are the squares of the original scores.) In Table 5.6, the variance and the standard deviation for the scores from the conformity study are computed using Computational Formula 5.7.

Table 5.7 illustrates how the computational formulas can be used to calculate the variance and standard deviation of the water polo data for Cucamonga College and Buckaroo University.

The easiest alternative to computing any statistic is to use a computer program or a calculator that will compute the values for you. However, before you use computer programs or calculators to compute the variance or standard deviation, make sure you know which formulas the machine uses. The formulas shown here

Table 5.6 Computation of the Variance Using the Computational Formula for the Study on Conformity

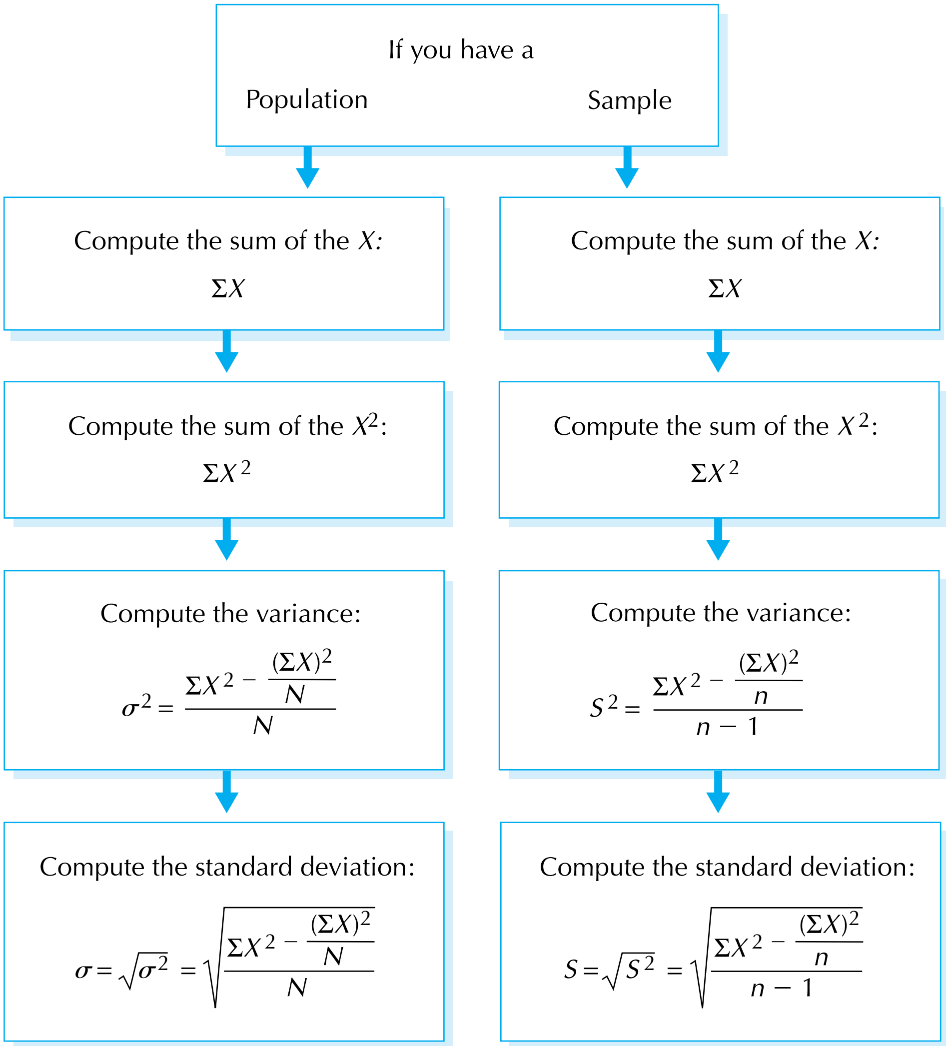
Participant	X	X^2
A	1	1
B	2	4
C	3	9
D	4	16
E	5	25
	$\Sigma X = 15$	$\Sigma X^2 = 55$

$$S^2 = \frac{\Sigma X^2 - \frac{(\Sigma X)^2}{n}}{n - 1} = \frac{55 - \frac{(15)^2}{5}}{5 - 1} = \frac{55 - \frac{225}{5}}{4} = \frac{55 - 45}{4} = \frac{10}{4} = 2.5$$

$$S = \sqrt{S^2} = \sqrt{2.5} = 1.581$$

VISUAL SUMMARY

Computational Formulas for the Variance and Standard Deviation



for the population variance and the sample variance are different. The population variance and standard deviation use only N in the denominator, whereas the sample standard deviation uses $n - 1$. Because there are two similar but very different formulas, most calculators are designed to display both a population value and a sample value. You will need to read the instruction manuals that come with your calculator or computer to determine which value is the one you want.

Table 5.7 Computation of the Variance and the Standard Deviation for the Cucamonga College and Buckaroo University Water Polo Teams

	Cucamonga College		Buckaroo University	
Game	X_1	X_1^2	X_2	X_2^2
1	6	36	0	0
2	5	25	11	121
3	6	36	13	169
4	7	49	0	0
5	6	36	12	144
6	6	36	0	0
7	6	36	0	0
8	8	64	14	196
9	5	25	10	100
10	5	25	0	0
	$\Sigma X_1 = 60$	$\Sigma X_1^2 = 368$	$\Sigma X_2 = 60$	$\Sigma X_2^2 = 730$

$$S_1^2 = \frac{638 - \frac{60^2}{10}}{10 - 1} = \frac{368 - 360}{9} = \frac{8}{9} = .889$$

$$S_1 = \sqrt{S_1^2} = \sqrt{.889} = .943$$

$$S_2^2 = S_1^2 = \frac{730 - \frac{60^2}{10}}{10 - 1} = \frac{730 - 360}{9} = \frac{370}{9} = 41.111$$

$$S_2 = \sqrt{S_2^2} = \sqrt{41.111} = 6.412$$

Concept Quiz

Suppose a group of personal injury attorneys hired you to conduct a study similar to the study done by Marti and Wissler (2000). In your study, you ask mock jurors to read a scenario involving a person seriously injured on a construction job and to award monetary damages between a minimum of $1,000 and a maximum of $100,000.

1. The variance in monetary awards is simply the _________________ of the squared deviations from the sample mean.
2. If the variance is $300,000, the standard deviation is the _________________ of the variance, or $_________________.
3. Suppose the following were the monetary awards separated for mock jurors in three different age groups. Which age group had the greatest variance?
   a. Senior jurors: $20,000, $30,000, $40,000, $50,000, $60,000
   b. Middle-age jurors: $20,000, $20,000, $20,000, $30,000, $60,000
   c. Young jurors: $20,000, $60,000, $60,000, $60,000, $60,000

4. Why is the standard deviation typically preferred over the variance when describing the variability of a distribution of scores?
5. If Billy Bob was injured on the job and had a choice of juries, which would you recommend if you knew the means and standard deviations of the monetary awards of Jury A and Jury B in a similar injury case? Why?
   a. Jury A: $\overline{X} = \$70{,}000$, $s = \$1{,}550$
   b. Jury B: $\overline{X} = \$70{,}000$, $s = \$18{,}000$
6. What is the difference between the computation of the value ΣX^2 and the value $(\Sigma X)^2$?

Answers

1. mean
2. square root; $547.72
3. c; The variance equals 17,888.544 (the variance of a equals 15,811.368; the variance of b equals 17,320.508).
4. The standard deviation is in the same units as the scores in the original distribution, whereas the variance is in squared units.
5. Jury A, because there is more consistency in their award
6. To compute the ΣX^2, you first square each score in the distribution then add the squared scores. To compute $(\Sigma X)^2$, you first add all the scores in the distribution and then square that sum.

Question: Is it possible to compute the variance and standard deviation from a grouped frequency distribution?

It is always preferable to use raw data to compute means, variances, and standard deviations. Nevertheless, it is possible to compute these values from a grouped frequency distribution if the raw data are not available. You will look at that procedure next.

Calculating Variability From Grouped Frequency Distributions

The procedure for computing the variance and standard deviation from a grouped frequency distribution is similar to that used to compute the mean from a grouped frequency distribution. In that procedure, you assume that the scores within each class interval are represented by the midpoint of that interval and use the following formula:

$$S^2 = \frac{\Sigma\left(F \cdot x^2\right)}{n - 1} \qquad (5.9)$$

where

F = the frequency in the class interval

x^2 = the squared deviation for the class interval (to compute this you subtract the mean from the midpoint of the class interval and then square the result)

n = the total number of scores in the entire frequency distribution

Because the standard deviation is the square root of the variance, the formula for the standard deviation of a grouped frequency distribution is

$$S = \sqrt{S^2} = \sqrt{\frac{\Sigma(F \cdot x^2)}{n - 1}} \qquad (5.10)$$

In Table 5.8, the variance and standard deviation from a simplified grouped frequency distribution have been computed using Formulas 5.9 and 5.10. Table 5.9 uses these formulas to compute the variance and standard deviation from the frequency distribution of the highway divider line estimates discussed in Chapter 4.

Question: It's so easy to get caught up in the mathematical formulas and calculations for the variance and standard deviation that you lose sight of what they are all about. Can you explain why we would use them without using mathematical jargon?

In plain terms, the variance and the standard deviation are statistics that measure how much the scores in the distribution deviate from the mean, and they

Table 5.8 Computation of the Variance and Standard Deviation From a Simplified Grouped Frequency Distribution

Apparent Limits	Frequency (F)	Midpoint	$F \cdot$ Midpoint	x	x^2	$F \cdot x^2$
12–14	1	13	13	6	36	36
9–11	2	10	20	3	9	18
6–8	4	7	28	0	0	0
3–5	2	4	8	−3	9	18
0–2	1	1	1	−6	36	36
	$n = 10$		$\Sigma(F \cdot \text{midpoint}) = 70$			$\Sigma(F \cdot x^2) = 108$

$$\overline{X} = \frac{70}{10} = 7$$

$$S^2 = \frac{\Sigma(F \cdot x^2)}{n - 1} = \frac{108}{10 - 1} = \frac{108}{9} = 12$$

$$S = \sqrt{S^2} = \sqrt{\frac{\Sigma(F \cdot x^2)}{n - 1}} = \sqrt{12} = 3.464$$

Table 5.9 Computation of the Variance and Standard Deviation From Estimated Line Lengths of Highway Divider Lines (in centimeters)

Apparent Limits	Frequency (F)	Midpoint	$F \cdot$ Midpoint	x	x^2	$F \cdot x^2$
300–324	10	312	3,120	115.275	13,288.326	132,883.26
275–299	25	287	7,175	90.275	8,149.576	203,739.40
250–274	69	262	18,078	65.275	4,260.826	293,996.99
225–249	146	237	34,602	40.275	1,622.076	236,823.10
200–224	247	212	52,364	15.275	233.326	57,631.52
175–199	206	187	38,522	−9.725	94.576	19,482.66
150–174	147	162	23,814	−34.725	1,205.826	177,256.42
125–149	104	137	14,248	−59.725	3,567.076	370,975.90
100–124	32	112	3,584	−84.725	7,178.326	229,706.43
75–99	14	87	1,218	−109.725	12,039.576	168,554.06

$n = 1{,}000$ $\quad\quad \Sigma(F \cdot \text{midpoint}) = 196{,}725$ $\quad\quad \Sigma(F \cdot x^2) = 1{,}891{,}049.74$

$$\bar{X} = \frac{196{,}725}{1{,}000} = 196.725$$

$$S^2 = \frac{\Sigma(F \cdot x^2)}{n - 1} = \frac{1{,}891{,}049.74}{1{,}000 - 1} = \frac{1{,}891{,}049.74}{999} = 1{,}892.942$$

$$S = \sqrt{S^2} = \sqrt{\frac{\Sigma(F \cdot x^2)}{n - 1}} = \sqrt{1{,}892.942} = 43.508$$

enable you to determine the extremes of the distribution. They indicate whether the scores are clustered close to the mean, as were the Cucamonga water polo team's, or spread out far from the mean, as were Buckaroo's. As standards, the variance and the standard deviation can be readily used to compare the variability of different distributions. For example, you could use them to compare the variability of a conformity study involving a heterogeneous sample of people with a separate study involving a sample of high-anxiety people.

The difference between the variance and the standard deviation is that the latter is the square root of the former. This difference is quite helpful when you are discussing the characteristics of various distributions because the standard deviation is in the same units as the original scores, whereas the variance is in squared units. For instance, suppose you were scouting out information to help you predict the outcome of the water polo match and someone told you that the variance of Buckaroo's scores was 41. This wouldn't help you much, especially in light of the fact that Buckaroo's scores range from 0 to 14. However, if you were told that the standard deviation was about six and a half compared to a standard deviation of close to one for Cucamonga, it would be easy to compare the two distributions, and you would almost certainly not bet on a 6-6 tie.

VISUAL SUMMARY

Finding the Variance and Standard Deviation of a Grouped Frequency Distribution

Before You Begin: Compute the mean of the grouped frequency distribution.

Compute the deviation score for each class interval by subtracting the mean from the midpoint: (x = midpoint − mean)

↓

Square each deviation score: (x^2)

↓

Multiply each squared deviation score by the frequency in that class interval: ($F \cdot x^2$)

↓

Sum these products: $\Sigma(F \cdot x^2)$

↓

Divide the sum by the number of scores minus 1 to compute the variance:

$$S^2 = \frac{\Sigma(F \cdot x^2)}{n - 1}$$

↓

The standard deviation is the square root of the variance:

$$S = \sqrt{S^2} = \sqrt{\frac{\Sigma(F \cdot x^2)}{n - 1}}$$

Concept Quiz

Recall the personal injury awards discussed in the previous Concept Quizzes. The grouped frequency distribution of these data is shown here.

Grouped Frequency Distribution of Monetary Award Data in Dollars

Apparent Limits	Frequency	Midpoint
90,000–99,000	2	94,500
80,000–89,000	5	84,500
70,000–79,000	5	74,500
60,000–69,000	10	64,500
50,000–59,000	12	54,500
40,000–49,000	16	44,500
30,000–39,000	20	34,500
20,000–29,000	25	24,500
10,000–19,000	25	14,500

1. Calculating estimates of variance and standard deviation from a grouped frequency distribution is similar to computing the estimated ________________ from a grouped frequency distribution.
2. The best single score used to represent each class interval in the grouped frequency distribution is the ________________, and it is used to calculate variance and standard deviation.
3. To calculate the variance from the grouped frequency distribution of monetary award, we first must compute the deviation score of each interval by subtracting the ________________ from the midpoint of each class interval. The deviation score for the class interval 60,000–69,000 is ________________.
4. After calculating the deviation scores, ________________ each score and multiply it by the ________________ in that interval.
5. The final step in calculating the variance is ________________ the products of deviation scores times the frequency of the class intervals and dividing by ________________.

Answers

1. mean
2. midpoint
3. mean ($38,833.33); $25,666.67
4. square; frequency
5. summing; $n - 1$

Summary

This chapter has focused on the measures of variation: the range, the average mean deviation, the variance, and the standard deviation. The range is simply the difference between the highest and lowest scores. Although it is easily computed, the range can be overly influenced by extreme scores.

The computation of the other measures of variation begins with the calculation of deviation scores. The variance and standard deviation are the most widely used measures of variability.

The variance is computed by squaring each deviation score, adding up all the squared deviations, and then dividing that sum by $n - 1$. The standard deviation is the square root of the variance. The major advantage of the standard deviation over the variance is that the standard deviation is in the same units as the original scores.

It is also possible to compute the variance and standard deviation from a grouped frequency distribution.

Key Terms

measures of variability
range
average mean deviation
deviation scores
variance
standard deviation

Formulas

$$\text{Range} = \text{high score} - \text{low score} \tag{5.1}$$

$$\text{Average mean deviation } (AMD) = \frac{\Sigma(X - \overline{X})}{n} = \frac{\Sigma x}{n} \tag{5.2}$$

Deviation Formulas

$$\text{Variance of a population} = \sigma^2 = \frac{\Sigma(X - \mu)^2}{N} = \frac{\Sigma x^2}{N} \tag{5.3}$$

$$\text{Variance of a sample} = S^2 = \frac{\Sigma(X - \overline{X})^2}{n - 1} = \frac{\Sigma x^2}{n - 1} \quad (5.4)$$

$$\sigma = \sqrt{\frac{\Sigma(X - \mu)^2}{N}} = \sqrt{\frac{\Sigma x^2}{N}} \text{ or } \sigma = \sqrt{\sigma^2} \quad (5.5)$$

$$S = \sqrt{\frac{\Sigma(X - \overline{X})^2}{n - 1}} = \sqrt{\frac{\Sigma x^2}{n - 1}} \text{ or } S = \sqrt{S^2} \quad (5.6)$$

Computational Formulas

$$\text{Variance} = S^2 = \frac{\Sigma X^2 - \frac{(\Sigma X)^2}{n}}{n - 1} \quad (5.7)$$

$$\text{Standard deviation} = S = \sqrt{\frac{\Sigma X^2 - \frac{(\Sigma X)^2}{n}}{n - 1}} \quad (5.8)$$

Grouped Frequency Distributions

$$S^2 = \frac{\Sigma(F \cdot x^2)}{n - 1} \quad (5.9)$$

$$S = \sqrt{S^2} = \sqrt{\frac{\Sigma(F \cdot x^2)}{n - 1}} \quad (5.10)$$

Problems

1. Find the mean, the variance, and the standard deviation of the following IQ scores:

120	134	74	79	98	88
124	129	106	143	106	92
134	119	111	76	128	96
129	97	85	89	100	92

2. Find the mean, the variance, and the standard deviation of the following welding aptitude test scores:

23	25	27	22	35	45
29	26	33	34	25	27
25	29	33			

3. Find the mean, the variance, and the standard deviation of the following sample of the number of items answered correctly on a memory test:

49	46	43	40	38	37
48	45	43	39	38	37
48	44	42	38	38	36

47	44	41	38	37	34
46	43	41	38	37	33

4. Find the mean, the variance, and the standard deviation of the following sample of the annual salaries of 15 college professors:

55,000	50,000	40,000	30,000	25,000
54,000	49,000	38,000	29,000	24,000
52,000	49,000	33,000	27,000	22,000

5. These 11 scores are the result of a simple reaction time experiment (in milliseconds). Compute the mean, the variance, and the standard deviation.

240	356	277	835	277	354
456	789	923	235	456	

6. The following data represent the number of times that 15 patients of a psychoanalyst used the defense mechanism of regression over the last year of therapy. Compute the mean, the variance, and the standard deviation.

23	7	11	21	6
6	17	45	23	67
25	22	11	56	6

7. A cognitive psychologist interested in short-term working memory has measured the capacity of the short-term memory for 18 students. Compute the mean, the variance, and the standard deviation.

9	3	5	6	7	7	7	6	8
9	10	13	5	6	4	7	7	7

8. A neuropsychologist has been studying the density of neurons in a structure called the hippocampus in the brain of the rat and has found the density of neurons vary from rat to rat. The following data represent the numbers of neurons found in equal-sized tissue samples from the brains of 13 rats. Compute the mean, the variance, and the standard deviation.

88	93	65	77	77	106	123
139	142	190	97	143	88	

9. A drug rehabilitation center has kept records of the number of days that 20 former patients have remained drug free. Compute the mean, the variance, and the standard deviation.

21	35	78	90	78	121	88	17	19	123
45	45	67	123	72	89	78	122	180	87

10. As the pressure to publish increases at colleges and universities, many psychologists meet this pressure by working with coauthors on more and more papers. The following scores are the numbers of authors for all research reports published in the journal *Psychological Science* for the year 1990. Compute the mean, the variance, and the standard deviation.

3	4	2	3	1
2	4	5	4	3
3	2	4	3	1
5	2	1	1	
4	2	5	2	

11. The work performance ratings for 18 employees of a small business are listed here. Compute the mean, the variance, and the standard deviation.

2	4	5	5	6	6	6	6	6
6	7	7	8	9	9	9	9	10

12. A psychologist studying the sensation of touch uses a vibrator to stimulate the nerve endings in the fingers of 11 participants. Because the rate of vibration can be changed, she is able to find the number of vibrations per second to which each participant is most sensitive. Compute the mean, the variance, and the standard deviation.

234	254	266	250	231	245
300	222	250	245	231	

13. Compute the mean, the variance, and the standard deviation of the frequency distribution of the following IQ scores.

Apparent Limits	Frequency	Cumulative Frequency	Midpoint
150–159	3	173	154.5
140–149	5	170	144.5
130–139	9	165	134.5
120–129	18	156	124.5
110–119	26	138	114.5
100–109	36	112	104.5
90–99	34	76	94.5
80–89	23	42	84.5
70–79	13	19	74.5
60–69	5	6	64.5
50–59	1	1	55.5

14. Compute the mean, the variance, and the standard deviation of the group frequency distribution of the following welding aptitude test scores.

Apparent Limits	Frequency	Cumulative Frequency	Midpoint
45–47	3	88	46
42–44	5	85	43
39–41	10	80	40
36–38	14	70	37
33–35	22	56	34
30–32	15	34	31
27–29	10	19	28
24–26	7	9	25
21–23	1	2	22
18–20	1	1	19

15. Compute the mean, the variance, and the standard deviation of the following grouped frequency distribution of the numbers of items answered correctly on a memory test.

Apparent Limits	Frequency
48–49	2
46–47	4
44–45	5
42–43	9
40–41	13
38–39	11
36–37	8
34–35	3
32–33	2
30–31	1

16. Compute the mean, the variance, and the standard deviation of the following grouped frequency distribution of student rankings of student services at a small college.

Apparent Limits	Frequency
90–99	10
80–89	16
70–79	28
60–69	34
50–59	49
40–49	25
30–39	39
20–29	32
10–19	11
0–9	6

17. Compute the mean, the variance, and the standard deviation of the following grouped frequency distribution of professors' annual salaries (in dollars):

Apparent Limits	Frequency
56,000–59,999	2
52,000–55,999	5
48,000–51,999	11
44,000–47,999	14
40,000–43,999	27
36,000–39,999	35
32,000–35,999	29
28,000–31,999	18
24,000–27,999	14
20,000–23,999	6

18. The midterm exam scores for 500 statistics students at a major university are shown here in a grouped frequency distribution. Compute the mean, the variance, and the standard deviation.

Apparent Limits	Frequency
95–99	6
90–94	9
85–89	55
80–84	67
75–79	189
70–74	47
65–69	29
60–64	22
55–59	21
50–54	22
45–49	13
40–44	9
35–39	7
30–34	3
25–29	1

19. The following grouped frequency distribution summarizes the numbers of errors made by 294 new student drivers in their first attempt at a driving simulation test. Compute the mean, the variance, and the standard deviation.

Apparent Limits	Frequency
70–76	12
63–69	23
56–62	24
49–55	43
42–48	56
35–41	54
28–34	44
21–27	23
14–20	12
7–13	3

20. The chairs of 117 psychology departments in the United States were asked how many students were currently employed by their departments as undergraduate research assistants. A grouped frequency distribution of the data is given here. Compute the mean, the variance, and the standard deviation.

Apparent Limits	Frequency
22–23	1
20–21	3
18–19	3
16–17	4
14–15	4
12–13	6
10–11	8
8–9	13
6–7	14
4–5	17
2–3	20
0–1	24

Problems From the Literature

The following problems refer to actual results from research studies that are cited at the end of the chapter. However, the data used to generate the problem sets are hypothetical.

Sometimes college students dislike group projects, especially when they believe the grading is not fair. This may hinder some students from enrolling in courses requiring group work. Hoffman and Rogelberg (2001) examined this issue and randomly assigned students to evaluate 1 of 12 versions of a hypothetical college syllabus from a course that required group projects. They found that students reported the highest intentions to enroll in courses and had the most positive perceptions of the grading procedure when both individual and group performance in the group project were evaluated. Suppose students were asked to evaluate two course syllabi at their college. One course graded group projects using the preferred grading method found in Hoffman and Rogelberg's study, and the other course did not. The evaluations of syllabi were made on a 1 (poor syllabus) to 7 (excellent syllabus) scale. Use the following data to solve Problems 21–25.

Course 1: Preferred Grading Method

3	5	4	6	7	1	4	6	6	5
2	7	4	5	1	6	7	5	4	7
4	6	2	7	3	3	6	5	7	7

Course 2: Nonpreferred Grading Method

1	3	2	5	6	3	4	4	6	1
4	5	3	4	1	1	1	2	4	5
4	5	6	4	3	3	3	5	4	2

21. What is the mean evaluation of the course syllabus for each course?
22. Compute the range of the evaluation scores for each course.
23. Compute the variance of the evaluation scores for each course.
24. Compute the standard deviation of evaluation scores for each course.
25. Evaluate the mean evaluation score relative to the size of the standard deviation.

In close relationships, people may be biased in their perceptions of each other. They may think their partner sees the world as they do. This may be because they assume their partners are similar to them. Researchers Kenny and Acitelli (2001) examined these issues, among others, in their study of a sample of heterosexual dating and married couples. Participants answered many questions, including those relating to feelings of closeness, feelings of caring, enjoyment of sex, and job satisfaction. Among their findings, the researchers discovered evidence for bias particularly when the topic was related to the relationship. Suppose another researcher decides to study perceived similarity among intimate heterosexual couples and collects evaluations of the partners' similarity using a similarity scale. Higher scores indicate more perceived similarity. Use the following data to solve Problems 26–30.

22	31	34	28	47	36	9	18	15
16	25	43	41	39	35	48	30	44

26. What is the mean of the scores?
27. Compute the range of the scores.
28. Compute the variance of the scores.
29. Compute the standard deviation of the scores.
30. Evaluate the mean relative to the size of the standard deviation of the scores.

Recall Benjamin and Bjork's (2000) study discussed in the Problems of Chapter 4. These researchers conducted several experiments examining the effect of recognition time pressure on learning new words using either rote or elaborative rehearsal. They found that time pressure during recognition decreased the accessibility to words learned via elaborative rehearsal compared to those learned via rote rehearsal. The frequency distribution from another researcher's similar study is shown here. Participants learned 35 new words. The total numbers of words recognized under time pressure were measured. Use the frequency data to solve Problems 31 and 32.

31. Find the estimated mean from the grouped frequency data.
32. Use the estimated mean to calculate the estimation of variance from the grouped frequency.

Frequency Distribution of Word Recognition Data

Apparent Limits	Frequency
22–23	18
20–21	4
18–19	3
16–17	6
14–15	5
12–13	5
10–11	5
8–9	5
6–7	7
4–5	6
2–3	7

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