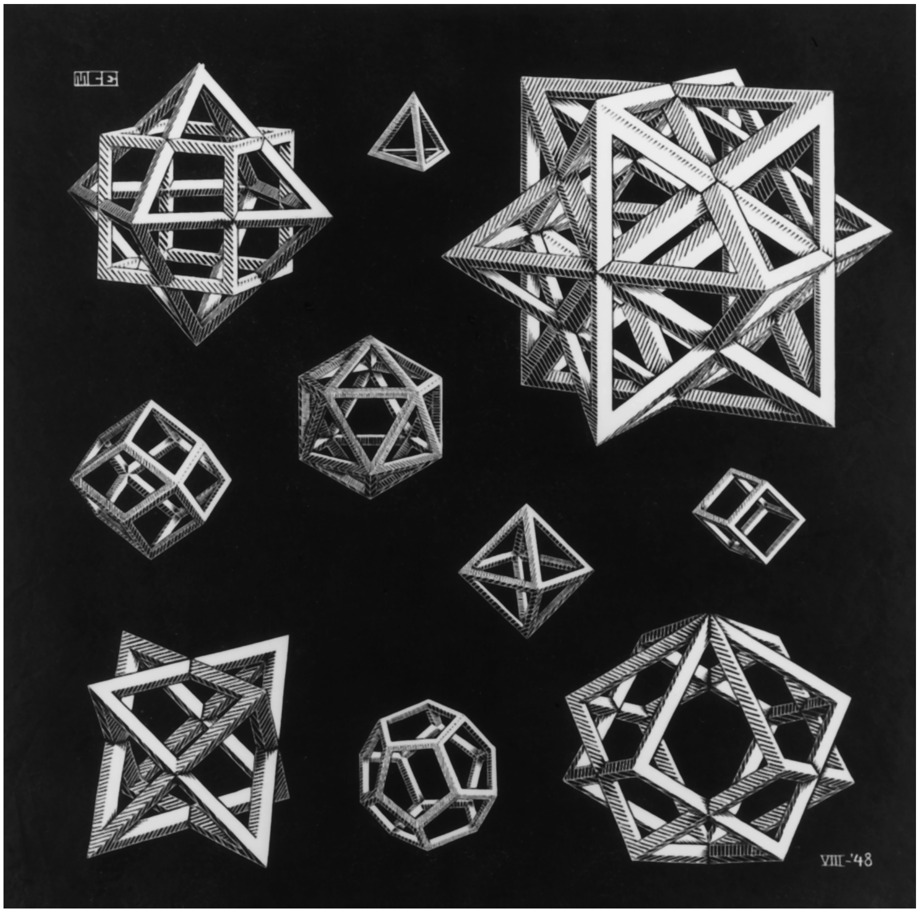
Activity Set 9.3 MODELS FOR REGULAR AND SEMIREGULAR POLYHEDRA

PURPOSE

To construct and use models in order to observe and examine the properties of regular and semiregular polyhedra.

MATERIALS

Scissors and patterns for constructing regular polyhedra on Material Cards 29 and 30. Two-centimeter grid paper from Material Card 31.

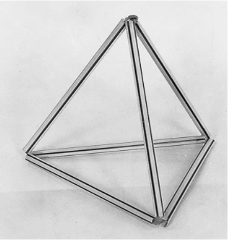
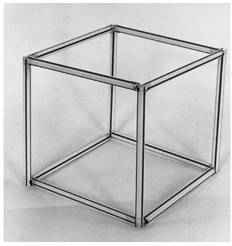
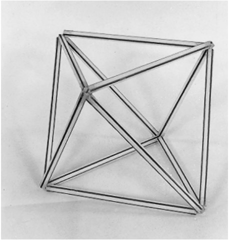
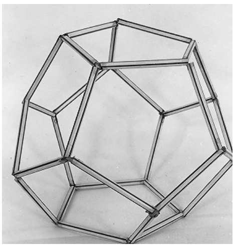
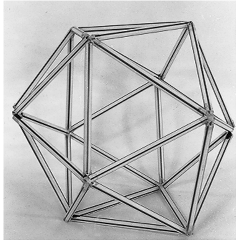


INTRODUCTION

The term polyhedron comes from Greek and means "many faces." A *regular polyhedron* is a convex polyhedron whose faces are regular polygons and whose vertices are each surrounded by the same number of these polygons. The regular polyhedra are also called *Platonic solids* because the Greek philosopher Plato (427–347 B.C.E.) immortalized them in his writings. Euclid (340 B.C.E.) proved that there were exactly five regular polyhedra. Can you identify the Platonic solids in M. C. Escher's wood engraving *Stars?*

Platonic Solids

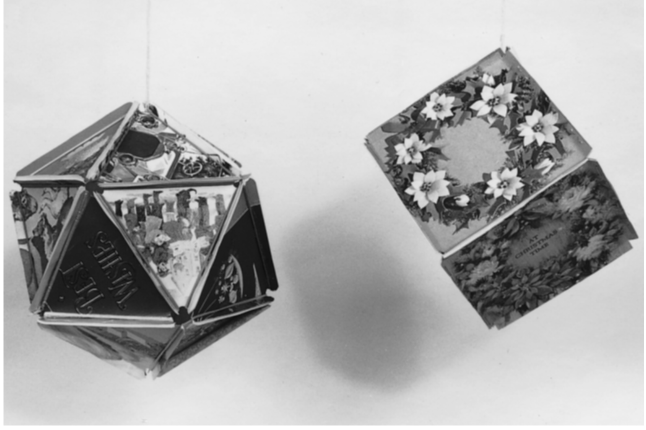
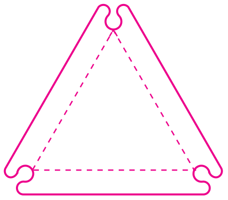
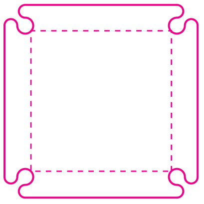
1. There are several common methods for constructing the five Platonic solids. The polyhedra pictured here are see-through models similar to those in Escher's wood engraving. They can be made from drinking straws that are threaded or stuck together.

Another method is to cut out regular polygons and glue or tape them together. Material Cards 29 and 30 contain regular polygonal patterns for making the Platonic solids. Cut out the patterns, use a ballpoint pen or sharp point to score the dashed lines, and assemble them by folding on the dashed lines and taping the edges. For each Platonic solid, record the number of faces (polygonal regions) and the name of the polygon used as the face.

	Number of Faces	Type of Face
Tetrahedron	______	______
Cube	______	______
Octahedron	______	______
Dodecahedron	______	______
Icosahedron	______	______

2. The faces of the icosahedron and cube in the photo were made from colorful greeting cards. (The triangular and square patterns for the faces are also shown. The corners of these patterns were cut with a paper punch and scissors, and the edges bent up along the dashed lines. The adjoining faces of the polyhedra are held together by rubber bands on their edges.)

a. If you were to make your Platonic solids as described on the previous page, how many rubber bands would you need to make each one? (One band is used for each edge.)

Tetrahedron ________

Cube ________

Octahedron ________

Dodecahedron ________

Icosahedron ________

b. You may enjoy planning colors for your Platonic solids so that no two faces with a common edge have the same color. For this condition to be satisfied, what is the least number of colors needed for the octahedron?

3. Euler's Formula

***a.** In the following table, record the numbers of vertices, faces, and edges for each of the Platonic solids.

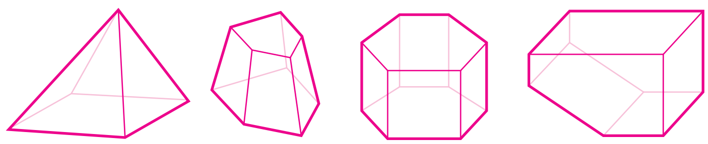
	Vertices (V)	Faces (F)	Edges (E)
Tetrahedron	________	________	________
Cube	________	________	________
Octahedron	________	________	________
Dodecahedron	________	________	________
Icosahedron	________	________	________

b. There is a relationship among the numbers of vertices, faces, and edges. Make and record a conjecture about this relationship and express it in terms of the letters V, F, and E.

Conjecture:

***c.** The following polyhedra are not Platonic solids. Count their vertices, faces, and edges and record your results. Test your conjecture from part b on these polyhedra.

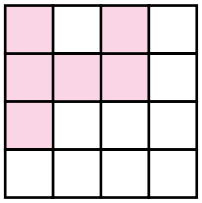
(1) (2) (3) (4)



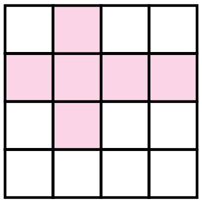
	(1)	(2)	(3)	(4)
Vertices	______	______	______	______
Faces	______	______	______	______
Edges	______	______	______	______

4. Net Patterns for Cubes: The shaded areas in parts a–f below were formed by joining 6 squares along their edges. A two-dimensional pattern that can be folded into a three-dimensional shape without overlap is called a *net* for the three-dimensional shape. Three of the following patterns are nets for a cube and three are not. Identify the patterns in parts a–f that are nets for a cube. Use 2-centimeter grid paper (Material Card 31) to create at least five more nets for a cube (there are 11 such nets in total). You may wish to cut your patterns out to test them. Sketch the nets in the provided grids.

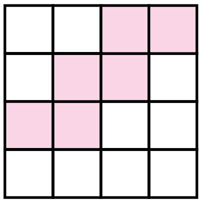
a. Net or not?



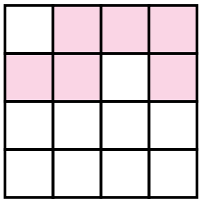
b. Net or not?



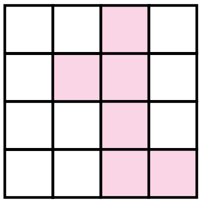
c. Net or not?



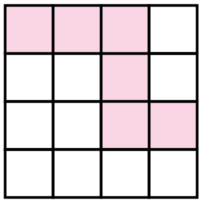
d. Net or not?



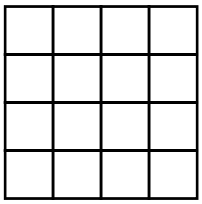
e. Net or not?



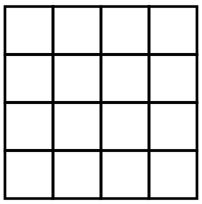
f. Net or not?



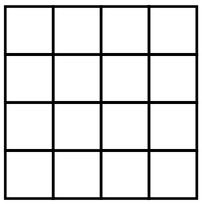
g. Sketch a net



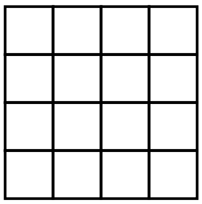
h. Sketch a net



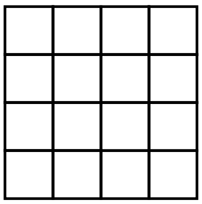
i. Sketch a net



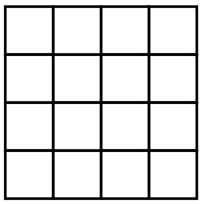
j. Sketch a net



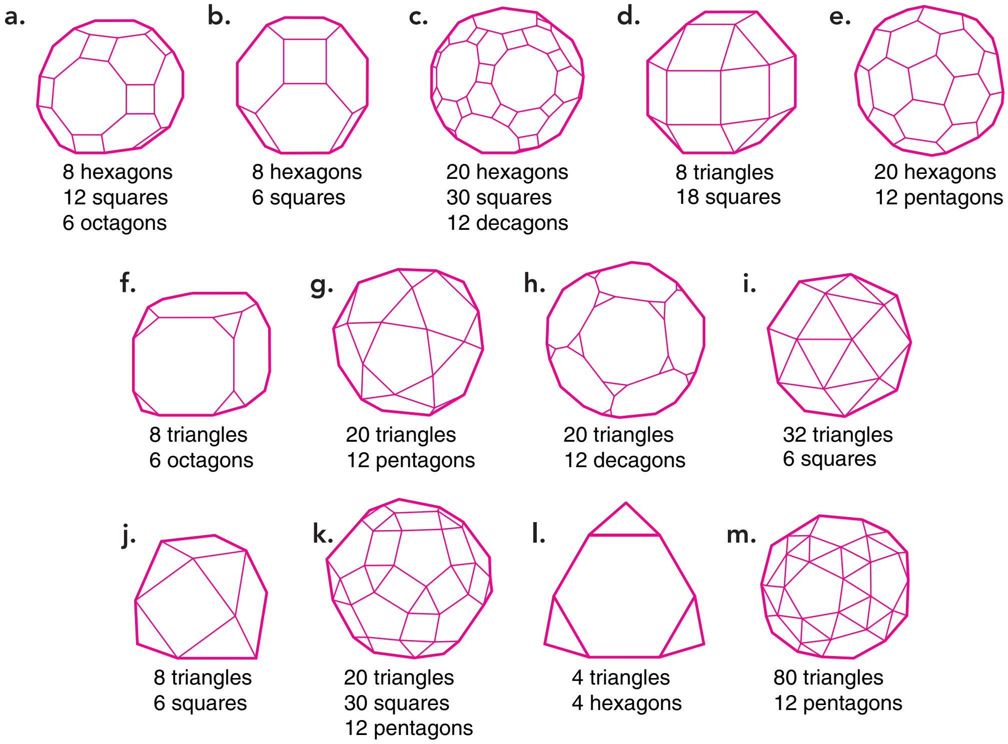
k. Sketch a net



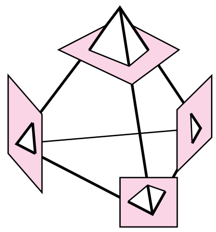
l. Sketch a net



5. **Archimedean Solids:** A *semiregular polyhedron* is a polyhedron that has as faces two or more regular polygons and the same arrangement of polygons about each vertex. There are a total of 13 semiregular polyhedra shown below, and they are called *Archimedean solids.* These were known to Archimedes (287–212 B.C.E.) who wrote a book on these solids, but the book has been lost.



The seven Archimedean solids shown in b, e, f, g, h, j, and l can be obtained by truncating Platonic solids. For example, if each vertex of a tetrahedron is cut off, as shown in the figure at the right, we obtain a truncated tetrahedron whose faces are regular hexagons and triangles, as shown in figure l above.


Tetrahedron

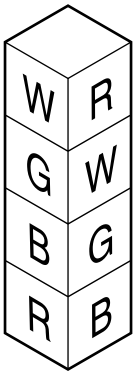
- Identify which Platonic solid is truncated to obtain each of the semiregular solids in b, e, f, g, h, and j.
- Note: Vertices may also be cut off all of the way to the midpoints of the edges.
- It may be helpful to look at the Platonic solids you constructed in activity 1.

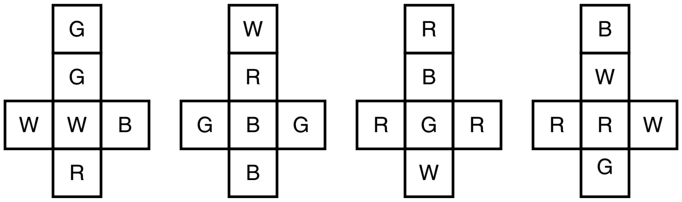
JUST FOR FUN

INSTANT INSANITY

Instant Insanity is a popular puzzle that was produced by Parker Brothers. The puzzle consists of four cubes with their faces colored either red, white, blue, or green. The object of the puzzle is to stack the cubes so that each side of the stack (or column) has one of each of the four colors.

Material Card 32 contains patterns for a set of four cubes. Cut out and assemble the cubes and try to solve the puzzle.

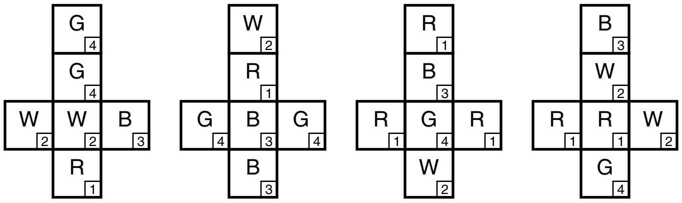




Cube A Cube B Cube C Cube D

Robert E. Levin described the following method for solving this puzzle.[5] He numbered the faces of the cubes, using 1 for red, 2 for white, 3 for blue, and 4 for green, as shown below. To solve the puzzle you must get the numbers 1, 2, 3, and 4 along each side of the stack. The sum of these numbers is 10, and the sum of the numbers on opposite sides of the stack must be 20.

Levin made the following table showing opposite pairs of numbers on each cube and their sums. For example, 4 and 2 are on opposite faces of cube A and their sum is 6. The



Cube A Cube B Cube C Cube D

bottom row of the table contains the sums of opposite faces for each cube. For one combination of numbers whose sum is 20, the sums have been circled. Notice that above these circled numbers, the numbers 1, 2, 3, and 4 each occur only twice. This tells you how to stack the cubes so that two opposite sides of the stack will have a total sum of 20. (Both of these sides will have all four colors.) The two remaining sides of the stack must also have a sum of 20. Find four more numbers from the bottom row (one for each cube) such that their sum is 20 and each of the numbers 1, 2, 3, and 4 occurs exactly twice among the faces. The remaining two faces of each cube will be its top and bottom faces when it is placed in the stack.

	Cube A	Cube B	Cube C	Cube D
Pairs of opposite faces	4 4 3	4 1 2	1 1 3	1 1 4
	2 1 2	4 3 3	1 4 2	3 2 2
Sums of pairs	6 5 5	8 4 5	2 5 5	4 3 6

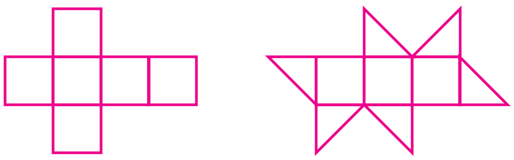
[5]R. Levin, "Solving Instant Insanity," *Journal of Recreational Mathematics* 2 (July 1969): 189–192.

Connections 9.3

MODELS FOR REGULAR AND SEMIREGULAR POLYHEDRA

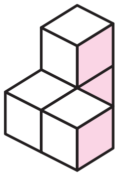
1. *School Classroom:* Suppose that your students are using sticks and gumdrops to create models of the five Platonic solids. How will you help them decide how many sticks and how many gumdrops they need and how to assemble their solids?

2. *School Classroom:* When asked to make a net for a cube, a drawing like net A is the usual response. Design an activity for middle school students that will lead them to draw nets like net B that are formed without using all squares. Describe your activity in such a way that another person could follow your directions. Submit a few designs for a net by sketching them on 2-centimeter grid paper. 2-centimeter grid paper is available for download at the Online Learning Center.



Net A Net B

3. *Math Concepts:* Design a two-dimensional net for the following figure that is composed of four cubes. Describe your procedure for constructing this net and submit a copy of the net with your response.



4. *Math Concepts:* Open the **Math Laboratory Investigation 9.3: Read Me—Pyramid Patterns Instructions** from the Online Learning Center and investigate the pyramid patterns described in question 1 of the *Starting Points for Investigations 9.3.* Show your procedures and explain your thinking.

5. *NCTM Standards:* Read over the **Geometry Standards** in the back pages of this book. Pick one *Expectation,* from each grade level that the activities in this section address. State the *Expectations* and the *Standards* they are under. Explain which activities address these *Expectations* and how they do so.

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