CHAPTER 6

Data Encryption Standard (DES)

(Solution to Odd-Numbered Problems)

Review Questions

- 1. The block size in DES is 64 bits. The cipher key size is 56 bits. The round key size is 48 bits.
- **3.** In the first approach, DES uses 16 mixers and 15 swappers in encryption or decryption algorithm; in the second (alternative approach), DES use 16 mixers and 16 swappers in encryption or decryption algorithm.
- 5. The total number of exclusive-or operations is $16 \times 2 = 32$, because each round uses two exclusive-or operations (one inside the function and one outside of the function).
- 7. The cipher key that is used for DES include the parity bits. To remove the parity bits and create a 56-bit cipher key, a parity drop permutation is needed. Not only does the parity-drop permutation drop the parity bits, it also permutes the rest of the bits.
- 9. Double DES uses two instances of DES ciphers for encryption and two instances of reverse ciphers for decryption. Each instance uses a different key, which means that the size of the key is 112 bits. However, double DES is vulnerable to meet-in-the-middle attack.

Exercises

11.

a.

	Input: 1 1011 1	\rightarrow	3, 11	\rightarrow	Output: 03 (0011)
b.					
	Input: 0 0110 0	\rightarrow	0, 6	\rightarrow	Output: 09 (1001)

c.

Input: $0\ 0000\ 0$ \rightarrow 0,0 \rightarrow Output: $04\ (0100)$

d.

Input: 1 1111 1 \rightarrow 3, 15 \rightarrow Output: 09 (1001)

13. The following table shows the output from all boxes. No pattern can be found:

Input	Box 1	Box 2	Вох 3	Box 4	Box 5	Box 6	Box 7	Box 8
111111	1101	1001	1100	1110	0011	0011	1101	1011

15.

a. The following shows that 2 bits will be changed in the output.

Input: 0 0110 0	\rightarrow	00, 06	\rightarrow	Output: 03 (0011)
Input: 0 0000 0	\rightarrow	00, 00	\rightarrow	Output: 15 (1111)

b. The following shows that 2 bits will be changed in the output.

Input: 1 1001 1
$$\rightarrow$$
 03, 09 \rightarrow Output: 06 (0110)
Input: 1 1111 \rightarrow 03, 15 \rightarrow Output: 09 (1001)

17. The following table shows 32 input pairs and 32 output pairs. The last column is the difference between the outputs.

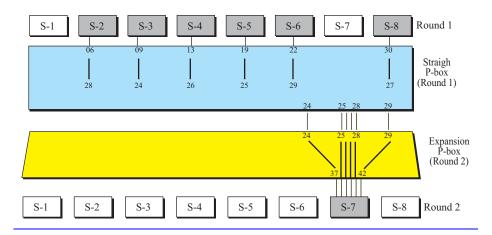
Input	Оитри	Output pairs		
000000	000001	0010	1110	1100
000010	000011	1100	1011	0111
000100	000101	0100	0010	0110
000110	000111	0001	1100	1111
001000	001001	0111	0100	0011
001010	001011	1010	0111	1001
001100	001101	1011	1101	0110
001110	001111	0110	0001	0111
010000	010001	1000	0101	1101
010010	010011	0101	0000	0101
010100	010101	0011	1111	1100
010110	010111	1111	1010	0101

Input	Outpu	Output pairs		
011000	011001	1101	0011	1110
011010	011011	0000	1001	1001
011100	011101	1110	1000	0110
011110	011111	1001	0110	1111
100000	100001	0110	1011	1101
100010	100011	0010	1000	1010
100100	100101	0001	1100	1101
100110	100111	1011	0111	1100
101000	101001	1010	0001	1011
101010	101011	1101	1110	0011
101100	101101	0111	0010	0101
101110	101111	1000	1101	0101
110000	110001	1111	0110	1001
110010	110011	1001	1111	0110
110100	110101	1100	0000	1100
110110	110111	0101	1001	1100
111000	111001	0110	1010	1100
111010	111011	0011	0100	0111
111100	111101	0000	0101	0101
111110	111111	1110	0011	1101

If we sort the table on last column (output differences), we get the following: two (0011)'s, five (0101)'s, four (0110)'s, three (0111)'s, three (0111)'s, one (1010), one (1011), one (1100), five (1100)'s, four (1101)'s, one (1110), and two (1111)'s. **None of the group has a size larger than eight.**

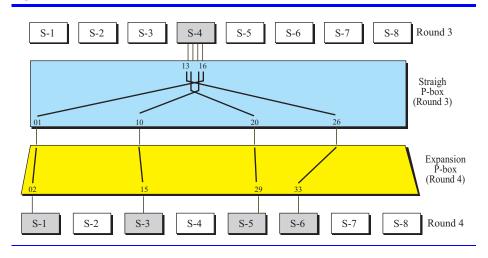
- **19.** Figure S6.19 shows the situation. The inputs to S-box 7 in round 2 comes from six different S-boxes in round 1.
 - **a.** The six inputs to S-box 7 in round 2 come from six outputs (37, 38, 39, 40, 41, 42) of expansion permutation box.
 - **b.** The above six outputs correspond to the six inputs (24, 25, 26, 27, 28, 29) in the expansion permutation box (See Table 6.2 in the textbook).
 - **c.** The above six inputs correspond to the six inputs (09, 19, 13, 30, 06, 22) inputs in the straight permutation box (See Table 6.11 in the textbook).
 - **d.** The above six inputs correspond to the outputs of six different S-Boxes (S-3, S-5, S-4, S-8, S-2, and S-6) in round 1.

Figure S6.19 Solution to Exercise 19



21. Figure S6.21 shows the situation. The outputs from S-box 4 in round 3 go to four S-boxes in round 4.

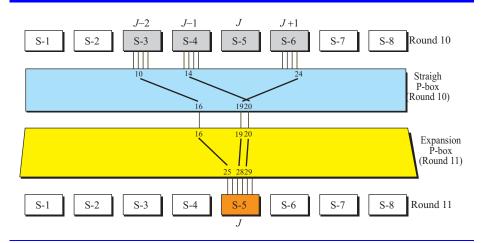
Figure S6.21 Solution to Exercise 21



- **a.** The four outputs from S-box 3 in round 4 go to six outputs (26, 20, 10, 01) of straight permutation box (See Table 6.1).
- **b.** The above four outputs correspond to four outputs (33, 29, 15, 02) in the expansion permutation box (See Table 6.11).
- **c.** The above four inputs corresponds to the inputs of four different S-Boxes (S-6, S-5, S-3, and S-1) in round 4.

23. Figure S6.23 shows the situation. We assume j = 5.

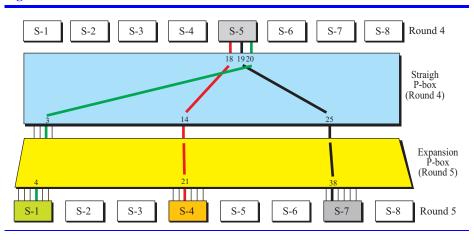
Figure S6.23 Solution to Exercise 23



- **a.** One output from S-box 3 (output 10) goes to the first input of S-box 5 (input 25).
- **b.** An output from S-box 4 (output 14) goes one the last two inputs of S-box 6 (input 29).
- **c.** An output from S-box 6 (output 24) goes to one of the middle input of S-box 5 (input 19).

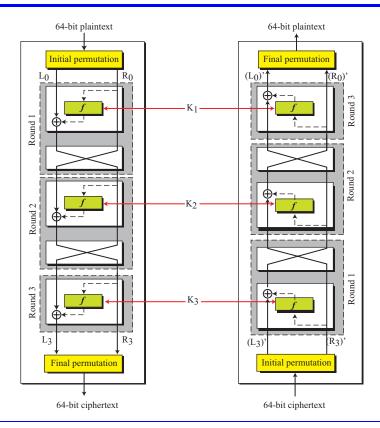
25. Figure S6.25 will help us in this problem.

Figure S6.25 Solution to Exercise 25



- **a.** Only output 19 form S-box 5 (in round 4) goes to S-box 7 (in round 5). However, the input 38 is not a middle input, so the criterion does not apply. This answers the question about this exercise, but we do some more investigations.
- b. Output 18 from S-box 5 goes a middle input (21) in S-box 4 in the next round. To check the criteria, we need to see if any input from S-box 4 goes to a middle input in next round. Looking at Figure S6.24, we can see that this is not the case. The criterion applies here.
- c. We also observe that output 19 from S-box 5 goes a middle input in S-box 1 (input 4). However, none of the inputs from S-box 1 in round 4 goes to a middle input of S-box 5 in the next round. Only one output from S-box 1 (output 2 goes to S-box 5, input 26, but it is not a middle input). The criterion applies here.
- 27. Figure S6.27 shows a three-round cipher. We prove the equalities between the L's and R's from bottom to top. We have labeled each left section in the encryption L_i and in the decryption (L_i) ; we have labeled each right sections R_i in the encrypting and (R_i) in decryption.

Figure S6.27 *Solution to Exercise 27*



a. Since final permutation and initial permutations are inverse of each other (if there is no corruption during transmission), we have

$$(\mathbf{L}_3)' = (\mathbf{L}_3)$$
 $(\mathbf{R}_3)' = (\mathbf{R}_3)$

b. Using the previous equalities and the relations in the mixers, we have

$$(R_2)' = (R_3)' = R_3 = R_2$$
 $(R_2)' = (L_3)' \oplus f[(R_3)', K_3]$ $(R_2)' = L_3 \oplus f[R_3, K_3]$ $(R_2)' = L_2 \oplus f[R_2, K_3] \oplus f[R_2, K_3]$ $(L_2)' = R_2$ $(R_2)' = L_2$

c. Using the previous equalities and the relations in the mixers, we have

$$(L_{1})' = (R_{2})' = L_{2} = R_{1}$$

$$(R_{1})' = R_{2} \oplus f[L_{2}, K_{2}]$$

$$(R_{1})' = R_{2} \oplus f[R_{1}, K_{2}]$$

$$(R_{1})' = L_{1} \oplus f[R_{1}, K_{1}] \oplus f[R_{1}, K_{1}]$$

$$(L_{1})' = R_{1}$$

$$(R_{1})' = L_{1}$$

d. Using the previous equalities and the relations in the mixers, we have

$$(L_0)' = (L_1)' \oplus f[(R_1)', K_1]$$

$$(L_0)' = R_1 \oplus f[L_1, K_1]$$

$$(L_0)' = L_0 \oplus f[L_0, K_1] \oplus f[L_0, K_1]$$

$$(R_0)' = R_0$$

$$(R_0)' = R_0$$

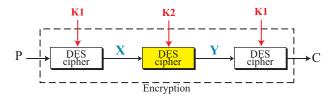
We have proved that $(\mathbf{L_1})' = \mathbf{L_0}$ and $(\mathbf{R_0})' = \mathbf{R_0}$. Since the final permutation and initial permutation are inverse of each other, the plaintext created at the destination is the same as the plaintext started at the source.

29. The following shows the result:

31. The first round key is

33. Figure S6.33 shows the encryption using 3DES with two keys, in which X or Y are the intermediate texts.

Figure S6.33 *Solution to Exercise 33*



Van Oorschot and Wiener have devised a meet-in-the-middle attack on the above configuration. The attack is basically a known-plaintext attack. It follows the steps shown below:

a. Eve intercepts *n* plaintext/ciphertext pairs and stores them in a table which is sorted on values of P as shown below:

Plaintext	Ciphertext
$egin{array}{c} ext{P}_1 \ ext{P}_2 \end{array}$	$egin{array}{c} C_1 \ C_2 \end{array}$
P_n	C_n

Table 1: n P/C pairs

b. Eve now chooses a value for X (see Figure S6.33) and uses the decryption algorithm, and all 2⁵⁶ possible K1's values to create 2⁵⁶ different P values as shown below:

$$P_1 = D(K1_1, X)$$
 $P_2 = D(K1_2, X)$... $P_m = D(K1_m, X)$ where $m = 2^{56}$

c. If a value of P created in step b matches one of the value of P in Table 1, Eve uses the corresponding value for K1 (from the list in step b) and the value of C from Table 1 and calculates a value for second intermediate text Y = D (K1, C). Now Eve creates a second table, Table 2, which is sorted on the value of Y. Eve has now r possible candidate for K1 keys.

Y	K1
\mathbf{Y}_{1}	K1 ₁
Y_2	K1 ₂
\mathbf{Y}_r	 K1 _r

Table 2: r Y/K1 pairs

d. Eve now searches for K2. For each 2^{56} possible values of K2, Eve uses the decryption algorithm and the value of X chosen in step b to create 2^{56} different values for Y's.

```
Y_1 = D(K2_1, X) Y_2 = D(K2_2, X) ... Y_m = D(K2_m, X) where m = 2^{56}
```

If a value of Y_i created in this step matches one of the value in Table 2, Eve have found a pair of keys: K1 is extracted from Table 2 and K2 is extracted from the decryption algorithm that matches the value of Y in Table 2.

e. Now Eve tests pairs of K1/K2 values on more intercepted plaintext/ciphertext. If there is matching, Eve has found the keys; if there is no match, Eve needs to repeat step *b* to *e* using a different value of X.

35.

```
 \begin{aligned} & \textbf{split} \; (n, m, \text{inBlock}[1 \; \dots n], \text{leftBlock}[1 \; \dots m], \text{rightBlock}[1 \; \dots m]) \\ & \{ & i \leftarrow 1 \\ & \textbf{while} \; (i \leq m) \\ & \{ & \text{leftBlock}[i] \leftarrow \text{inBlock}[i] \\ & \text{rightBlock}[i] \leftarrow \text{inBlock}[i + m] \\ & i \leftarrow i + 1 \\ & \} \\ & \textbf{return} \\ & \} \end{aligned}
```

37.

39. We have added one extra parameter ED (encrypt/decrypt). If ED = E, we do encryption; if ED = D, we do decryption.

```
cipher (ED, plainBlock[1...64], RoundKeys[1...16][1...48], cipherBlock[1... m64])
{
         permute (64, 64, plainBlock, inBlock, InitialPermutationTable)
         split (64, 32, inBlock, rightBlock, leftBlock)
         for (round = 1 to 16)
         {
                   if (ED = E)
                             mixer (leftBlock, rightBlock, RoundKey[round]
                   if(ED = D)
                             mixer (leftBlock, rightBlock, RoundKey[16 - round]
                   if (round!= 16)
                             swapper (leftBlock, rightBlock)
          }
         combine (32, 64, leftBlock, rightBlock, outBlock)
         permute (64, 64, outBlock, cipherBlock, FinalPermutationTable)
}
```