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Problem Solving

In grades 3–5, students should investigate numerical and geometric patterns and express them mathematically in words or symbols. They should analyze the structure of the pattern and how it grows or changes, organize this information systemically, and use their analysis to develop generalizations about the mathematical relationships in the pattern.¹

Activity Set 1.1 SEEING AND EXTENDING PATTERNS WITH PATTERN BLOCKS

PURPOSE

To recognize, describe, construct, and extend geometric patterns.

MATERIALS

Pattern blocks and color tiles from the Manipulative Kit or from the Virtual Manipulatives.

INTRODUCTION

In this first activity set colored geometric shapes called pattern blocks will be used to recognize, study, and extend geometric patterns. The set of pattern blocks consists of six different polygons: a green triangle, an orange square, a red trapezoid, a blue rhombus, a tan rhombus, and a yellow hexagon.

Human beings are pattern-seeking creatures. Babies begin life’s journey listening for verbal patterns and looking for visual patterns. Scientists in search of extraterrestrial intelligence send patterned signals into the universe and listen for incoming patterns on radio telescopes. Mathematics is also concerned with patterns. Many mathematicians and educators involved in reforming mathematics teaching and learning at the elementary and middle school levels are suggesting that the notion of mathematics as the study of number and shape needs to be expanded. Some suggest that “mathematics is an exploratory science that seeks to understand every kind of pattern.”²

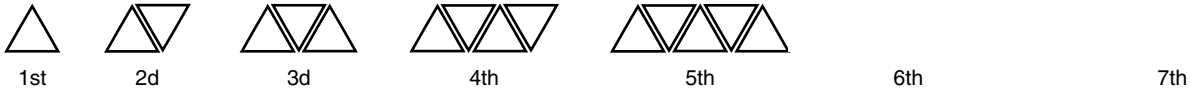
In this set we will look at a variety of sequences. A *sequence* is an ordered set of mathematical objects. There are many possibilities for sequences. A few examples of sequences

¹*Principles and Standards for School Mathematics* (Reston, VA: National Council of Teachers of Mathematics, 2000): 159.

²Lynn A. Steen, *On the Shoulders of Giants: New Approaches to Numeracy* (Washington, DC: National Academy Press, 1990): 1–8.

are a sequence of pattern block figures, a sequence of tile figures, a sequence of letter groupings, a sequence of whole numbers, and a sequence of fractions.

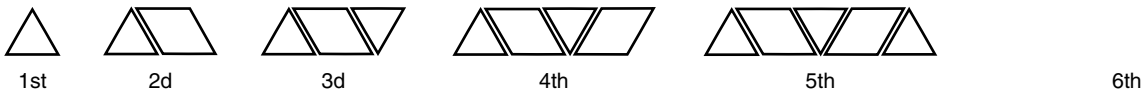
1. The pattern block figures shown here form the first five figures of a sequence. Use your green triangles to construct the sixth and seventh figures that you think extend the given pattern and sketch these figures.



- *a. Describe in writing at least three ways that the seventh figure in your sequence differs from the sixth figure.

- *b. Describe in writing what the 15th figure in this sequence would look like so that someone reading your description, who had not seen this sequence, could build the same figure.

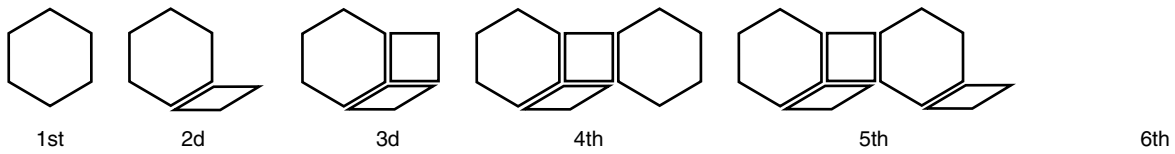
2. Use your pattern blocks to construct the sixth figure of the sequence below and sketch that figure.



- a. Describe in writing how new figures are created as this sequence is extended.
- b. Will the 10th figure in an extended sequence have a green triangle or a blue rhombus on the right end? Explain your reasoning.
- c. How many triangles and how many rhombuses are in the 25th figure of the extended sequence? Explain how you arrived at your answer.
- d. Complete the following statement that will enable readers to determine the number of triangles and rhombuses in any figure they choose.

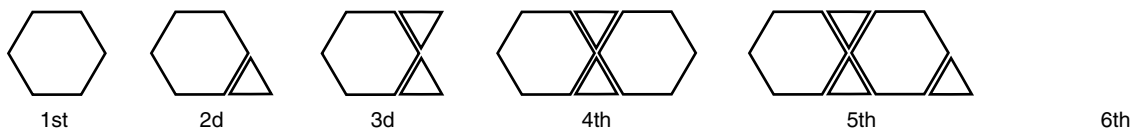
In the n th figure, where n is an even number, there will be $n \div 2$ triangles and $n \div 2$ rhombuses. If n is an odd number, the n th figure will contain _____ triangles and _____ rhombuses.

3. The pattern block sequence started below uses three different types of pattern blocks. Use your pattern blocks to build and sketch the next figure in the extended sequence.



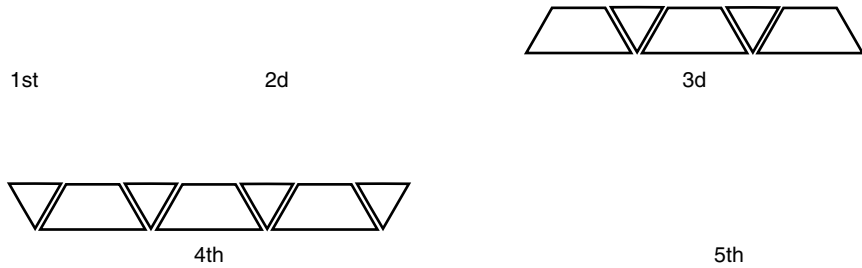
- *a. Describe in writing how new figures are created as this sequence is extended.
- *b. What pattern block will be attached to the right end of the 16th figure to obtain the 17th figure in this sequence?
- *c. Determine the number of hexagons, squares, and rhombuses in the 20th figure of the sequence. Explain how you thought about it.
- *d. Repeat part c for the 57th figure in the sequence.

4. Use your pattern blocks to build and sketch the sixth figure of the sequence here.



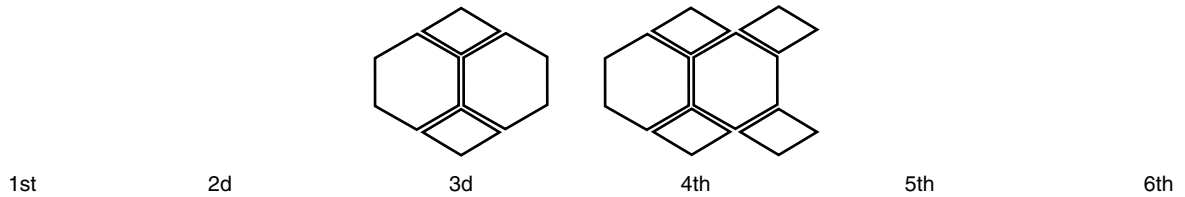
- a. Determine the number of triangles and hexagons in the 10th figure of the extended sequence. Do the same for the 15th figure.
- b. Any figure number that is a multiple of 3 has $\frac{1}{3}$ that number of hexagons and $\frac{2}{3}$ that number of triangles. Explain how you can determine the number of hexagons and triangles if the figure number is 1 more than a multiple of 3. One less than a multiple of 3.

5. The third and fourth figures of a pattern block sequence are given below. Use your pattern blocks to construct and sketch the first, second, and fifth figures in this sequence.



- a. Describe how the odd-numbered figures differ from the even-numbered figures.

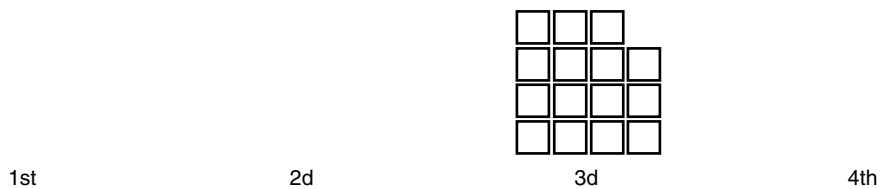
- b. Sketch the missing figures for the next sequence. Explain how you can determine the number of hexagons in any even-numbered figure of the sequence, then explain it for any odd-numbered figure.



Explanation:

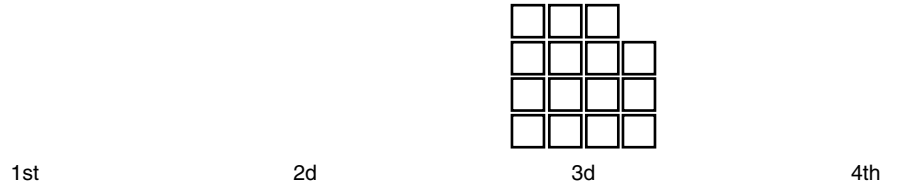
- *6. The third term of a color tile sequence is shown below. Use your color tiles to create more than one sequence for which the given figure is the third figure. Sketch diagrams of the first, second, and fourth figures. Write a rule for extending each pattern you create so that the reader is able to build the next few figures in the sequence.

Sequence I



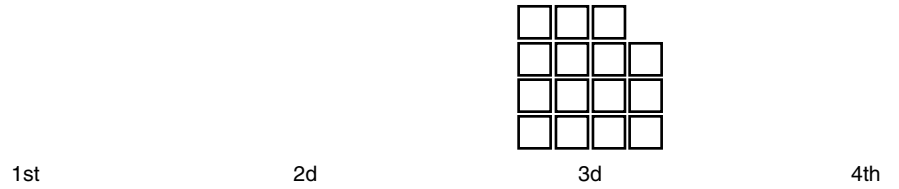
Rule:

Sequence II



Rule:

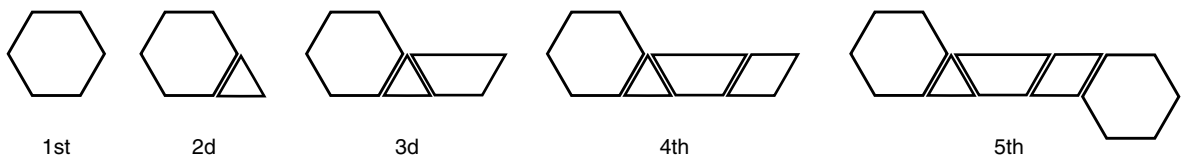
Sequence III



Rule:

7. Pattern block sequences I and II begin repeating in the fifth figure and pattern block sequence III begins repeating in the sixth figure. Build and sketch the next figure in each sequence with your pattern blocks. For the 38th figure in each sequence determine which pattern block is at its right end and how many of each type of pattern block the 38th figure contains. Describe how you reached your conclusion in each case.

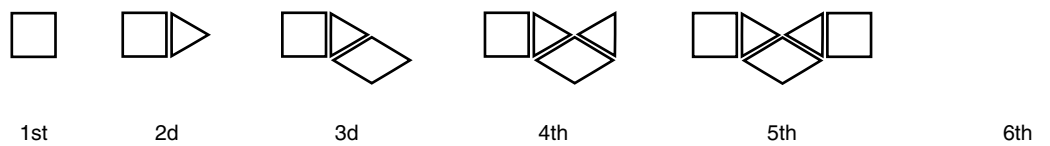
Sequence I



Explanation

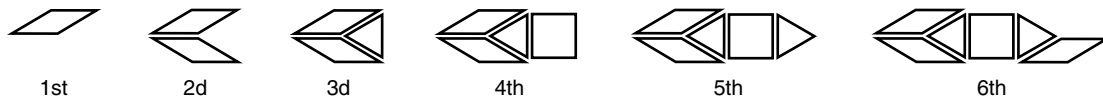
6th

***Sequence II**



Explanation:

Sequence III



Explanation:

7th

8. Devise your own sequence of figures with pattern blocks. Pose at least three questions about your sequence. Ask another person to build your sequence and answer your questions. Sketch at least the first four figures from your sequence and record your questions.



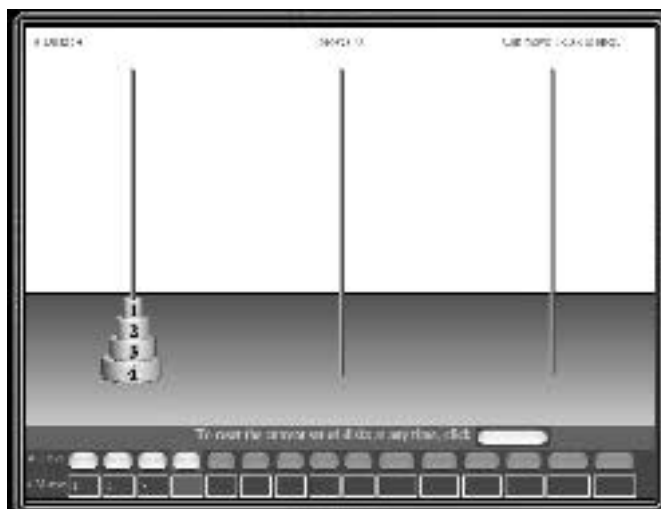
JUST FOR FUN

TOWER PUZZLE

This ancient puzzle is sometimes referred to as the “Tower of Brahma.” The story says that at creation, the priests were given three golden spindles. One golden spindle has 64 golden disks with the largest disk at the bottom of the spindle and each successive disk getting smaller up to the top smallest disk. Day and night the priests were to transfer disks from one spindle to another, moving the disks one at a time but never placing a larger disk on a smaller one until all the disks are transferred to another spindle—in the original

order. When the priests finished transferring the spindle of 64 disks, the world was to come to an end.

Use a model, or the interactive applet from the Online Learning Center, and the problem-solving strategies of simplifying, making a table, and looking for a pattern as you try to form a conjecture about the minimum number of moves to transfer all 64 disks from one spindle to another spindle. By experimenting with special cases, such as 2 disks, 3 disks, etc., data can be gathered that lead to conjectures for predicting the number of moves for transferring 64 disks.



Tower Puzzle Applet, Chapter 1

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Connections 1.1

SEEING AND EXTENDING PATTERNS WITH PATTERN BLOCKS

1. *School Classroom:* A second-grade teacher started the following pattern block sequence and asked her students to continue the pattern by adding five more pattern blocks.



- a. One student looked at the sequence and said he did not know what to do. Describe what you would say or do as the teacher.
- b. Another student continued the pattern block sequence as follows. Describe what you believe this student was thinking. What questions can you ask to encourage the student to reveal how she perceived and extended the pattern?



2. *School Classroom:* Design a pattern block sequence that you believe is appropriate for an elementary school student and write a few questions that you can ask about your sequence. Try this activity on an elementary school age child of your choice. Record your sequence, your questions, and the student's responses.
3. *Math Concepts:* Make up two secret pattern block or color tile sequences that have the same first four figures but different figures after that. Show the first four figures to a partner and challenge them to find both of your secret sequences. Illustrate your sequences and explain the results of your challenge.
4. *NCTM Standards:* The National Council of Teachers of Mathematics (NCTM) is a professional organization whose goal is to improve the teaching and learning of mathematics. Go to the NCTM website, www.nctm.org, to locate information on their publication, *The Principles and Standards for School Mathematics*. Write a short description of the purpose of this publication and give the specific URL where you found this information.
5. *NCTM Standards:* Go to the **Pre-K–2 Standards** in the back pages of this book and find the *Expectation* where studying patterns is recommended. State the *Expectation* and the *Content Standard* the *Expectation* is under and explain why you think the study of patterns is under this particular *Standard*.



ONLINE LEARNING CENTER

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Virtual Manipulatives
Interactive Chapter Applet
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Color Transparencies
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